# *ANALOG CIRCUIT DESIGN ORAL NOTES*

## *By Giacomo Tombolan*



***Guide to these notes:***

- If you payed for these notes, **you've been scammed**. I always give my notes for free.
- There are some additional notes at the end of the documents that could make some questions (like the orchard theorem) clearer to you, or at least, they did to me.
- I tried to explain and justify every step of each question, I hope you can follow my reasonings. If there's something that's "taken for granted" (except for filter theory that's purely mathematical and it's not explained in this course), there's probably an additional note in the end of the document
- This is a helpful guide to be read together with the lectures PDFs. I don't recommend to fully rely on the stupid things I wrote. Btw, PDFs contain typos so beware of that too
- Questions #24,#25,#1 are a bit meh, I didn't really know what so say about those. I really recommend not to use those as reference
- Question everything you read because during the oral you will be asked exactly that. Just to make few examples:
  - (Question #22 on 1/f noise) why do we take $\beta = \frac{1}{4}$ ? Can't we generalize the reasoning to all energy levels? Of course, but it's more complex. We analyze the traps at Fermi level just to demonstrate that if we consider multiple $\tau$ we end up with several lorentian shapes. Of course, on a more general view, there will be different families of traps (ions, defects on lattice, etc..) + different energy levels that will lead to something like $\frac{1}{f^{\beta}}$
  - (questions #14 and #15) How do we size the square length and area $\Lambda, A_0$? Is there any reasoning behind that? (See additional notes at the end of the document)
- NEVER EVER TAKE ANY FORMULA OR SYMBOL FOR GRANTED. The exact moment you memorize anything without a clear understanding of what you learned you will fail the oral. It's guaranteed. You must be able to justify anything that gets out your mouth or pen.
- Tip that helped me the most: don't memorize every step but memorize the first and last steps. Then memorize the track you need to follow to get from start to the end, it helps the flow of your speech and it won't take too much brain space
- It took me about 7 days to write these notes from scratch, so if my writing isn't clear, well, I'm sorry C: hope it helps anyway. Also, I speak maccheroni and I'm well aware of the English mistakes I made. However I didn't have enough time to be my grammar nazi
- There are two questions #19, the given pdf with the oral topics had two #19 and I didn't see that until I was done writing everything. Just ignore this mistake

If you're having any issue with this document just send an email to giacomo.tombolan@mail.polimi.it

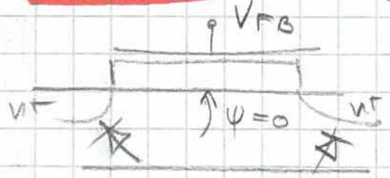
# Topics for the orals

1. MOSFET's figures of merit: maximum voltage gain and cut-off frequency. Dependences on bias (strong, moderate and weak inversion).
2. Independent/interacting capacitors and poles. Extension of the time constant method. Middlebrook's theorem. Examples with RC networks.
3. The prototypical differential stage: from resistive to active loads. Common mode feedback and single ended option.
4. Single ended differential stage with mirror: Bias, input and output voltage swings, differential gain, Common mode gain.
5. Two-stage CMOS OTA: topology, frequency response, Miller compensation.
6. Two-stage CMOS OTA: frequency compensation with the nulling resistor. Implementing the nulling resistor
7. Two-stage CMOS OTA: frequency compensation with ideal voltage and current buffers. Impact of the buffer finite resistance.
8. Nested Miller Compensation
9. OTA Linear response. In-band zero-pole doublets and features of the settling response.
10. The slew rate limit. Impact on settling time. CMOS-OTA: Internal and external slew rate limits. Improving SR with class AB output stages.
11. CMOS single stage amplifiers: telescopic and folded cascode structures. Motivations, performance and linear swing
12. Output stages: Class A vs. class B output stage. Efficiency and distortion.
13. Output stages: Total harmonic distortion and feedback
14. Variability and matching: Relative matching of resistors. Pelgrom's formula
15. Variability and matching: Relative matching of threshold voltage values. Pelgrom's formula
16. OTA: Offset. Deterministic and statistical contributions to input referred offset.
17. OTA: Common-mode rejection ratio. Deterministic and statistical limits to CMRR
18. Quantitative description of noise: the power spectral density concept, thermal noise in resistors and MOSFETs
19. Input referred noise sources of a two-port network. Definitions and derivation. Extension to the differential stage
19\. Noise models: The Nyquist argument for the thermal noise power spectral density
20. Noise models: Shot noise model. Application to p-n junctions and MOSFETs in weak inversion
21. Trapping noise: trapping noise in a resistor
22. McWorther model of the 1/f noise in MOSFETs. Tvidis formula.
23. Introduction to analog filters: Ideal performance. Limits of the causal response. Group delay and signal distortion.
24. LP filters: Filter mask and numerical parameters (selectivity, discrimination). Families of LP filter functions (Butterworth, Chebyshev, Bessel, …) and their properties.
25. Mapping: Motivations, HP to LP transformation, BP to LP transformation
26. Active cells: The Sallen-key. Sizing options and sensitivity.
27. The universal cell. Deriving the block-diagram. Properties. The Tow-Thomas cell.
28. Ladder networks: Orchard theorem, implementation with active integrators. Flow-graph derivation procedure. Denormalization.
29. Ladder networks: implementations with gyrators. Topologies of gyrators (OP-AMP, OTA based). Properties and limitations
30. Dynamic range in filters: number of equivalent bits, guidelines to reduce the noise floor.
31. Impact of OP-AMP non-idealities on reference radial frequency and Q factor of a biquad cell.
32. Switched capacitor filters: Motivations, concepts, implementation of the ideal integrator. Stray insensitive topologies
33. SC filters: sampling, transfer function, output spectrum. Anti-aliasing filter and clock frequency.
34. SC filters: trade-off between settling time and charge sharing in sizing the switches.

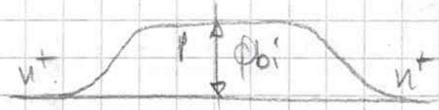
## 1) MOS FoM + $f_{\omega T}$. Dependences on strong, moderate + weak inv.

Let's see first some characteristics of the mosfet:

- $V_G = V_{FB}$ → off state → $\psi_s = 0$




Substrate forms diodes with both S, D. We therefore just have

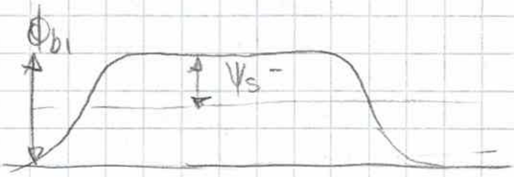



a $\Phi_{bi}$ (built-in potential) related to the diodes.

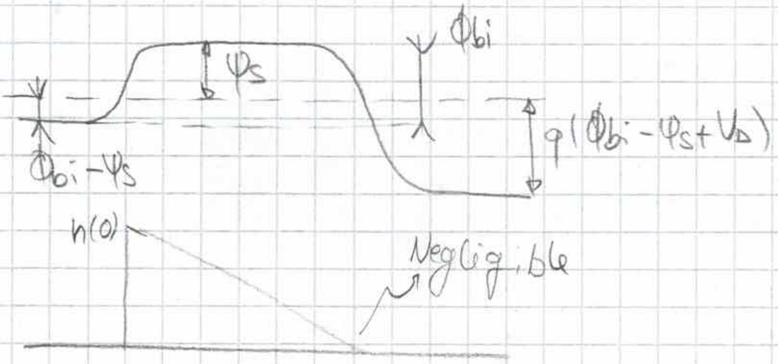
For $V_G < V_T$ we are sub-threshold

- $V_G < V_T$ and $V_S = V_D = 0$

Energy barrier is now lower




- $V_{FB} < V_G < V_T$ and $V_{DS} > 0$ ($V_S = 0$)



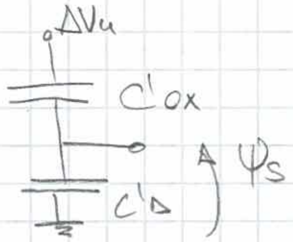

Since current is low → no ohmic drop → potential energy along the channel remains fairly constant. However, on the source we have a $\Phi_{bi}$ that (Boltzmann) leads to a carrier concentration of $n(0) = N_S e^{-q\frac{\Phi_{bi}}{kT}}$ while on the drain side, $V_D > 0$ will lead to a negligible concentration.

This concentration leads to a bipolar-like diffusion current that is

$$I_{DIFF} = I_S e^{\frac{q(V_{GS} - V_T)}{nkT}} \qquad g_D = \frac{\partial I}{\partial V_{GS}} = I_S \cdot \frac{q}{nkT} e^{\frac{q(V_{GS} - V_T)}{nkT}} = q\frac{I_{DS}}{nkT}$$

$$= \frac{I_{DS}}{n V_{TH}}$$

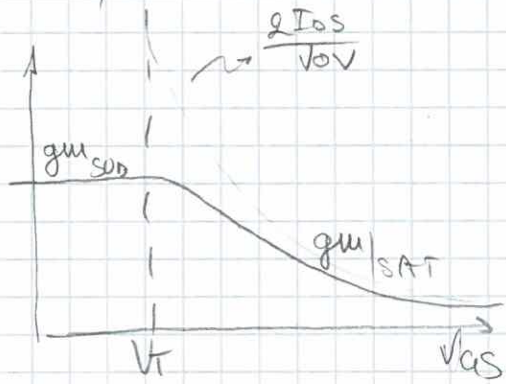
n :




$$\Delta\psi_S = \Delta V_G \cdot \frac{C'_{ox}}{C'_{ox}+C'_D} \rightarrow \Delta\psi_S = \frac{\Delta V_G}{n}$$

with $n = 1 + \frac{C'_D}{C'_{ox}} \sim 1,5$

## Maximum gain

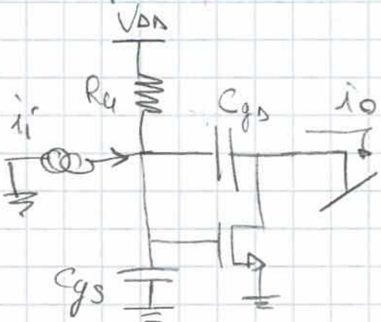



$$g_m|_{SUB} = \frac{I_{DS}}{nV_{TH}} \qquad g_m|_{SAT} = \frac{2I_{DS}}{V_{OV}}$$

$$\mu|_{SUB} = \frac{\cancel{I_{DS}}}{nV_{TH}} \cdot \frac{V_A}{\cancel{I_{DS}}} = \frac{V_A}{nV_{TH}} \rightsquigarrow \text{MAX } \mu \text{ ACHIEVABLE}$$

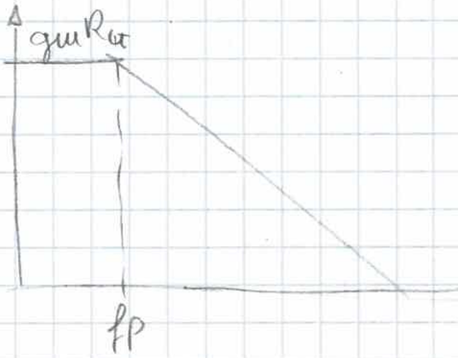
This will be the max achievable gain

## Cutoff frequency




$$G_{DC} = -g_m R_G = \frac{i_{OUT}}{i_{in}}$$

$$f_p = \frac{1}{2\pi C_{ox}\cdot R_G}$$



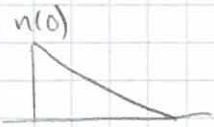

$$g_m \cancel{R_G} \cdot \frac{1}{2\pi \cancel{R_G} C_{ox}} = \frac{g_m}{C_{ox}\cdot 2\pi}$$

We define the gain-BW product as $f_T = \frac{g_m}{C_{ox}} \cdot \frac{1}{2\pi}$

$$f_T = \frac{2 I V_{ov}}{2\pi W L C'_{ox}} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}\mu \cancel{C'_{ox}} \frac{\cancel{W}}{L} V_{ov}}{2\pi \cancel{W} L \cancel{C'_{ox}}} = \frac{\mu V_{ov}}{2\pi L^2} = \frac{\bar{v}}{2\pi L} = \frac{1}{2\pi t_{TR}}$$

$\frac{V_{ov}}{L}$ = electric field $\quad \bar{v} = \mu E = \mu \frac{V_{ov}}{L}$

This is not true for sub-thresh:




$$J_{DIFF} = q D_n \frac{n(0)}{L} \rightsquigarrow \frac{\Delta n}{\Delta x}$$ diffusion density along the channel

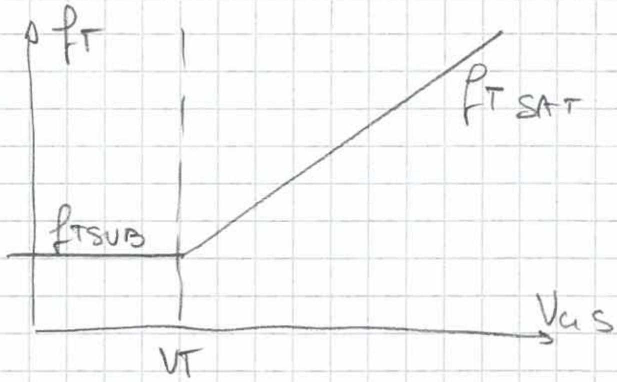
Stored charge will be $Q' = q \cdot \frac{n(0)\cdot L}{2}$ ⇝ area of the triangle

So $t_{DIFF} = \frac{Q'}{J_{DIFF}} = \frac{1}{2} \frac{q\, \cancel{n(0)}\cdot L}{\cancel{q} D_n \frac{\cancel{n(0)}}{L}} = \frac{L^2}{2 D_n}$

So

$$f_T\big|_{SUB} = \frac{1}{2\pi\, t_{DIFF}} = \frac{D_n}{\pi L^2}$$

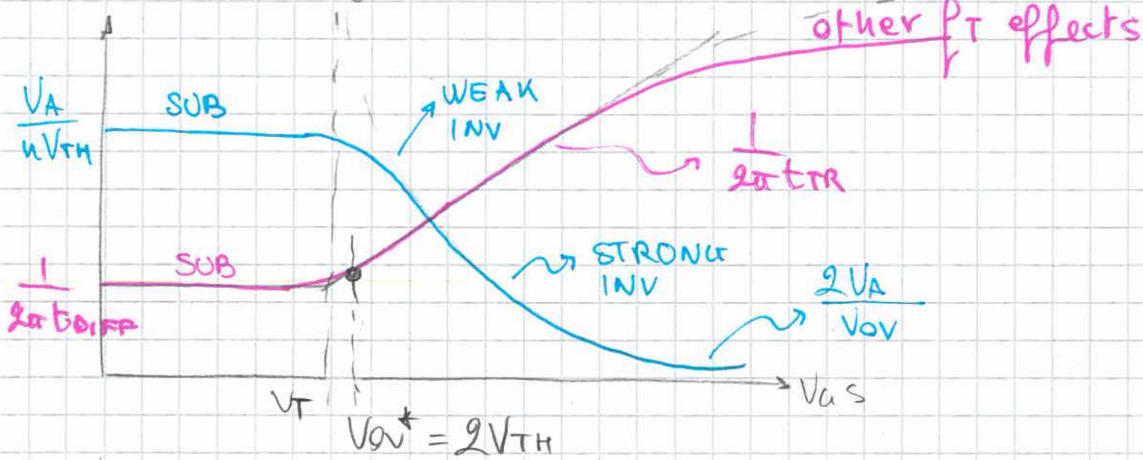
$$f_T\big|_{SAT} = \frac{g_m}{2\pi\, C'_{ox}} = \frac{\mu V_{OV}}{2\pi L^2}$$




$$f_{SUB} = f_{SAT} \rightarrow \frac{L^2}{\mu V_{OV}} = \frac{L^2}{2 D_n} \Rightarrow \frac{2 D_n}{\mu} = V_{OV}^*$$

Since $D_n = \frac{KT}{q} \cdot \mu$ $\quad V_{OV}^* = 2\frac{KT}{q} \simeq 51\ mV$ @ 300K

Real, final graph



## 2) Time constant method + examples + extension

For a LTI network the transfer function is:

$$T(s) = T(0)\frac{N(s)}{D(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + 1}{b_n s^n + b_{n-1} s^{n-1} + \dots + 1}$$

$N(s)$ are zeros $D(s)$ are poles. For a 3 capacitors network

$$b_1 = C_1 R_1^{(0)} + C_2 R_2^{(1)} + C_3 R_3^{(0)}$$

$$b_2 = C_1 C_2 R_1^{(0)} R_2^{(1)} + C_1 C_3 R_1^{(0)} R_3^{(1)} + C_2 C_3 R_2^{(1)} R_3^{(2)}$$

$$b_3 = C_1 C_2 C_3 R_1^{(0)} R_2^{(1)} R_3^{(1,2)}$$

$$a_1 = C_1 R_{01}^{(0)} + C_2 R_{02}^{(0)} + C_3 R_{03}^{(0)}$$

$$a_2 = C_1 C_2 R_{01}^{(0)} R_{02}^{(1)} + C_1 C_3 R_{01}^{(0)} R_{03}^{(1)} + C_2 C_3 R_{02}^{(0)} R_{03}^{(2)}$$

$$a_3 = C_1 C_2 C_3 R_{01}^{(0)} R_{02}^{(1)} R_{03}^{(1,2)}$$

$$D(s) = b_3 s^3 + b_2 s^2 + b_1 s + 1 \qquad N(s) = a_3 s^3 + a_2 s^2 + a_1 s + 1$$

Since the coefficients are related to the circuit topology, order of capacitors doesn't matter

We can denote roots of the denominator so that

$$D_3(s) = \left(1 - \frac{s}{p_3}\right)\left(1 - \frac{s}{p_2}\right)\left(1 - \frac{s}{p_1}\right) = b_3 s^3 + b_2 s^2 + b_1 s + 1$$

By comparing the expressions we find that:

$$b_1 = -\left(\frac{1}{p_3} + \frac{1}{p_2} + \frac{1}{p_1}\right)$$

If there's a dominant pole out of the three, $\frac{1}{p}$ will be much higher; for example $\frac{1}{p_1} \gg \frac{1}{p_2}, \frac{1}{p_3}$, so

$$b_1 = C_1 R_1^{(0)} + C_2 R_2^{(0)} + C_3 R_3^{(0)} \sim -\frac{1}{p_1}$$

On the other hand, for high frequency

$$b_3 s^3 + b_2 s^2 + b_1 s + 1 \approx b_3 s^3 + b_2 s^2 = s(s b_3 + b_2)$$

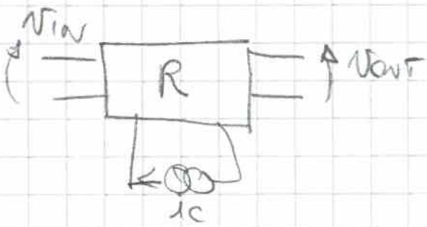
Therefore $p_H = -\frac{b_2}{b_3} = \frac{C_1C_2R_1^{(1)}R_2^{(1)} + C_1C_3R_1^{(1)}R_3^{(1)} + C_2C_3R_2^{(1)}R_3^{(1)}}{C_1C_2C_3R_1^{(1)}R_2^{(1)}R_3^{(1)}}$

We can simplify the terms and find that

$$p_H = -\frac{b_2}{b_3} = \frac{-1}{C_3R_3^{(1,2)}} - \frac{1}{C_2R_2^{(1,3)}} - \frac{1}{C_1R_1^{(2,3)}} =$$

$$= -\left(\frac{1}{\tau_3^{\infty}} + \frac{1}{\tau_2^{\infty}} + \frac{1}{\tau_1^{\infty}}\right) = -\frac{1}{\tau^{\infty}}$$

## Zero of a network with a single pole




LTI network → $v_{OUT} \propto$ linear to $v_{IN}$, $i_c$
→ $v_c \propto$ linear to $v_{IN}$, $i_c$

$$\begin{cases} v_{OUT} = A_0 v_{IN} + R_m i_c \\ v_c = B_0 v_{IN} + R_1 i_c \end{cases}$$

Since  $v_c = -\frac{i_c}{sC} \longrightarrow i_c = -sCv_c$

$v_c = B_0 v_{in} - sCR_1 v_c \longrightarrow v_c = \frac{B_0 v_{IN}}{(1+sR_1C)}$ then $v_{out}$ will be

$$v_{OUT} = v_{IN}\left[A_0 - s\frac{R_mCB_0}{1+sR_1C}\right]$$

$$= v_{IN}A_0 \frac{1+sC\left[R_1 - \frac{R_mB_0}{A_0}\right]}{1+sCR_1}$$ pole is $\tau_p = R_1C$

For the zero to occur, it must be $A_0 v_{IN} + R_m i_c = 0$

Therefore at the same time

$$v_c\Big|_{ZERO} = -\frac{B_0R_m}{A_0} i_c\Big|_{ZERO} + R_1 i_c\Big|_{ZERO}$$

Resistance seen on C during zero condition will be

$$\frac{v_c}{i_c}\Big|_{ZERO} = \left(R_1 - \frac{R_mB_0}{A_0}\right)$$

Results :

- $f_{pole} = \dfrac{1}{2\pi R_1 C}$

  → There's a pole

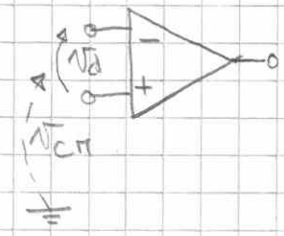
- $f_z = \dfrac{1}{2\pi R_z C}$

  → there's a zero

- DC gain is $\dfrac{V_{OUT}}{V_{IN}}$ with C open

## 3) Diff stage: from resistive to active load. CM feedback + single ended option

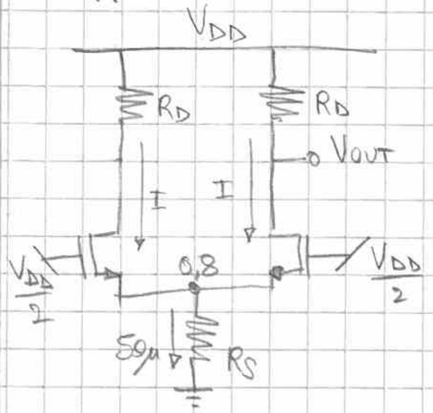
$$v_d = v^+ - v^-$$




$$V_{OUT} = G_D \cdot v_d + G_{CM} v_{CM} \qquad v_{CM} = (v^+ + v^-)/2$$

Goal: - high differential gain ($\infty$ ideally)
- low common mode gain (0 ideally)

## Differential stage → resistors




Put example with: $R_L = 10K\Omega$ $R_S = 16K\Omega$

$V_B = \frac{V_{DD}}{2}$ $2I = 50\mu A$

$$V_{OUT}\Big|_{v_d} = g_m \frac{v_d}{2} \cdot R_D = \frac{2 I_D}{V_{OV}} \cdot \frac{R_D}{\cancel{2}} v_d$$

$25\mu$

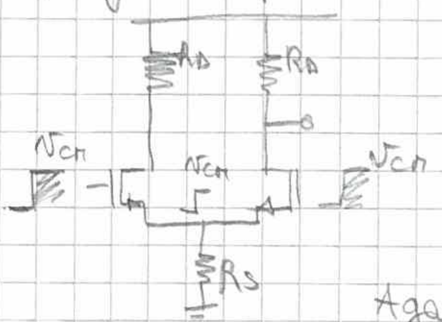
$$G_D = \frac{I_D R_D}{\boxed{V_{OV}}}$$ where $I_D R_D$ = bias voltage drop on $R_D$

Issue: gain is limited by voltage drop on resistor:

To increase $G_D$: → $R_D \nearrow\nearrow$ → $M_1, M_2$ go out of saturation

→ $V_{OV} \nearrow\nearrow$ → $g_m$ can saturate (EKV model)

Solution: load that shows high impedance with low bias voltage drop.




$$V_{out}\Big|_{v_{CM}} = -\frac{v_{CM}}{R_S} \cdot \frac{1}{2} \cdot R_D \rightarrow G_{CM} = -\frac{R_D}{2R_S}$$

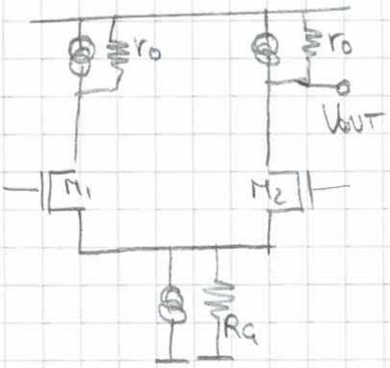
$$G_{CM} = \underbrace{\frac{I_D}{I_D} \frac{R_D}{2R_S}}_{} = \frac{\Delta V_{R_D}}{\boxed{\Delta V_{R_S}}}$$

Again: gain is limited by voltage drops on resistors (ideally $G_{CM} \searrow\searrow$)

# Differential stage → active loads

High impedance + low voltage drop → current generators!

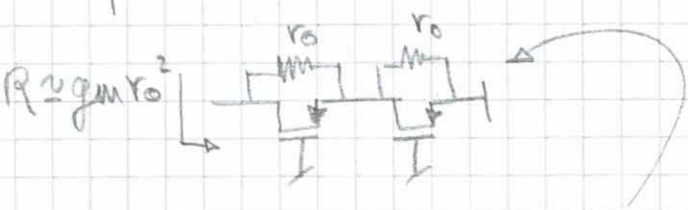





$$G_D = g_m \cdot \frac{1}{2} \cdot r_o = \frac{\mu}{2} \rightarrow \text{max transistor gain}$$

$$G_{CM} = \frac{r_o}{2R_G}$$

Current generators are implemented using mosfets.

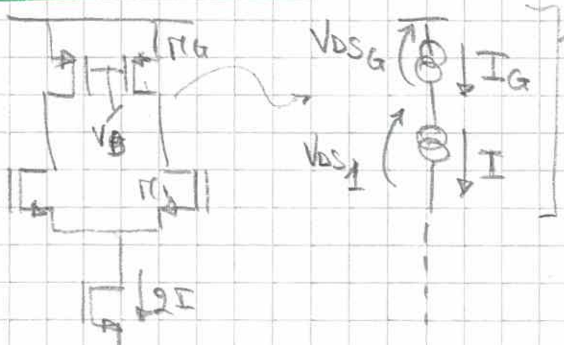




Since we have low voltage drop, we can use cascode structures in order to increase load impedance

We can do the same for tail generators so that Gcm can be decreased at the expense of burning more power supply → High CMRR means more power consumption

CMRR = Common Mode Rejection Ratio

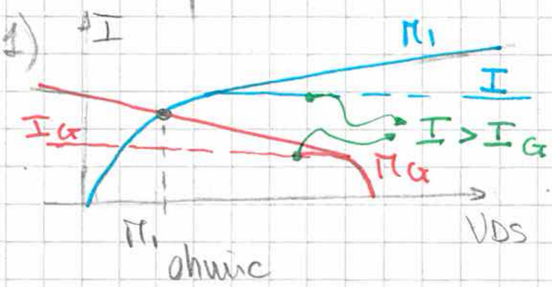
# Active bias issue



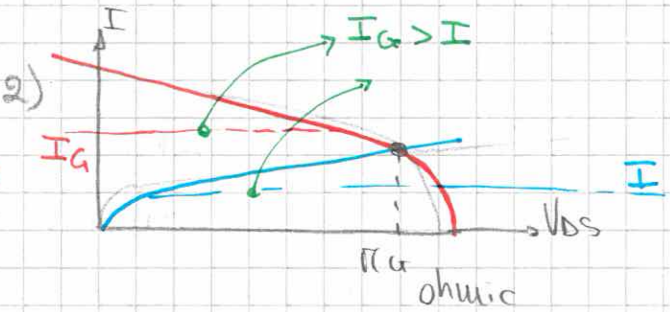

Currents need to be exactly matched, otherwise:

1. $I_G < I \rightarrow M_1$ ohmic
2. $I_G > I \rightarrow M_G$ ohmic

1)

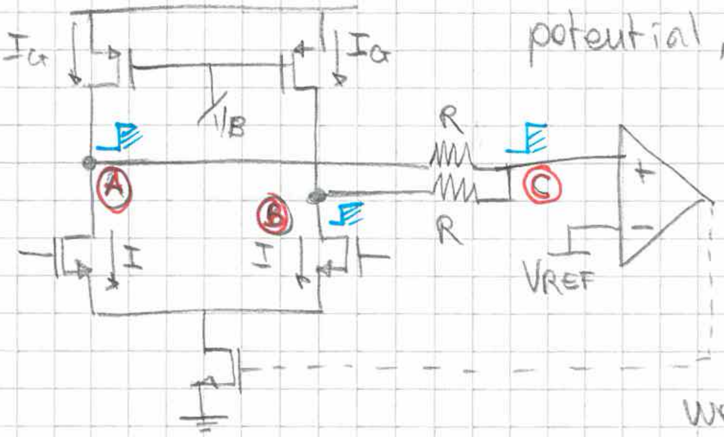



2)



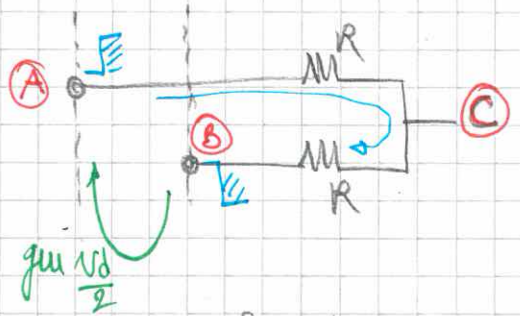

# Common mode feedback

This helps rebalancing the currents so that they can be matched



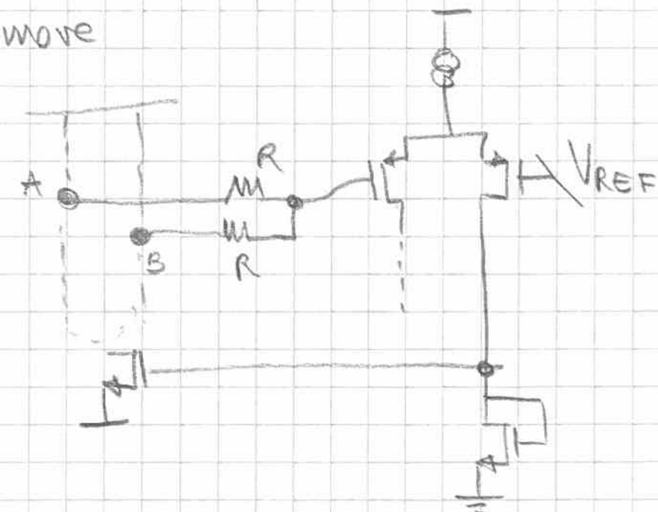

If $I_G > I \rightarrow$ A, B will rise in potential, thus C will rise too

It's easy to see that two resistors are used to generate the average between A, B because we need not to change the common mode bias during differential stimula:

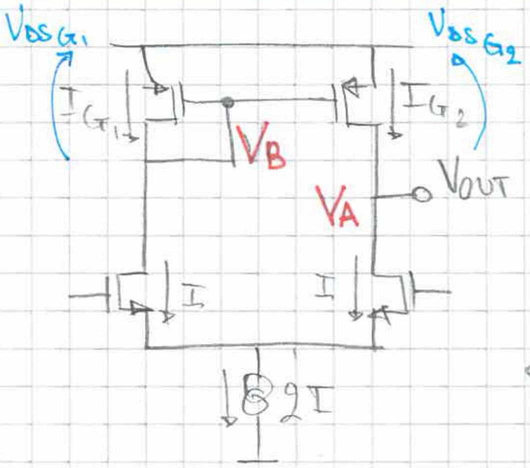



C doesn't move

Example of implementation



## Another compensation: Single ended opamp



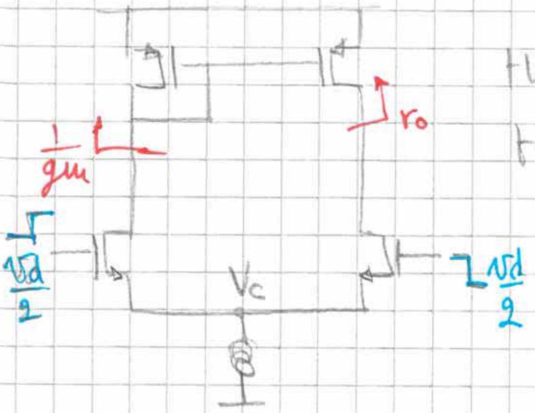

By using a current mirror we can eliminate the need for a CM feedback configuration. Cost:

- No fully differential stage
- Symmetry ($\rightarrow$ CMRR deterministic contribution)

If $V_B = V_A \rightarrow I_{G_1} = I_{G_2} = I$ to first order $(V_{DS_{G_1}} = V_{DS_{G_2}})$

When $2I$ is changed, $V_B$ changes as well, the transdiode automatically adjusts its current in order to match $I_G = I$

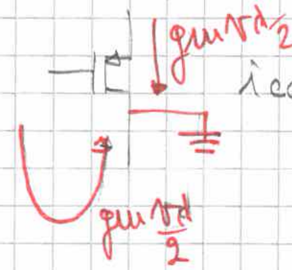
# 1) Single ended differential stage with mirrors: bias, dynamics, $G_D$, $G_{CM}$




We see $\frac{1}{g_m}$, $r_o$ → Stage is unbalanced

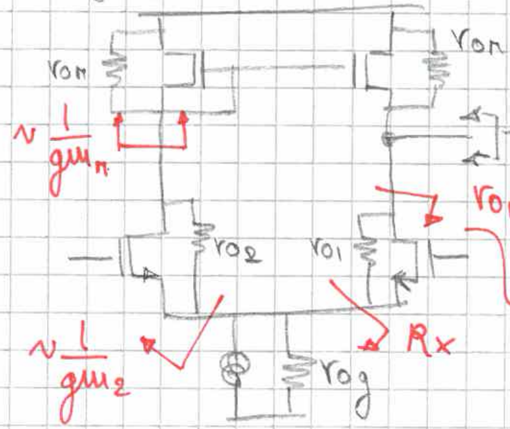
this means $V_c$ is changing with respect to $v_d$ → $v_d$ isn't split symmetrically

Turnaround: Norton theorem




$i_{cc} = g_m v_d$

By shorting $V_{out}$, stage will now see $\frac{1}{g_m}$ and $0\,\Omega$ → we "recovered" symmetry, therefore we can easily compute $G_d$ using Norton

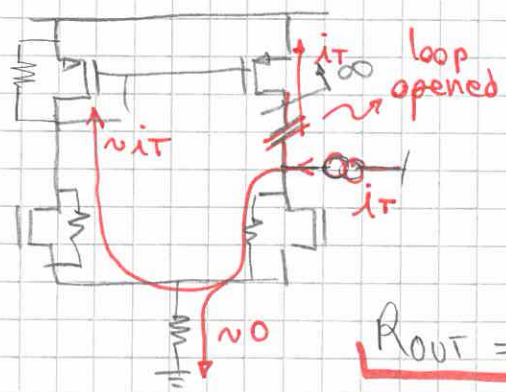



$$\underline{R_{OUT}} = R_{up} /\!/ R_{DOWN} = \underbrace{r_{op} /\!/ \frac{R_{DOWN}^{(0)}}{1 - G_{loop}}}$$

$$r_{o1}(1 + g_m R_x) \simeq r_{o1}\left(g_m\left[r_{og} /\!/ \frac{1}{g_{m_2}}\right] + 1\right) \simeq r_{o1}\left(1 + g_{m_1} \cdot \frac{1}{g_{m_2}}\right)$$

$$\to \sim 2 r_{o1} = R_{DOWN}^{(0)}$$

We have a feedback loop:




$$\underline{G_{loop} = -1} \to R_{DOWN} = \frac{2 r_{o1}}{1 - G_{loop}} =$$

$$R_{DOWN} = \frac{r_{o1} \cdot \cancel{2}}{\cancel{2}}$$

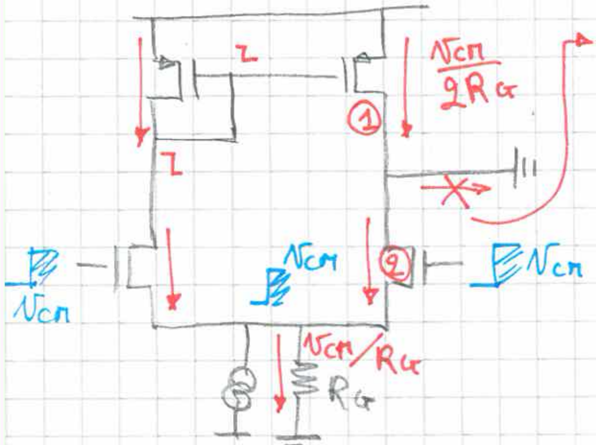
$$\boxed{R_{OUT} = r_{op} /\!/ r_{o1}}$$

$r_{op} = r_{o1}$

$$\boxed{G_D\Big|_{Norton}} = i_{cc} \cdot R_{OUT} = g_m \cdot (r_{op} /\!/ r_{o1}) = \underbrace{\frac{g_m r_o}{2} = \frac{\mu}{2}}_{\to 7\,V}$$

We can simplify $G_D = \frac{\cancel{2} I}{V_{ov}} \cdot \frac{1}{\cancel{\lambda}} \cdot \frac{V_A'}{\cancel{I}} = \frac{V_A'^{(0)}}{V_{ov_1}} \cdot \frac{L}{L_{min}} \to 0{,}35\mu$

# Common mode gain



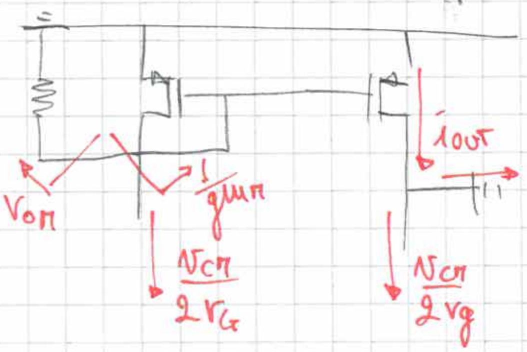

Ideally, current generated in ① would be sucked in ② so $i_{cc} = 0$

We can say that ① is slightly mismatched with respect to ② by an error $\varepsilon$, so:

$$i_{cc} = \varepsilon \cdot \frac{v_{CM}}{2R_G} \longrightarrow G_{CM} = i_{cc} R_{OUT} = \boxed{\varepsilon \frac{R_{OUT}}{2R_G}}$$

$$CMRR = \frac{G_D}{G_{CM}} = \frac{g_m \cancel{R_{OUT}}}{\frac{\varepsilon \cancel{R_{OUT}}}{2R_G}} = \boxed{\frac{2 g_m R_G}{\varepsilon}}$$

Let's briefly quantify one contribution of $\varepsilon$:



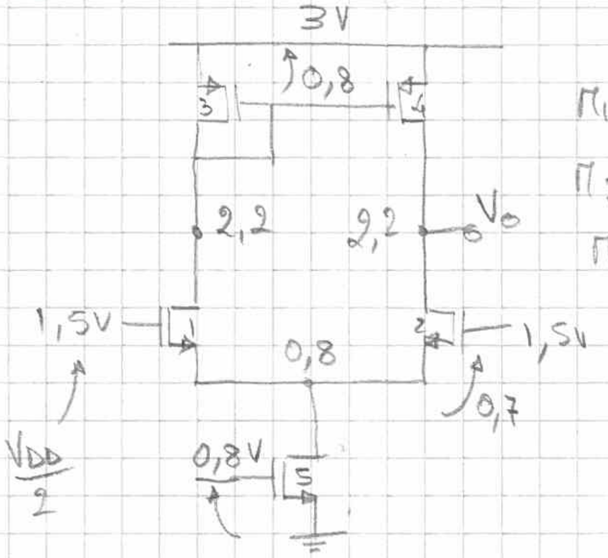

$$i_{OUT} = \frac{v_{CM}}{2r_G} \cdot \frac{r_{oM}}{r_{oM} + 1/g_{mM}} = \frac{v_{CM}}{2r_G} \cdot \frac{1}{1 + \frac{1}{g_{mM} r_{oM}}} \simeq$$

$$= \frac{v_{CM}}{2r_G}\left(1 - \frac{1}{g_{mM} r_{oM}}\right) = \frac{v_{CM}}{2r_G}(1 - \varepsilon) \qquad \frac{1}{1+x} \sim 1 - x \text{ for } x \to 0$$

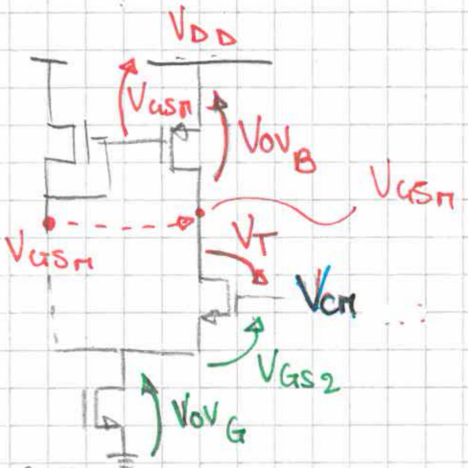
So $\varepsilon \simeq \frac{1}{\mu} \simeq 1\%$ since $\mu = 100$

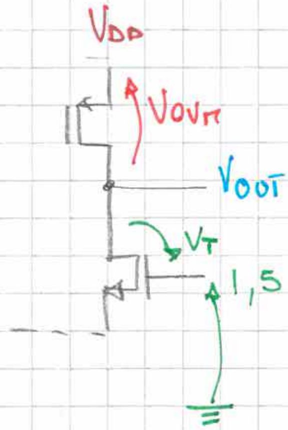
Therefore $CMRR = \boxed{\frac{2 g_m r_G}{\varepsilon}} = \frac{2 \cdot 100}{1/100} \simeq 86\text{dB}$

# Bias and dynamics (IN/OUT)




| | I | $V_{ov}$ | W/L | $g_m$ | $r_o$ |
|---|---|---|---|---|---|
| $M_{1,2}$ | 25µ | 0,1 | 50 | 0,5 m | 560 k |
| $M_{3,4}$ | 25µ | 0,2 | 25 | 0,25 m | 560 k |
| $M_5$ | 50µ | 0,2 | 25 | 0,25 m | 140 k |




Input dynamics:

$$V_{IN}\Big|_{max} = V_{DD} - V_{GSM} + V_T \simeq 2,8$$

$$V_{IN}\Big|_{min} = V_{OV_G} + V_{GS_2} \simeq 0,9$$

$$\rightarrow |\Delta V_{IN}^+| = 2,8 - 1,5 = 1,3\,V$$

$$|\Delta V_{IN}^-| = 1,5 - 0,9 = 0,6\,V$$

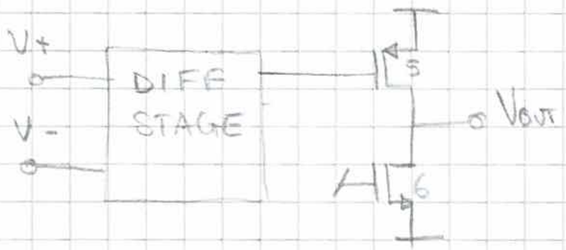
$$V_{OUT}\Big|_{max} = V_{DD} - V_{OVM} \simeq 2,8$$

$$V_{OUT}\Big|_{min} = V_{CM} - V_T \simeq 0,9$$

$$\rightarrow |\Delta V_{OUT}^+| = 2,8 - 2,2 = 0,6\,V$$

$$|\Delta V_{OUT}^-| = 2,2 - 0,9 = 1,3\,V$$

## 5) Two stages OTA: topology, freq response, Miller comp

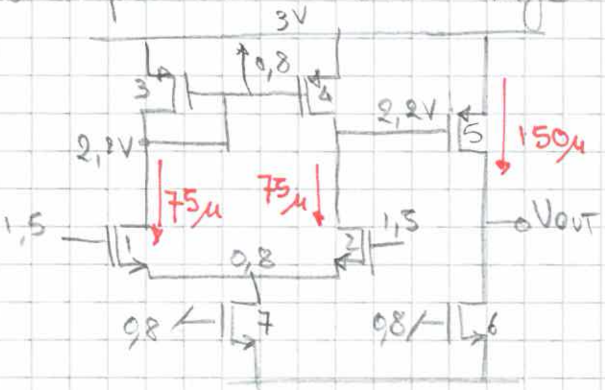



$G_1$ = diff stage gain $\simeq$ 40 dB, not enough for the > 85dB spec

$G_2$ = 2nd stage gain

$$G_D = G_1 G_2 = \underbrace{g_{m1}(r_{o1}/\!/r_{o2})}_{140}\ \underbrace{g_{m5}(r_{o5}/\!/r_{o6})}_{140} \simeq 86\,dB$$

We met the $G_D$ requirement at the cost of more power dissipation → 2nd stage requires bias:




$M_6$ has to match $M_5$ in order to get proper bias and Vout set at $V_{DD}/2$ (therefore $K_5 = K_6$)

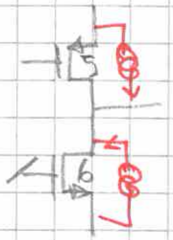
$$G_D = \frac{\cancel{2I_1}}{V_{ov1}} \cdot \frac{V_{A1}}{\cancel{2I_1}} \cdot \frac{\cancel{2I_5}}{V_{ov5}} \cdot \frac{V_{A5}}{\cancel{2I_5}} = \frac{V_{A1}}{V_{ov1}} \frac{V_{A5}}{V_{ov5}}$$

$V_{ov5}$ has to be the same of $V_{ov3,4}$ because of symmetry

We can therefore choose proper $L_5, L_6$ to match gain by finding the $V_{A5} = \frac{L_5}{L_{min}} \cdot 7V$

Note: when $M_5 \neq M_6$ we end up with a systematic offset that can be input translated to: $V_{os,in} = \left| \frac{V_{DD}/2 - V_{OUT}}{G_D} \right|$

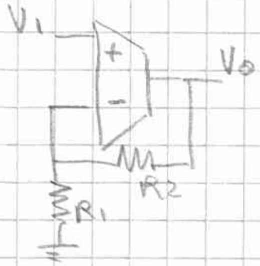
## Noise consideration on 2nd stage




$$\overline{S_{IN}}\Big|_{TOT} = \frac{8KT\gamma}{gm_1}\left(1+\frac{1}{2}\right) + 8KT\gamma\, gm_5 \cdot \frac{1}{G_1^{\,2}\, gm_5^{\,2}}$$

Division by $G_1^2$ makes 2nd stage noise negligible with respect to 1st stage

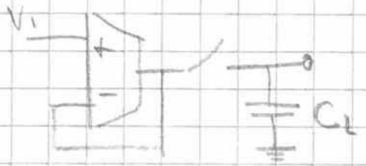
## OTA applications




⟶ Resistive loads kill the loop gain

$$G_{loop} = gm \cdot \frac{R_{OUT}}{R_{OUT}+R_1+R_2} \cdot R_1$$

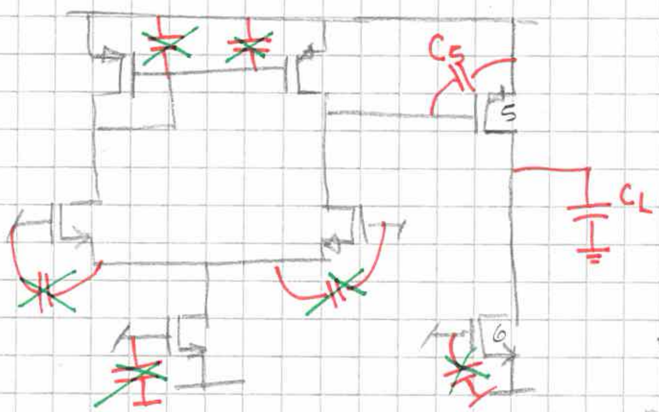
We should use a buffer between Gm stage and the resistive load → OPAMP




We could use the OTA as a S&H:

- No resistive load
- Capacitive load → could generate unstability

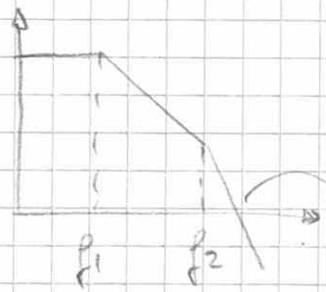
## Frequency response




$C_{1,2,3,4}$ see low impedance → pole is at high frequency

$C_7, C_6$ do not affect signal response.

$C_{gs5}$, $C_L$ affect frequency response by introducing two poles well beyond $G_{loop}=1$:



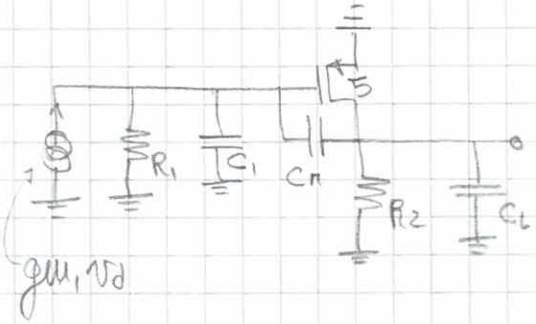

$$f_1 = \frac{1}{2\pi\, C_L (r_{o5} /\!/ r_{o6})} \qquad f_2 = \frac{1}{2\pi\, C_{gs5} (r_{o4} /\!/ r_{o2})}$$

→ We are at 0dB with −40dB → UNSTABLE

This means that if we used the OTA as a buffer with a capacitive load, it would be unstable. or UNCOMPENSATED at unity gain

# Miller compensation

Let's simplify the stage and let's add a comp. capacitor:




$R_1$ = impedance of 1st stage

$R_2$ = " " 2nd "

$C_1 = C_{gs5}$

$C_M$ = Miller comp. capacitor

$$T(s) = G_D(0) \cdot \frac{s^2 a_2 + s a_1 + 1}{s^3 b_3 + s^2 b_2 + s b_1 + 1} \quad \text{where}$$

$b_1 = C_1 R_1^{(0)} + C_M R_M^{(0)} + C_L R_L^{(0)}$

$b_2 = C_M C_1 R_M^{(0)} R_1^{(M)} + C_1 C_L R_1^{(0)} R_L^{(1)} + C_M C_L R_M^{(0)} R_L^{(M)}$

$b_3 = C_M C_1 C_L R_M^{(0)} R_1^{(M)} R_L^{(M,1)}$

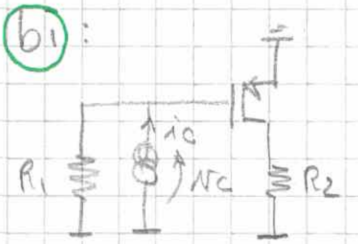
$a_1 = C_1 R_{o1}^{(0)} + C_M R_{oM}^{(0)} + C_L R_{oL}^{(0)}$

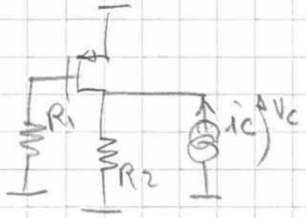
$a_2 = C_M C_1 R_{oM}^{(0)} R_{o1}^{(M)} + C_M C_L R_{oM}^{(0)} R_{oL}^{(M)} + C_1 C_L R_{o1}^{(0)} R_{oL}^{(1)}$

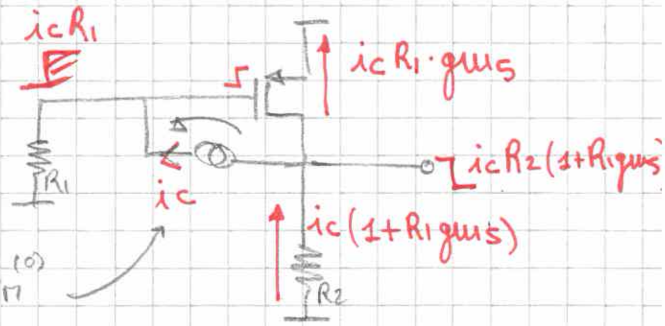
We can already say that there's no $a_3$ because $C_L$ is directly tied to output $\rightarrow \infty$ zero

Since capacitors interact (they're dependent on each other), we can already say that $b_3 = 0$ for whatever combination

So $\rightarrow$ max 2 poles, max 2 zeros (b1) :




$R_1^{(0)} = R_1$ $\qquad R_L^{(0)} = R_2$ $\qquad R_\pi^{(0)}$

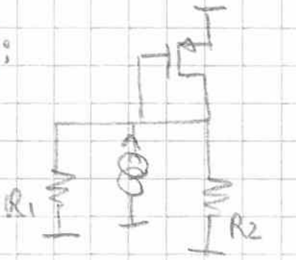
$R_\pi^{(0)}$: $v_C = i_C R_1 - (-i_C R_2 (1 + R_1 g_{m5}))$

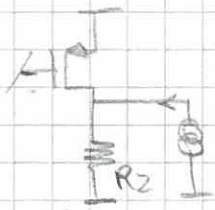
$$R_\pi^{(0)} = \frac{v_C}{i_C} = R_1 + R_2 (1 + R_1 g_{m5}) = R_2 + R_1 (1 + g_{m5} R_2)$$

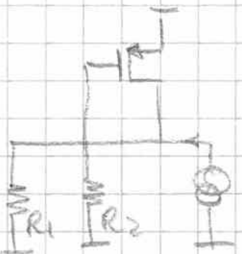
Remember that because of Miller effect $R_\pi = R_2 + R_1 (1 + G_2)$ where $G_2$ is the voltage gain between terminals of $C_\pi$

$$b_1 = \underbrace{C_1 R_1}_{\sim 90n} + \underbrace{C_L R_2}_{\sim 450n} + \underbrace{C_\pi [R_2 + R_1 (1 + g_{m5} R_2)]}_{\sim 8\mu s} \approx C_\pi R_1 g_{m5} R_2$$

(b2):



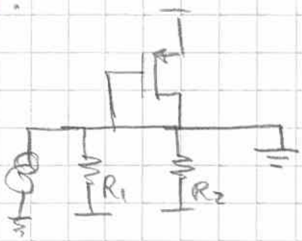



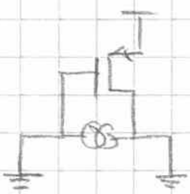

$R_1^{(\pi)} = R_1 // R_2 // \frac{1}{g_{m5}} \simeq 1/g_{m5}$ $\qquad R_L^{(1)} \; R_2$ $\qquad R_L^{(\pi)} = R_1 // R_2 // \frac{1}{g_{m5}} \simeq 1/g_{m5}$

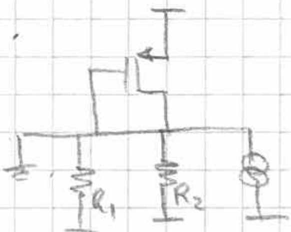
$$b_2 \simeq C_\pi C_1 R_1 R_2 \cancel{g_{m5}} \cdot \frac{1}{\cancel{g_{m5}}} + C_1 C_L R_1 R_2 + C_\pi C_L R_1 R_2 \cancel{g_{m5}} \cdot \frac{1}{\cancel{g_{m5}}}$$

$$= R_1 R_2 [C_\pi C_1 + C_1 C_L + C_\pi C_L] = C_1 C_L R_1 R_2 + [C_1 + C_L] C_\pi R_1 R_2$$

(b3) :

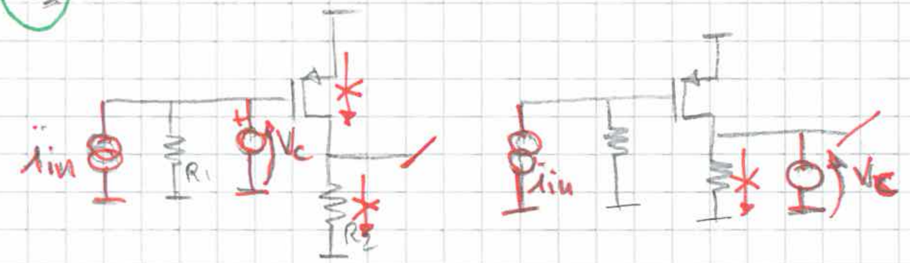





$b_3 = 0$ for every possible combination

$a_2$ :

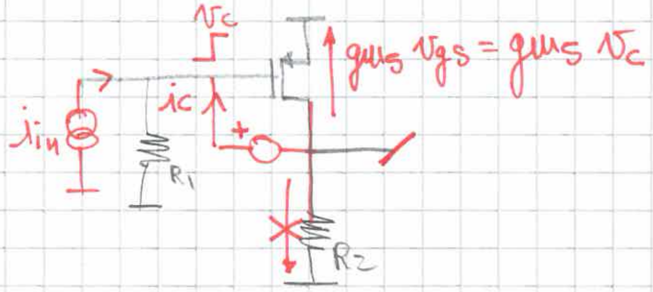



Setup for zeros:

- Vout can't move
- Input generator is ON
- Capacitor generator outputs $i_c$ and $v_c$

$R_{o1}^{(0)}$: Vout can't move $\rightarrow V_{R_2} = 0 \rightarrow I_{R2} = 0 \rightarrow I_{M5} = 0 \rightarrow$
$\rightarrow V_{gs5} = 0 \rightarrow V_c$ can't move $\Big] \rightarrow \underline{R_{o1}^{(0)} = 0}$

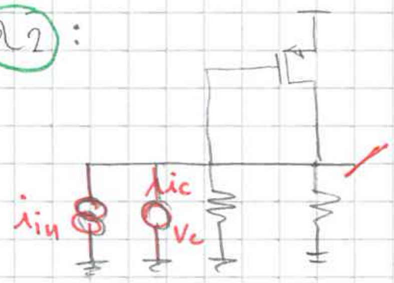
$R_{oL}^{(0)}$: Vout can't move $\rightarrow V_c$ can't move $\Big] \rightarrow \underline{R_{oL}^{(0)} = 0}$




Vout can't move $\rightarrow V_{R2}, I_{R2} = 0 \rightarrow i_c + i_{M5} = 0 \rightarrow i_c + g_{m5} v_c = 0$

$\frac{v_c}{i_c} = \underline{R_{o\pi}^{(0)}} = \underbrace{\frac{-1}{g_{m5}}}$ while $i_{in} = \frac{v_c}{R_1} - i_c$

$a_2$ :




$\underline{a_2 = 0}$ it's easy to derive that $R_{o1}^{(\pi)}, R_{o2}^{(\pi)} = 0$ because Vout can't move and both $C_1$, $C_L$ are tied to Vout when $C_\pi$ is short

---

$$G_D(s) = G_D(0) \cdot \frac{1 - s\, C_\pi / g_{m5}}{s^2 b_2 + s b_1 + 1}$$

## Frequency results estimation

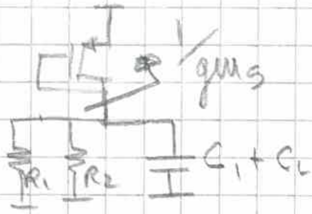
by seeing the large $\tau_{C\pi}$, we can say that the resulting two poles will differ by more than a decade. Approx:

$$p_L \approx -\frac{1}{b_1} = \frac{-1}{C_\pi R_1 g_{ms} R_2}$$ ⇝ $C_\pi$ contributes the most

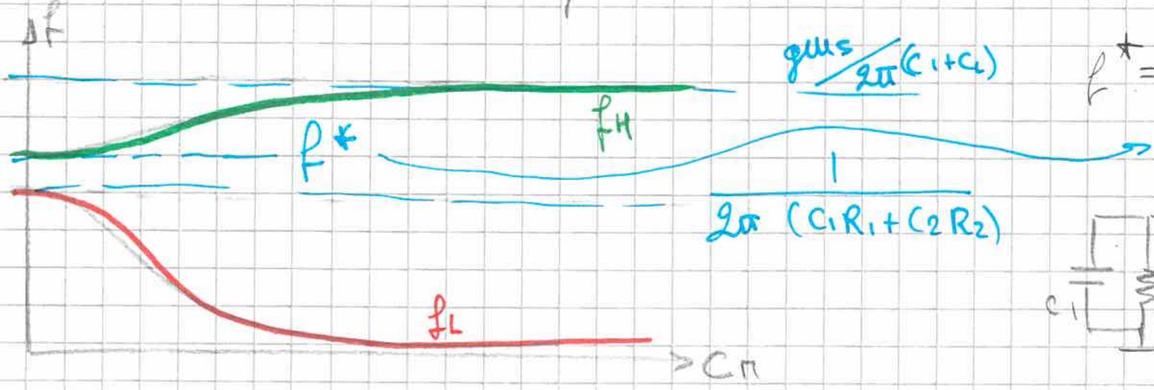
$$p_H \approx -\frac{1}{b_2} = \frac{-C_\pi g_{ms} \cancel{R_1 R_2}}{[C_1(C_\pi + C_L) + C_1 C_\pi] \cancel{R_1 R_2}} = \frac{-g_{ms} C_\pi}{C_\pi(C_1 + C_L) + C_1 C_L}$$

Note: for large $C_\pi \rightarrow p_H \simeq -\frac{g_{ms}}{C_1 + C_L}$

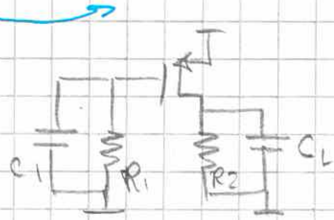
$p_H$ makes sense because if $C_\pi$ is short:




$$f_H = \frac{1}{2\pi (C_1 + C_L)[R_1 // R_2 // \frac{1}{g_{ms}}]} \simeq \frac{g_{ms}}{2\pi (C_1 + C_L)}$$



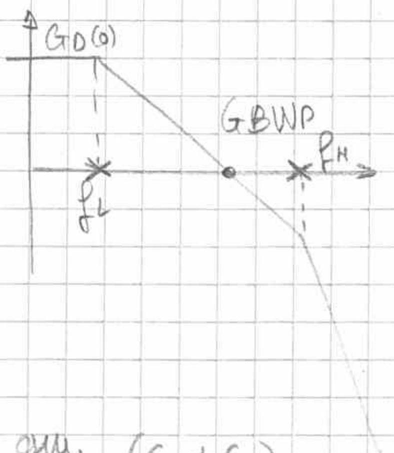

$$f^\star = \frac{1}{2\pi}\left[\frac{1}{C_1 R_1} + \frac{1}{C_2 R_2}\right]$$




for $C_\pi \rightarrow 0$ only $C_1$, $C_L$ take place in the circuit

for $C_\pi \rightarrow \infty$ $f_L \rightarrow 0$ and $f_H$ saturates to $\frac{g_{ms}}{2\pi (C_1 + C_L)}$

$$\text{GBWP} = G_D(0)\, f_L = \frac{g_{m1} \cancel{g_{ms} R_1 R_2}}{2\pi C_\pi \cancel{g_{ms} R_1 R_2}} = \frac{g_{m1}}{2\pi C_\pi}$$




In order not to cut with $-40\,\text{dB/dec}$:

$$f_H \geq \text{GBWP} \qquad \frac{g_{ms}}{\cancel{2\pi}(C_1 + C_L)} \geq \frac{g_{m1}}{\cancel{2\pi} C_\pi}$$

Therefore compensation should be $C_\pi \geq \frac{g_{m1}}{g_{ms}}(C_1 + C_L)$

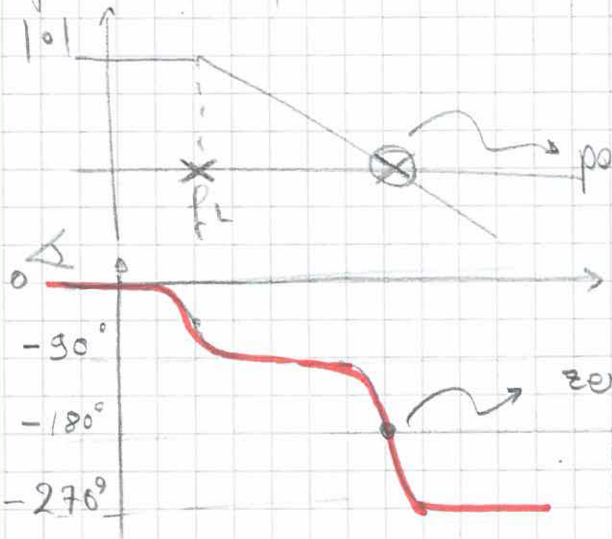
If we need more phase margin, $f_H \geq 2\,\text{GBWP}$

## Phase margin analysis

$f_L$ already contributed with $90°$

$$f_H = \frac{g_{m5}}{2\pi (C_1 + C_L)} \qquad GBWP = \frac{g_{m1}}{2\pi C_\pi}$$

$$f_z = \frac{g_{m5}}{2\pi C_\pi}$$




pole + zero

zero is RHP → phase behaves like a pole → $\Phi_\pi = 0°$

If this OTA is used like a buffer → very close to instability

To recover same margin, $f_H, f_z = 2\,GBWP$ so that

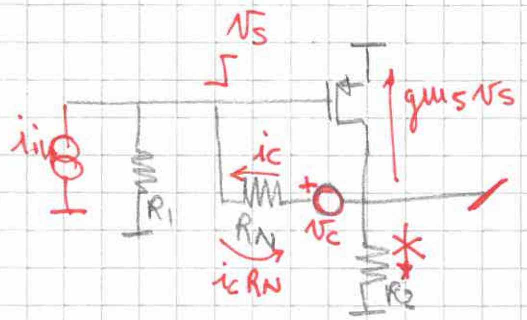
$$\Phi_\pi = 180° - 90° - tg^{-1}\underbrace{\left(\frac{GBWP}{f_z}\right)}_{1/2} - tg^{-1}\underbrace{\left(\frac{GBWP}{f_H}\right)}_{1/2} = 90° - 27° - 27° = \boxed{35°}$$

To obtain this we can: $\uparrow C_\pi$ or $\uparrow g_{m5}$:

1) Increasing $C_\pi$ shifts GBWP to the left but also $f_L$ is shifted → BW ↘
2) Increasing $g_{m5}$ shifts $f_H, f_z$ to the right at the cost of more power burned

## 6) Nulling resistor: compensation + implementation

We may think of a solution that changes the $f_z$ independently or moves it to LHP ($+90°$ contribution).
We need to place something in the path of the zero:

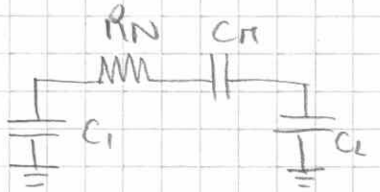


$$v_s = v_c - i_c R_N \qquad g_{m5} v_s = -i_c$$

$$R_{OT}^{(0)} = \frac{v_c}{i_c} = R_N - \frac{1}{g_{m5}} \rightarrow \text{new } f_z!$$

We can see that we can set $R_N = \frac{1}{g_{m5}}$ to have $f_z = \infty$, or we can move $f_z$ to the LHP so that we can have proper $\phi_m$

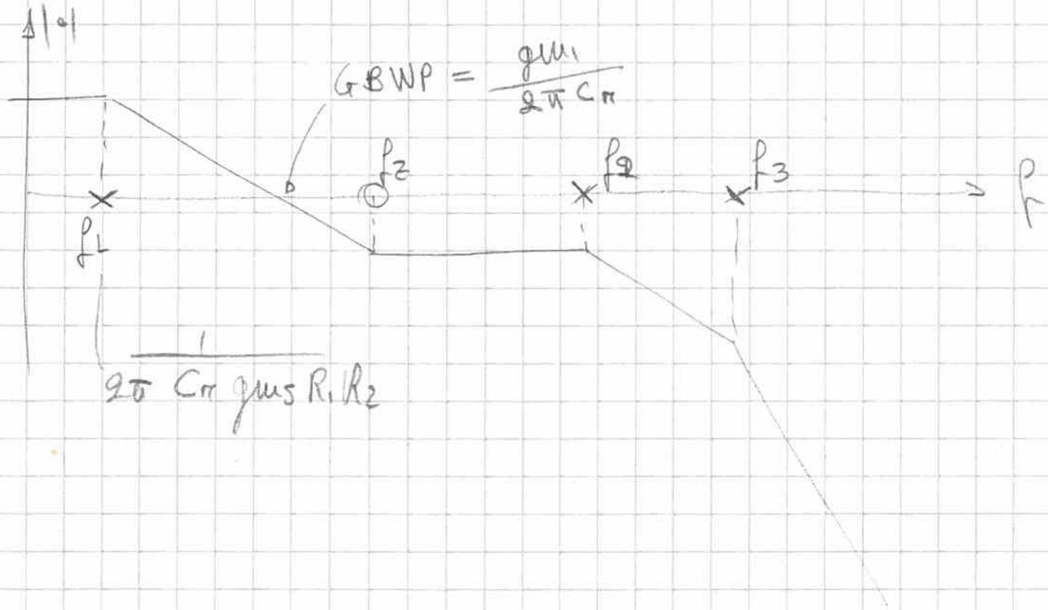
Note:



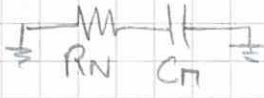
Caps are now independent → → additional pole.

$$f_L \simeq \frac{1}{2\pi} \frac{1}{C_1 R_1 + C_L R_2 + C_T [R_1 + R_2 + g_{m5} R_1 R_2 + R_N]}$$

It's easy to derive the new $b_1$ term
$R_N$ is usually $\sim 2/g_m$ → $f_L$ has negligible change and so does GBWP



Let's compute $f_3$:



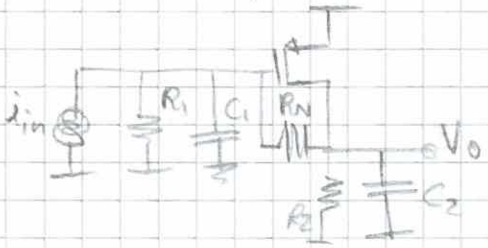
$C_L, C_\pi$ short $\qquad$ $C_1, C_L$ short $\qquad$ $C_1, C_\pi$ short

$$f_3 = \sum_i \frac{1}{\tau_i^\infty} = \frac{1}{2\pi}\left[\frac{1}{C_1(R_1/\!/R_N)} + \frac{1}{C_\pi R_N} + \frac{1}{C_L(R_2/\!/R_N)}\right] \sim$$

$$\sim \frac{1}{2\pi R_N (C_1 /\!/ C_\pi /\!/ C_L)} \sim \frac{1}{2\pi R_N C_1}$$

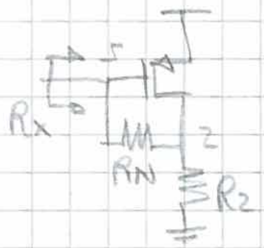
Note: it's not important to compute the precise value, hence this just gives us a hint on the order of magnitude

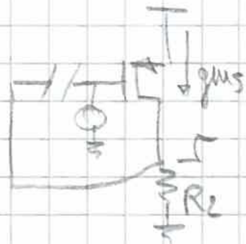
Compute $f_2$: consider $C_\pi$ as a short




$$\underline{R_1^{(0)}: R_1 /\!/ R_x}$$

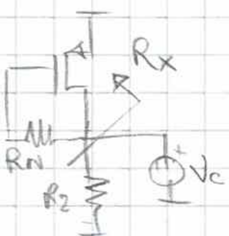
$$R_x = \frac{R_2 + R_N}{1 - G_{loop}}$$




$G_{loop} = -g_{ms} R_2$ so $R_x \sim 1/g_{ms}$ if $R_2 \gg R_N$ (which it is)

$R_L^{(0)}$:

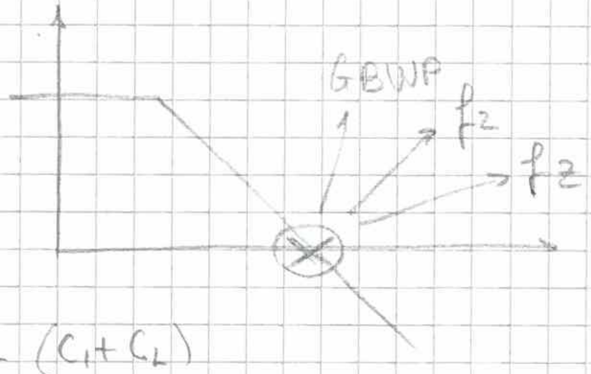



$\underline{R_L^{(0)} = R_2 /\!/ R_x}$ where $\underline{R_x = \frac{1}{g_{ms}}}$

So $$f_2 = \frac{1}{2\pi\left[\frac{C_1}{g_{ms}} + \frac{C_L}{g_{ms}}\right]} \sim \underline{\frac{g_{ms}}{2\pi(C_1 + C_L)}}$$

$f_2$ is roughly in the same position as before

## Zero-pole-compensation



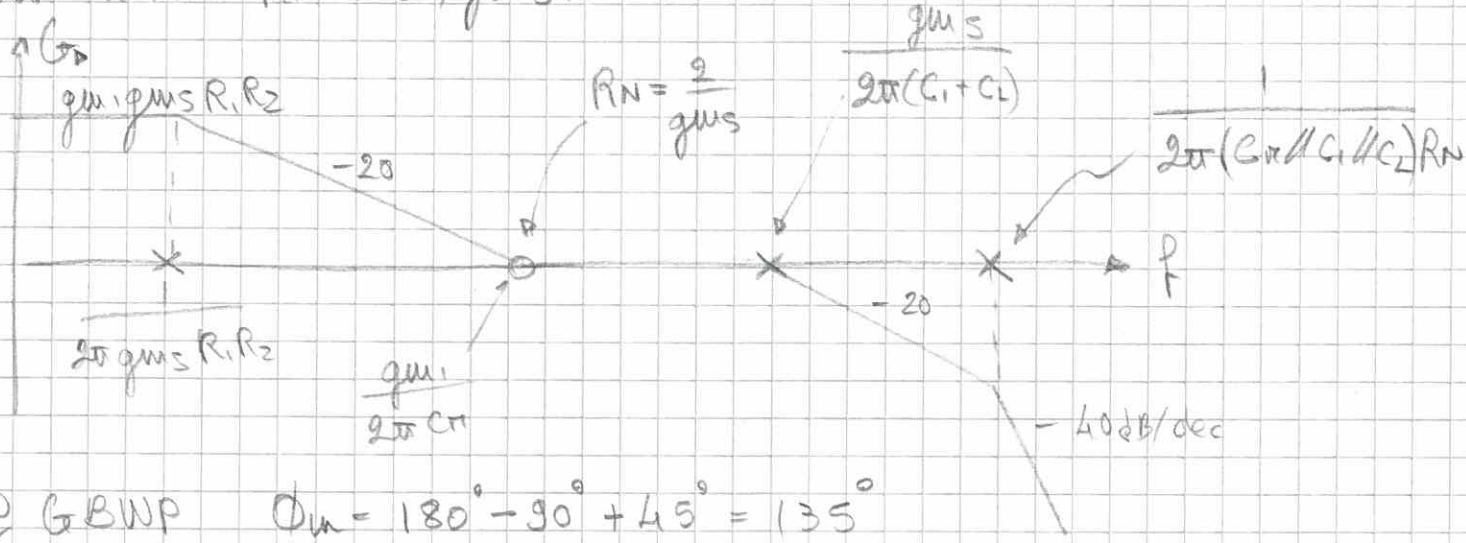

First approach:

GBWP @ $f_2$ and $f_z = f_2$

$$\frac{g_{m1}}{2\pi C_\pi} = \frac{g_{m5}}{2\pi(C_1+C_L)} \longrightarrow C_\pi = \frac{g_{m1}}{g_{m5}}(C_1+C_L)$$

$$\frac{1}{\cancel{2\pi C_\pi}\left(R_N - \frac{1}{g_{m5}}\right)} = \frac{g_{m1}}{\cancel{2\pi C_\pi}} \longrightarrow R_N \cong \frac{1}{g_{m5}}\left(1 + \frac{g_{m5}}{g_{m1}}\right)$$

Now with $R_N = 2/g_{m5}$:



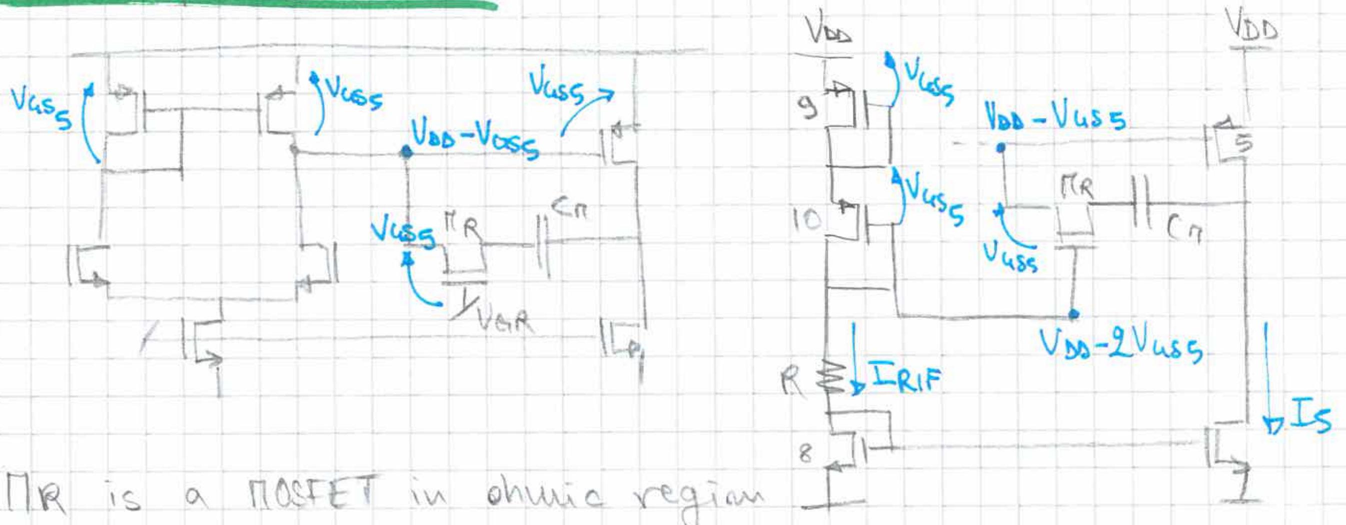

@ GBWP $\quad \Phi_m = 180^\circ - 90^\circ + 45^\circ = 135^\circ$

This way we can have the same bias, the same BW at the cost of implementing another resistor

Recap:

| | GBWP | $f_z$ | $f_2$ | $f_3$ |
|---|---|---|---|---|
| Miller | $g_{m1}/2\pi C_\pi$ | $g_{m5}/2\pi C_\pi$ RHP | $\frac{g_{m5}}{2\pi(C_1+C_L)}$ | $\infty$ |
| $R_N$ | $g_{m1}/2\pi C_\pi$ | $\frac{1}{2\pi\left[R_N - \frac{1}{g_{m5}}\right]C_\pi}$ | $\frac{g_{m5}}{2\pi(C_1+C_L)}$ | $> \frac{1}{2\pi C_1 R_N}$ |

## RN implementation




$M_R$ is a MOSFET in ohmic region

$$I_D = 2K'\left(\frac{W}{L}\right)\left(V_{ov}V_{DS}-\frac{V_{DS}^2}{2}\right) \qquad g_{oR}=\left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{DS}=0} = 2K'\left(\frac{W}{L}\right)(V_{ov}-V_{DS}) =$$

$$g_{oR} = 2K'\left(\frac{W}{L}\right)_R V_{ovR} = g_{mR}$$

Assume $R_N = \frac{1}{g_{m5}}(1+\alpha)$ where $\alpha$ is a variability of the process

Then $\frac{g_{m5}}{g_{mR}} = (1+\alpha) \longrightarrow \frac{(W/L)_5}{(W/L)_R}\frac{V_{ov5}}{V_{ovR}} = (1+\alpha)$

We can have $V_{ovR} = V_{ov5}$ so we can better set $\frac{W}{L}$ ratios.

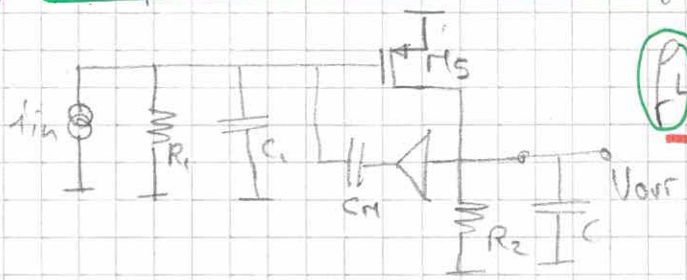
We can size $I_{RIF} = \frac{1}{m} I_S$ so we can have lower power dissip.

To have the $V_{GS5}$ listed on 9, 10, we need $\left(\frac{W}{L}\right)_{9,10} = \frac{1}{m}\left(\frac{W}{L}\right)_5$

While $\left(\frac{W}{L}\right)_8 = \frac{1}{m}\left(\frac{W}{L}\right)_6$

Note: $M_R$ is in ohmic region because at bias C is open and therefore $V_{DS}=0$ because no current flows during bias

# 7) Compensation with Voltage/current buffers (ideal/real)

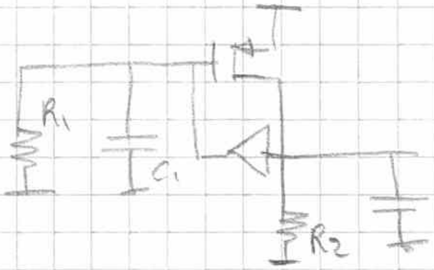


$$f_L = \frac{1}{2\pi C_\pi g_{m5} R_1 R_2} \qquad GBWP = \frac{g_{m1}}{2\pi C_\pi}$$

## IDEAL Voltage buffer

$f_3 = \infty$ with ideal voltage buffer (it can be verified by shorting two caps and seeing that $R_{eq}^{(\infty)} = 0$)

$f_2$ = say that $C_\pi$ is basically short:



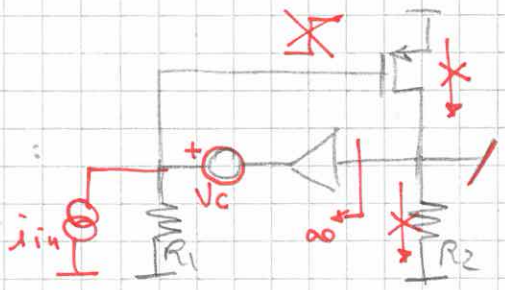
$R_1^{(0)} = 0$ because of the ideal buffer

$R_L^{(0)} = R_2 // R_x \simeq \frac{1}{g_{m5}}$

$R_x = \frac{1}{g_{m5}}$

$$f_2 \simeq \frac{g_{m5}}{2\pi C_L}$$

Let's see $f_z$:



Vout can't move, $I_{R_2} = 0$

therefore $I_{M5} = 0$

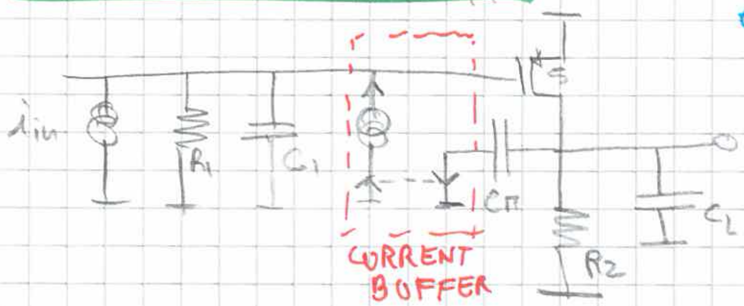
this way $V_{GS5} = 0$ so

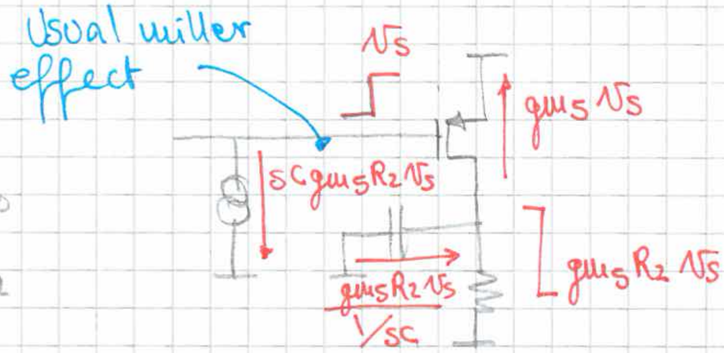
$V_C = 0 \rightarrow f_z = +\infty$

Recap

| | GBWP | $f_z$ | $f_2$ | $f_3$ |
|---|---|---|---|---|
| Voltage Buffer IDEAL | $\frac{g_{m1}}{2\pi C_\pi}$ | $\infty$ | $\frac{g_{m5}}{2\pi C_L}$ | $\infty$ |

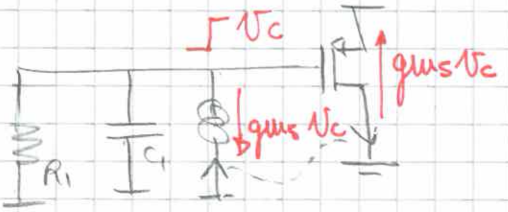
# Ideal current buffer




As always we can expect the same $f_1$, GBWP because we preserved the Miller effect

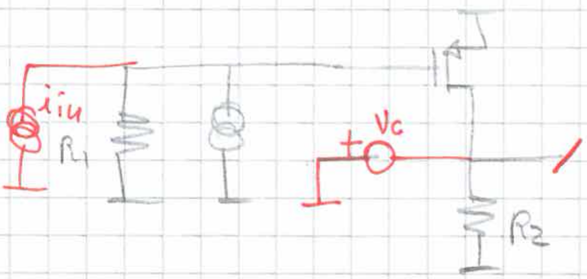
HF pole: $C_\pi$ is short $\rightarrow$ $R_2$ and $C_L$ are shorted




$$R_1^{(0)} = R_1 // \frac{1}{g_{m5}} \simeq \frac{1}{g_{m5}}$$

$$f_2 = \frac{g_{m5}}{2\pi C_1}$$

Important: $f_2$ is independent from $C_L$ $\rightarrow$ large loads do not affect frequency response!

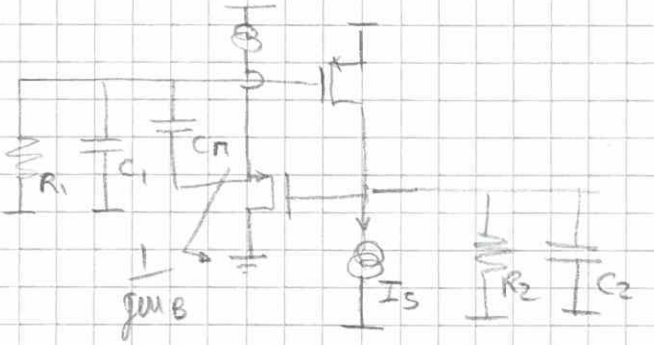



$f_z = \infty$ because $v_c$ sees 0V on both its ends

| | GBWP | $f_z$ | $f_2$ | $f_3$ |
|---|---|---|---|---|
| Current Buffer IDEAL | $\frac{g_{m1}}{2\pi C_\pi}$ | $\infty$ | $\frac{g_{m5}}{2\pi C_1}$ | $\infty$ |

$f_3$ can be easily derived by shorting the other two capacitors, we will see that result is the same of $f_2$

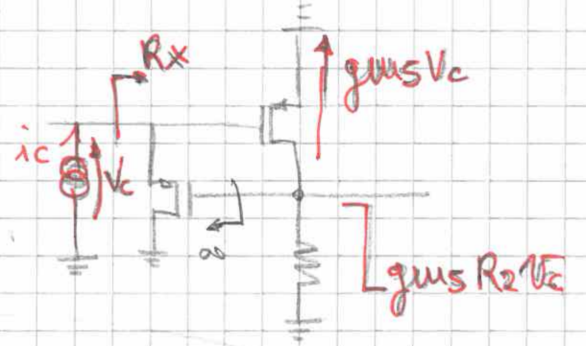
# Real voltage buffer implementation




We now have a zero:

$$f_z = \frac{g_{mB}}{2\pi C_\pi}$$

$f_2$: $C_\pi$ is short



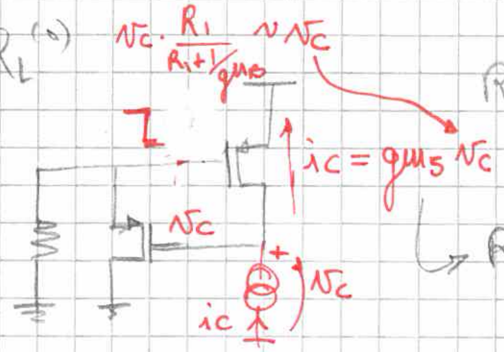

$$f_2 = \frac{1}{2\pi\left[C_1 R_1^{(0)} + C_L R_L^{(0)}\right]}$$

$R_1^{(0)} = R_1 \,//\, R_x \qquad i_c = g_{mB}(v_c + g_{mS} R_2 v_c)$

$$\frac{v_c}{i_c} = R_x = \frac{1}{g_{mB}(1 + g_{mS} R_2)} = \frac{1}{g_{mB}\, g_{mS} R_2} \simeq R_1^{(0)}$$

$R_1$ is neglig.

$R_L^{(0)}$

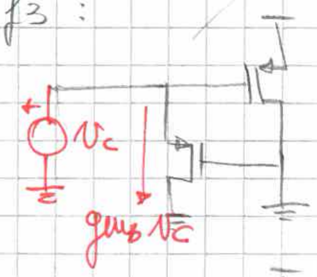



$R_L^{(0)} = R_2 \,//\, R_x \simeq 1/g_{mS}$

$R_x \sim \frac{1}{g_{mS}}$

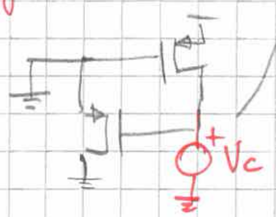
$$f_2 \simeq \frac{g_{mS}}{2\pi C_L} \quad \text{since} \quad \frac{1}{g_{mB}\, g_{mS} R_2} \ll \frac{1}{g_{mS}}$$

$f_3$:




$R_1^{(L)} = \frac{1}{g_{mB}} // R_1 \simeq \frac{1}{g_{mB}}$

$R_L^{(1)} = R_2$



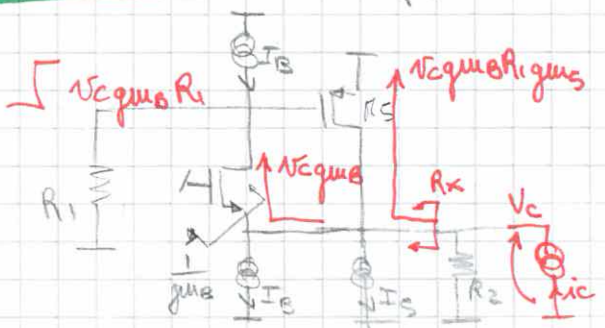

$$f_3 = \frac{1}{2\pi}\left[\frac{g_{mB}}{C_1} + \cancel{\frac{1}{C_L R_2}}\right] = \frac{g_{mB}}{2\pi C_1}$$

| | GBWP | $f_z$ | $f_2$ | $f_3$ |
|---|---|---|---|---|
| Voltage buffer Real | $\frac{g_{m1}}{2\pi C_\pi}$ | $\frac{g_{mB}}{2\pi C_\pi}$ | $\frac{g_{m5}}{2\pi C_L}$ | $\frac{g_{mB}}{2\pi C_1}$ |

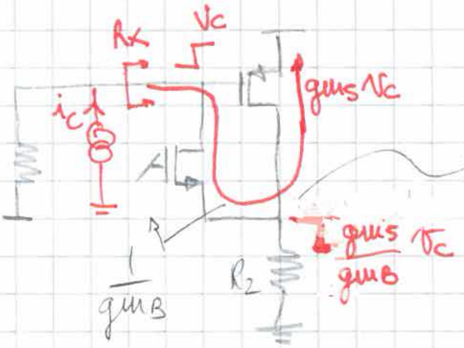
Since $C_L \gg C_1$, $f_3 > f_2$ (typically), better estimate would be using the full time constant method

We typically put $g_{mB} = g_{m1}$ → We basically have the same result of a RN comp but at the cost of more power consumption → $\text{FoM} = \frac{GBWP \cdot C_L}{I_{TOT}}$ decreases

## Real current buffer




$$R^{(0)} = R_2 // R_x \simeq \frac{1}{g_{m5}\, g_{mB} R_1}$$

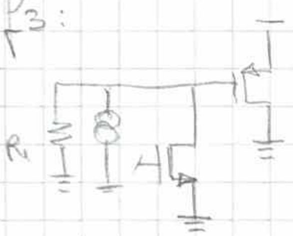


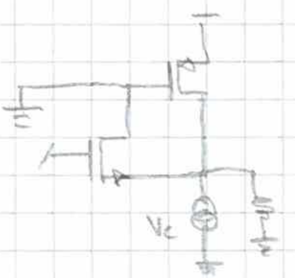

$$R_1^{(0)} = R_1 // R_x \simeq \frac{1}{g_{m5}}$$

$\frac{1}{g_{mB}} \ll R_2$ so $g_{m5} v_c$ is sucked by current buffer

$$f_2 = \frac{1}{2\pi}\left[\frac{C_1}{g_{m5}} + \underbrace{\frac{C_L}{g_{m5} g_{mB} R_1}}_{\to 0}\right]^{-1} \simeq \frac{g_{m5}}{2\pi C_1}$$

$f_3$:



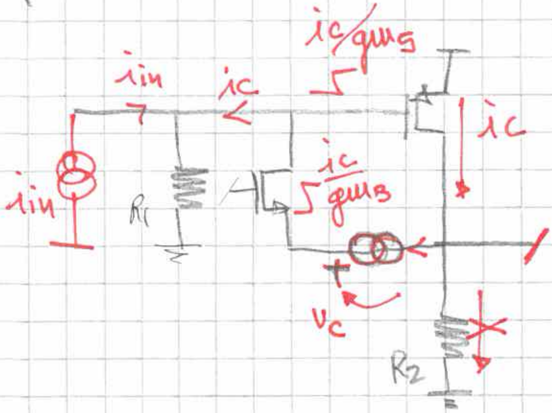




$$R_L^{(1)} = R_2 // \frac{1}{g_{mB}} \simeq \frac{1}{g_{mB}}$$

$$R_1^{(2)} = R_1$$

$$f_3 = \frac{1}{2\pi}\left[\underbrace{\frac{1}{C_1 R_1}}_{\to 0} + \frac{g_{mB}}{C_2}\right] \simeq \frac{g_{mB}}{2\pi C_2}$$

$f_Z$ :




$V_{OUT}$ can't move $\rightarrow V_{R_2} = 0 \rightarrow I_{R_2} = 0$

$i_c \neq 0$ so it must be collected by $M_S$ (thus $i_c/g_{mS}$ at the gate).

All $i_c$ flows through $g_{mB}$ so:

$$V_c = \frac{i_c}{g_{mB}} - 0 \longrightarrow R_{0\pi}^{(0)} = \frac{1}{g_{mB}}$$

$$f_Z = \frac{g_{mB}}{2\pi C_\pi}$$

| | GBWP | $f_Z$ | $f_2$ | $f_3$ |
|---|---|---|---|---|
| Current buffer Real | $\frac{g_{m1}}{2\pi C_\pi}$ | $\frac{g_{mB}}{2\pi C_\pi}$ | $\frac{g_{mS}}{2\pi C_1}$ | $\frac{g_{mB}}{2\pi C_L}$ |

WARNING! If $g_{mS} \sim g_{mB}$, $C_L > C_1$ then $f_3 > f_2 \longrightarrow$

This means that the method we used can't work $\longrightarrow$

We need the full Middlebrook theorem.

$$T(s) = G_D(0) \frac{1}{b_2 s^2 + b_1 s + 1} \longrightarrow T(s) = 1$$

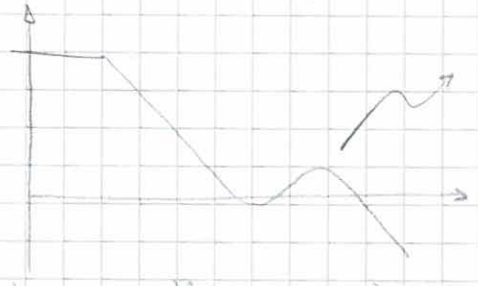
$$b_1 = C_1 R_1^{(0)} + C_L R_2^{(0)} = \frac{C_1}{g_{mS}} + \frac{C_L}{g_{mS} g_{mB} R_1}$$

$$b_2 = C_1 C_L R_1^{(0)} R_2^{(1)} = \frac{C_1}{g_{mS}} \cdot \frac{C_L}{g_{mB}}$$

$$s^2 \underbrace{\frac{C_1 C_L}{g_{mS} g_{mB}}}_{\frac{1}{\omega_0^2}} + s \underbrace{\left( \frac{C_1}{g_{mS}} + \frac{C_L}{g_{mS} g_{mB} R_1} \right)}_{\frac{1}{\omega_0 Q}} + 1 = 0$$

$$\omega_0 = \sqrt{\frac{g_{mS} g_{mB}}{C_1 C_L}}$$ and since $\frac{C_1}{g_{mS}} \gg \frac{C_L}{g_{mS} g_{mB} R_1} \longrightarrow Q\Big|_{approx} = \sqrt{\frac{g_{mS} C_L}{g_{mB} C_1}}$

Note: be careful when placing the zeros

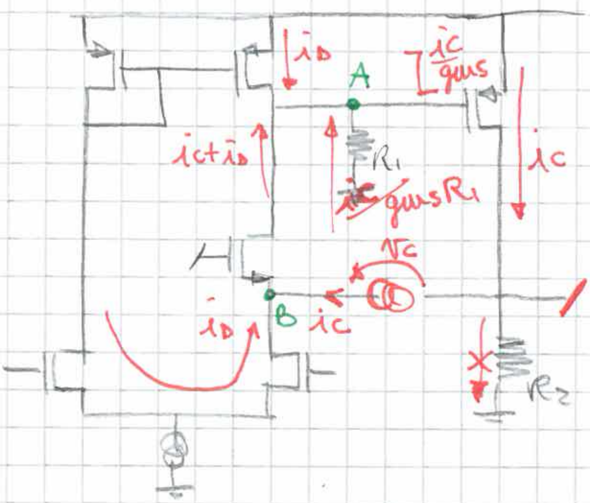


Peaking of the pole pair can go above 0dB. We may want to shift $f_Z$ right in order to keep the pealing under 0dB

## Current buffer in the differential structure

### Calculating the zeros

$f_Z$:




Vout can't move $\rightarrow V_{R2}=0$ and

$I_{\pi S}=i_C \rightarrow v_{gS_S}=i_C/g_{mS}$

Balance current at nodes A, B:

$$\begin{cases} +\dfrac{i_C}{g_{mS}R_1} = -i_C - 2i_D \\ \dfrac{v_C}{g_{mB}} = i_C + i_D \end{cases} \rightarrow$$

$$\frac{i_C}{g_{mS}R_1} + 2\frac{v_C}{g_{mB}} = i_C \rightarrow \frac{v_C}{i_C} = R_{O\pi}^{(0)} = \frac{g_{mB}}{2}\left(1-\frac{1}{g_{mS}R_1}\right)$$

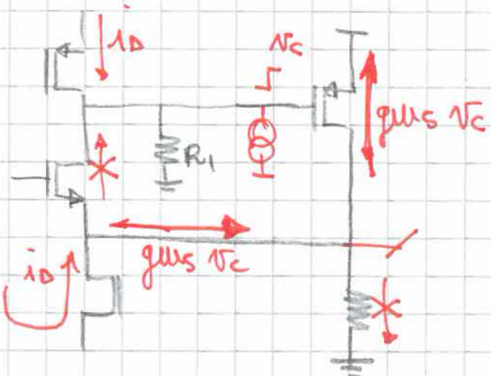
$a_1 = C_\pi R_{O\pi}^{(0)} \rightarrow C_1, C_L$ do not introduce zeros

$a_2 = C_1 C_\pi R_{O1}^{(0)} R_{O\pi}^{(1)}$ : $R_{O1}^{(0)} = 0$

while $R_{O3}^{(1)} = R_{O\pi}^{(0)}\Big|_{R_1=0} = \frac{g_{mB}}{2}\left(1-\frac{1}{0}\right) = -\infty$

$a_2 = 0\cdot(-\infty) \rightarrow$ indefinite form $\rightarrow$ shuffle terms

$a_2 = C_\pi C_1 R_{O\pi}^{(0)} R_{O1}^{(\pi)} \rightarrow R_{O1}^{(\pi)}$




$g_{mS}v_C + i_D = 0 \quad i_D = \frac{v_C}{R_1} - i_C$

$g_{mS}v_C + \frac{v_C}{R_1} = i_C \rightarrow R_{O1}^{(\pi)} \simeq -\frac{1}{g_{mS}}$

$$a_2 = C_\pi C_1 \frac{1}{g_{mB}}\cdot\frac{-1}{g_{mS}} = -\frac{C_\pi C_1}{g_{mB}g_{mS}}$$

# Poles of cascoded Ahuja compensation

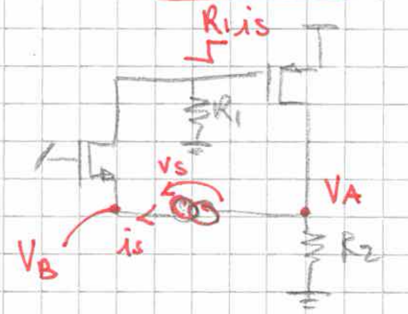
Since there are 3 independent capacitors, we know:

$b_1 = C_1 R_1^{(0)} + C_L R_L^{(0)} + C_\pi R_\pi^{(0)}$

$b_2 = C_1 C_L R_1^{(0)} R_L^{(1)} + C_1 C_\pi R_1^{(0)} R_\pi^{(1)} + C_L C_\pi R_L^{(0)} R_\pi^{(L)}$

$b_3 = C_1 C_L C_\pi R_1^{(0)} R_L^{(1)} R_\pi^{(1,L)}$

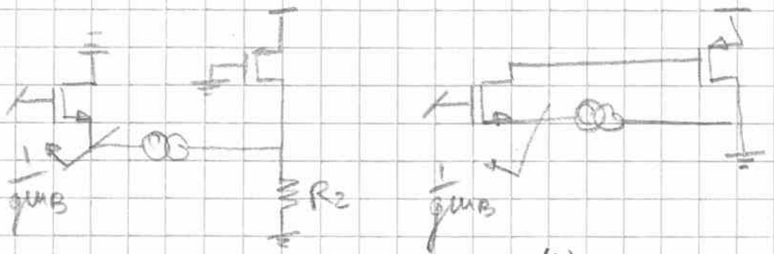
$b_1$ : $R_1^{(0)} = R_1$ $\quad R_L^{(0)} = R_2$ as always, while $R_\pi^{(0)}$:




$R_3^{(0)} = \frac{1}{g_{mB}} + R_2 + g_{mS} R_1 R_2$

$b_1 = C_1 R_1 + C_L R_2 + C_\pi \left(\frac{1}{g_{mB}} + R_2 + g_{mS} R_1 R_2\right) \simeq C_\pi g_{mS} R_1 R_2$

$b_2$ :




$R_\pi^{(1)} = \frac{1}{g_{mB}} + R_2 \qquad R_\pi^{(L)} = \frac{1}{g_{mB}}$

$b_2 \simeq C_1 C_L R_1 R_2 + C_1 C_\pi R_1 \left(R_2 + \underbrace{\frac{1}{g_{mB}}}_{\text{NEGLIGIBLE}}\right) + C_L C_\pi \frac{R_2}{g_{mB}}$

$= R_1 R_2 (C_1 C_L + C_1 C_\pi) + C_L C_\pi \frac{R_2}{g_{mB}}$

$b_3$ : $R_\pi^{(1,L)}$ is exactly like $R_\pi^{(L)} = \frac{1}{g_{mB}}$

$R_L^{(1)} = R_2$ intuitive

$b_3 = C_1 C_L C_\pi \frac{R_1 R_2}{g_{mB}}$

# Complete Ahuja transfer function

$$T(s) = g_{m1} g_{mS} R_1 R_2 \quad \frac{a_2 s^2 + a_1 s + 1}{b_3 s^3 + b_2 s^2 + b_1 s + 1}$$

$b_1 \cong C_\pi \, g_{mS} R_1 R_2$

$b_2 \simeq R_1 R_2 (C_1 C_L + C_1 C_\pi) + C_L C_\pi \dfrac{R_2}{g_{mB}}$

$b_3 = C_1 C_L C_\pi \dfrac{R_1 R_2}{g_{mB}}$

$a_1 = C_\pi \dfrac{g_{mB}}{2} \left(1 - \dfrac{1}{g_{mS} R_1}\right) \simeq \dfrac{g_{mB}}{2}$

$a_2 = - \dfrac{C_\pi C_1}{g_{mS} \, g_{mB}}$ probably not asked at the oral

For large $g_{mS} R_2 C_\pi$ (read lectures pdf) + some algebra:

Denominator: $\left[ s^2 \dfrac{C_1 C_L}{g_{mB} g_{mS}} + s \dfrac{C_1}{g_{mS}} \left( \dfrac{C_\pi + C_L}{C_\pi} \right) + 1 \right] \left( \dfrac{s}{g_{mS} R_1 R_2 C_\pi} + 1 \right)$

So: $f_L = \dfrac{1}{2\pi \, C_\pi \, g_{mS} R_1 R_2}$

additional poles at $\omega_0 = \sqrt{\dfrac{g_{mS} \, g_{mB}}{C_1 C_L}}$ $\quad Q = \dfrac{C_\pi}{C_\pi + C_L} \sqrt{\dfrac{g_{mS} C_1}{g_{mB} C_L}}$

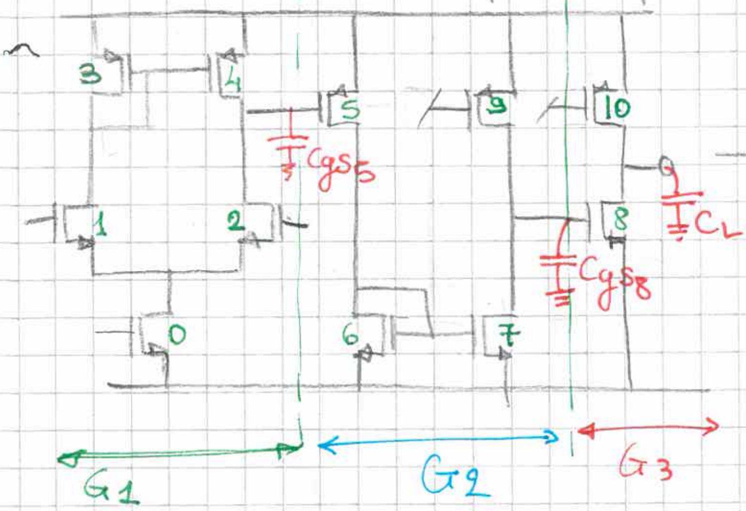
Zeros: $S_{12} = \dfrac{g_{mS}}{2 C_1} \left[ 1 \pm \sqrt{1 + 8 \dfrac{g_{mB} C_1}{g_{mS} C_\pi}} \right]$

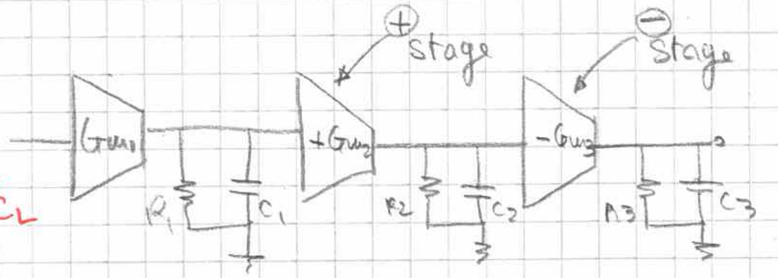
For large $C_L$ $\quad z_1 = -2 \dfrac{g_{mB}}{C_\pi} \quad z_2 = \dfrac{g_{mS}}{C_1}$

$z_1$ sits close to GBWP, $z_2$ is around a $f_T$ because $C_1$ is typically a $C_{gs}$

# 8) Nested Miller compensation

As technology scales → $V_{DD}$ ↘ → We can't pile up transistors. Therefore for large gains → multiple stages




$|G_{DC}| = G_{m1} R_1 \, G_{m2} R_2 \, G_{m3} R_3 = (200)^3 = 138\,dB$

$C_L$, $C_{gs5}$, $C_{gs8}$ load the OTA the most (high impedance nodes)

$C_{gs5}$ → $C_1$; $C_{gs8}$ → $C_2$

critical for input noise

low not to have high power burned

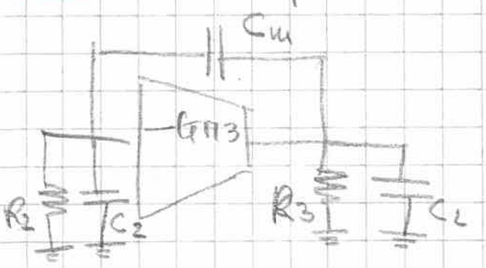
high to drive capacitive loads

| | | | | | |
|---|---|---|---|---|---|
| $G_{m1}$ | 100 μA | $G_{m2}$ | 30 μA/V | $G_{m3}$ | 1 mA/V |
| $R_1$ | 2 MΩ | $R_2$ | 6,7 MΩ | $R_3$ | 200 kΩ |
| $C_1$ | 10 fF | $C_2$ | 167 fF | $C_L$ | 100 pF |

$f_1 = 8\,MHz$ $\quad f_2 = 142\,kHz$ $\quad f_3 = 200\,Hz$

With this configuration → cut 0dB with −60dB/dec → NOPE

Miller compensation for $C_2$, $C_L$ (classic)




$$GBWP_{23} = \frac{G_{m2} \cancel{G_{m3} R_2 R_3}}{2\pi C_M \cancel{G_{m3} R_2 R_3}} = \frac{G_{m2}}{2\pi C_M}$$

$$f_3' = \frac{G_{m3}}{2\pi (C_L + C_2)} \simeq \frac{G_{m3}}{2\pi C_L}$$

$$f_z' = \frac{G_{m3}}{2\pi C_M} = \text{positive zero}$$

$$f_2' = \frac{1}{2\pi C_M G_{m3} R_2 R_3}$$

Neglect the zero → $\phi_m = 60°$ if $f_3' > 2\,GBWP$ :

$$\frac{G_{m3}}{\cancel{2\pi} C_L} = 2 \frac{G_{m2}}{\cancel{2\pi} C_M} \rightarrow C_M = 2 \frac{G_{m2}}{G_{m3}} C_L$$

At this point, the whole OTA response will be

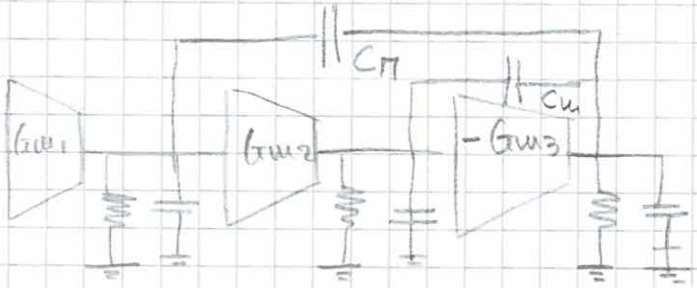
$$f_2' = \frac{1}{2\pi C_m G_{m3} R_2 R_3} = 20\,Hz \qquad f_3' = 2\,GBWP_{23} = \frac{G_{m3}}{2\pi C_L} = 1{,}6\,MHz$$

$$f_1 = \frac{1}{2\pi C_1 R_1} = 8\,MHz \qquad f_z' = 26{,}5\,MHz$$

$f_z' \gg GBWP$ we assumed its contribution is negligible

We still cut with $-40\,dB/dec$ at $0\,dB$ → UNSTABLE

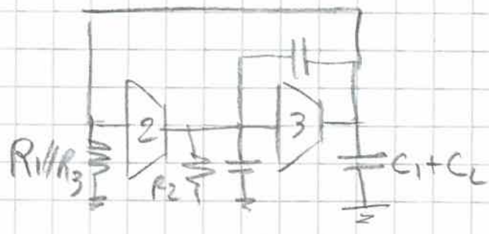
## Nested Miller compensation



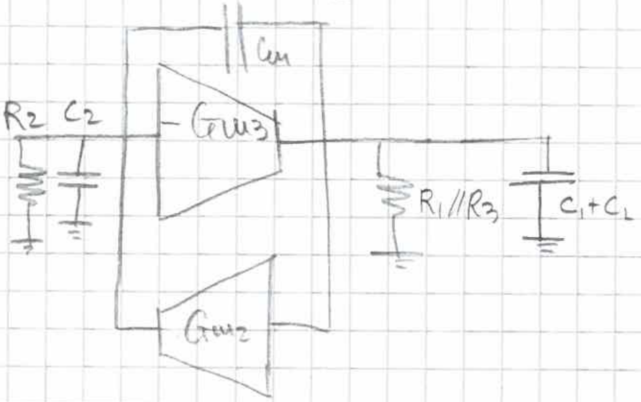

→ $G_{m2}$ has positive gain because we need overall negative gain on $C_M$ in order to exploit Miller!

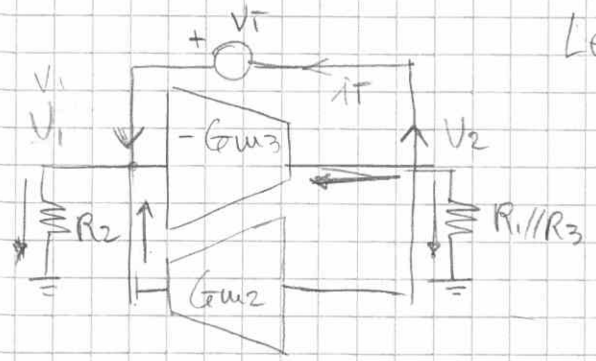
Since $G_2 = G_{m2} R_2$, $G_3 = G_{m3} R_3$ $C_M$ sees $G_2 G_3$ while $C_m$ only sees $G_3$ → lowest pole is given by $C_M$

$GBWP\big|_{TOT} = \frac{G_{m1}}{2\pi C_M}$ → we can now short $C_M$ (do not consider zeros for now)




$$f_2'' = \frac{1}{2\pi \left[ C_2 R_2^{(0)} + C_m R_m^{(0)} + (C_L + C_1) R_L^{(0)} \right]}$$






Let's calculate $R_{in}^{(0)}$:

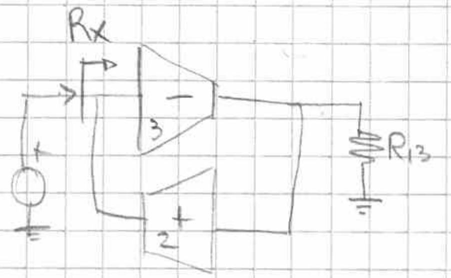
$$\frac{V_1}{R_2} - V_2 G_{m2} = I_T$$

$$\frac{V_2}{R_{13}} - V_1 G_{m3} = I_T$$

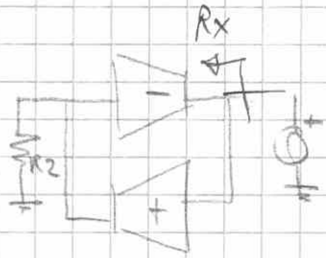
Neglect these

$$V_2 = -\frac{I_T}{G_{m2}} \qquad V_1 = -\frac{I_T}{G_{m3}} \qquad V_T = V_1 - V_2 = I_T\left(-\frac{1}{G_{m3}} + \frac{1}{G_{m2}}\right)$$

$$R_{in}^{(0)} = \frac{G_{m3} - G_{m2}}{G_{m3} G_{m2}}$$




$R_2^{(0)}$




$R_L^{(0)}$

$$R_2^{(0)} = R_2 // R_x \simeq \frac{1}{G_{m2} G_{m3} R_{13}}$$

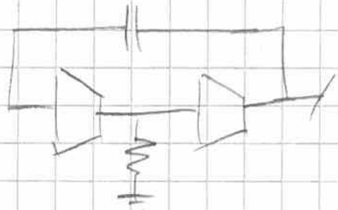
$$R_L^{(0)} = R_{13} // R_x \simeq \frac{1}{G_{m3} G_{m2} R_2}$$

$$f_2'' = \frac{1}{2\pi} \cdot \left[\frac{C_2}{G_{m2} G_{m3} R_{13}} + \frac{G_{m3} - G_{m2}}{G_{m2} G_{m3}} C_m + \frac{C_1 + C_L}{G_{m3} G_{m2} R_2}\right]^{-1} = 0{,}8\ \text{MHz}$$

As we did before, $f_2'' = 2\ \text{GBWP}$ so $C_\pi = 2 \dfrac{G_{m1}}{2\pi f_{p2}''} \simeq 40\text{pF}$

$$\text{GBWP} = \frac{G_{m1}}{2\pi C_m} = 400\ \text{kHz}$$

Estimate the zero:



$$f_z'' = \frac{G_{m2} R_2 G_{m3}}{2\pi C_\pi}$$

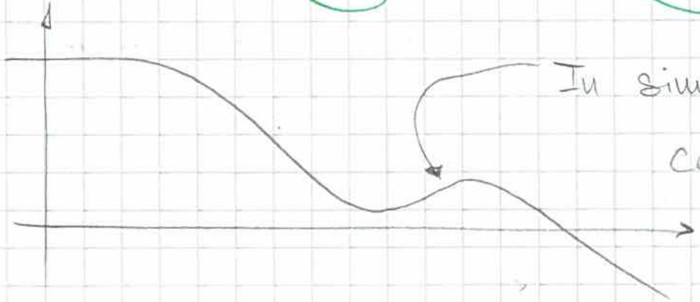
$f_z''$ is far away from GBWP. This though isn't totally correct, since $C_\pi$ and $C_m$ can contribute at most with two zeros (SPICE will tell)

## comments on TF

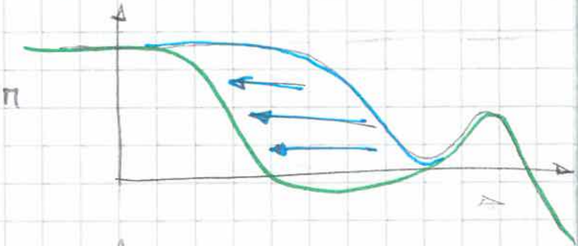
It is possible to derive the polynomial at the denominator:

$$s^2 b_2 + s b_1 + 1 = s^2 \frac{C_L C_m}{G_{m2} G_{m3}} + s \frac{G_{m3} - G_{m2}}{G_{m2} G_{m3}} + 1 = 0$$

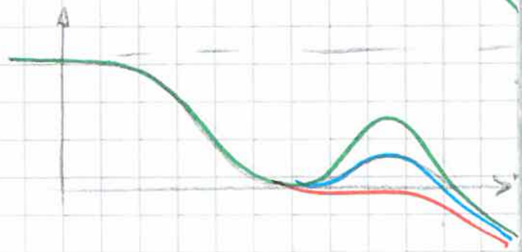
$$\omega_0 = \sqrt{\frac{G_{m2} G_{m3}}{C_L C_m}} \qquad Q = \frac{1}{G_{m3} - G_{m2}} \sqrt{G_{m2} G_{m3} \frac{C_L}{C_m}}$$



In simulations we see peaking. How can we solve this? look at $G_{m3}$, $C_m$



- Increase $C_M$ → BW is reduced but $C_M$ plays no role in Q factor → bad idea

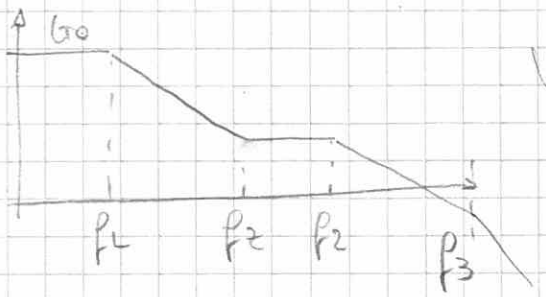


- Increase $G_{m3}$ → peak is quenched down

Fine tuning is done on SPICE

## 9) OTA linear response. In band doublets + settling response

We can have something like: ($R_N$ compensation)

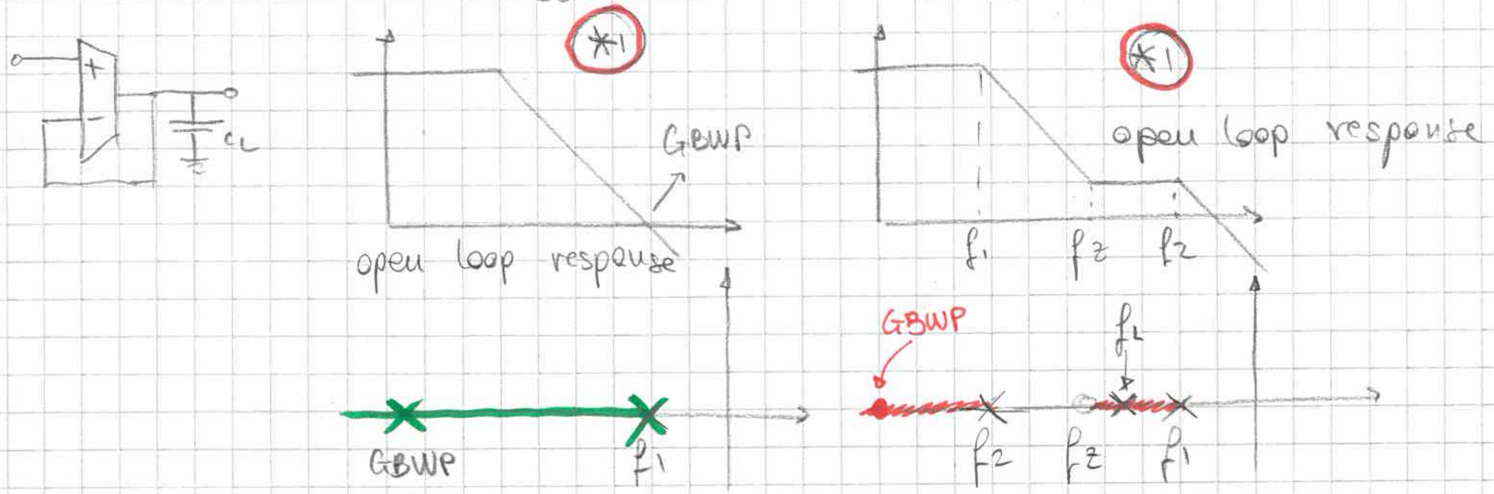



Where $f_2 \simeq \frac{g_{ms}}{2\pi C_L}$ $f_z = \frac{1}{2\pi \left(R_N - \frac{1}{g_{ms}}\right) C_n}$

$f_2 - f_z$ cancellation has negligible impact on GBWP and phase margin

This way, since $f_2 \propto \frac{1}{C_L}$ we could think of driving large capacitive loads without degrading the amplifier stability.

There are however some drawbacks to this:




*1 Compensated plot where $f_z = f_2 = GBWP$

*2 Same circuit used to drive larger $C_L$ ($f_2$ moves left) without major changes on compensation ($f_z$ moves left too).

If load isn't precise for zero-pole cancellation, we have *2

Analyze Gloop of *2:

$$G_{loop}(s) = -G_0 \frac{1 + s\tilde{\tau}_z}{(1+s\tilde{\tau}_1)(1+s\tilde{\tau}_2)}$$ by looking at the root locus:

$f_L$ for a large $G_0$ approaches $f_z$

$f_H$ will be $\sim$ GBWP

Let's find the closed loop singularities:

$G_{loop}(s) = 1 \qquad G_0(1+s\tau_z) = (1+s\tau_1)(1+s\tau_2)$

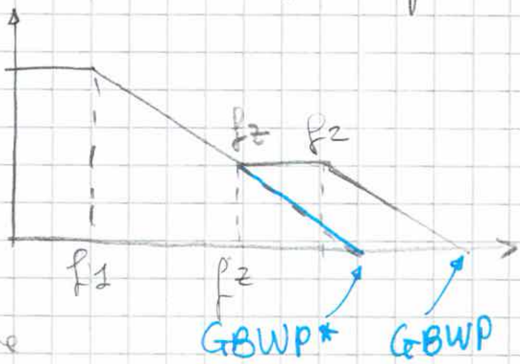
$s^2(\tau_1\tau_2) + s(\tau_1+\tau_2+G_0\tau_z) + (G_0+1) = 0$ → rough estimation:

- for HF $s^2 \gg s \gg G_0+1 \qquad s^{\cancel{2}}(\tau_1\tau_2) + \cancel{s}(\tau_1+\tau_2+G_0\tau_z) = 0$

$$S_{HIGH} = -\frac{\tau_1+\tau_2+G_0\tau_z}{\tau_1\tau_2} \approx \frac{G_0\tau_z}{\tau_1\tau_2}$$

- for LF $s^2 \ll s \ll G_0+1 \qquad s(\tau_1+\tau_2+G_0\tau_z) + G_0+1 = 0$

$$S_{low} = -\frac{G_0 \cancel{+1}}{\cancel{\tau_1+\tau_2}+G_0\tau_z} \rightarrow -\frac{1}{\tau_z}$$ $f_{low}$ approaches $f_z$ for high $G_0$

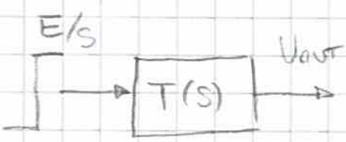



Let's analyze this a little better:

$$p_{low} = \frac{\tau_1+\cancel{\tau_2}+G_0\tau_z}{G_0+\cancel{1}} \approx \tau_z + \frac{\tau_1}{G_0}$$

It's interesting to note that $\frac{\tau_1}{G_0}$ is the GBWP*, the 0dB cut if $f_z$ wasn't there. In reality, the GBWP is $GBWP = \frac{f_2}{f_z} GBWP^*$

We now ask ourselves what is the linear response:




$$T(s) = \frac{1+s\tau_z}{(1+s\tau_L)(1+s\tau_H)} \qquad V_{OUT}(s) = \frac{E}{s}T(s)$$

Use Heaviside $v_{OUT}(t) = \mathcal{L}^{-1}\left[\frac{E}{s}\left(\frac{A}{1+s\tau_L} + \frac{B}{1+s\tau_H}\right)\right]$

Use limit theorem

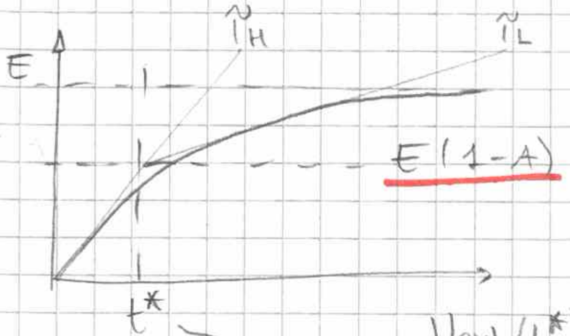
$$A = \lim_{s\to -\frac{1}{\tau_L}} \frac{(1+s\tau_z)\cancel{(1+s\tau_L)}}{\cancel{(1+s\tau_L)}(1+s\tau_H)} = \frac{1-\frac{1}{\tau_L}\tau_z}{1-\tau_H/\tau_L} = \frac{\tau_L-\tau_z}{\tau_L-\tau_H}$$

$$B = \lim_{s\to -\frac{1}{\tau_H}} \frac{(1+s\tau_z)\cancel{(1+s\tau_H)}}{(1+s\tau_L)\cancel{(1+s\tau_H)}} = \frac{\tau_H-\tau_z}{\tau_H-\tau_L} = \frac{\tau_z-\tau_H}{\tau_L-\tau_H}$$

Since $\tau_z > \tau_H$, we adjust to get $B>0$

$$V_{out}(t) = E\left[A\left(1-e^{-t/\tau_L}\right) + B\left(1-e^{-t/\tau_H}\right)\right] = E\left[1-Ae^{-t/\tau_L} - Be^{-t/\tau_H}\right]$$

$$A+B = \frac{\tau_L - \cancel{\tau_Z} + \cancel{\tau_Z} - \tau_H}{\tau_L - \tau_H} = 1$$




B term ($\tau_H$) quickly vanishes, leading to $V_{out} \simeq E\left[1 + Ae^{-t/\tau_L}\right]$

This means that the fast curve starts at a hight

$$V_{out}(t^*) = E\left[1 - A\underbrace{e^{-t^*/\tau_L}}_{\to 1}\right] = \boxed{E(1-A)}$$

Where $A = \dfrac{\tau_L - \tau_Z}{\tau_L - \tau_H} = \dfrac{\cancel{\tau_Z} + \frac{\tau_1}{G_0} - \cancel{\tau_Z}}{\tau_Z + \frac{\tau_1}{G_0} - \tau_H} \to \boxed{A} = \dfrac{\tau_1}{\underbrace{G_0 \tau_Z}_{GBWP^*}} = \dfrac{f_Z}{GBWP^*}$ (*3)

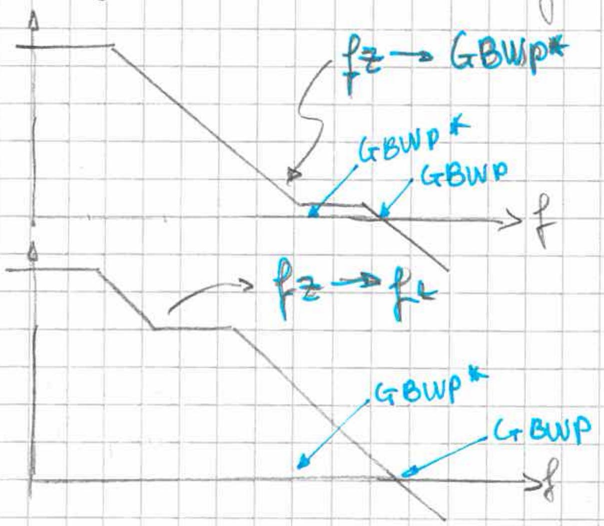
Since $\tau_L \simeq \tau_Z + \dfrac{\tau_1}{G_0}$

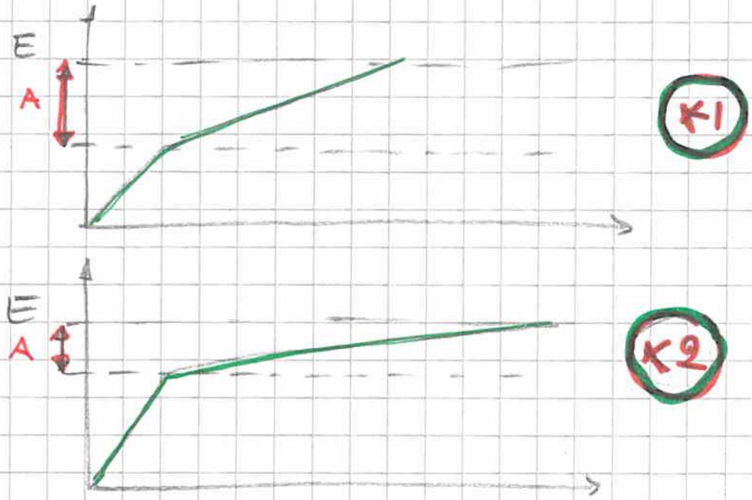
Suppose $f_Z = 5$ MHz GBWP = 50 MHz $f_Z / GBWP^* = 10\%$

Therefore, we end up with → $A \simeq 0$ when $f_Z \to f_L$

→ $A \simeq 1$ when $f_Z \to GBWP^*$

These two limits generate two situations

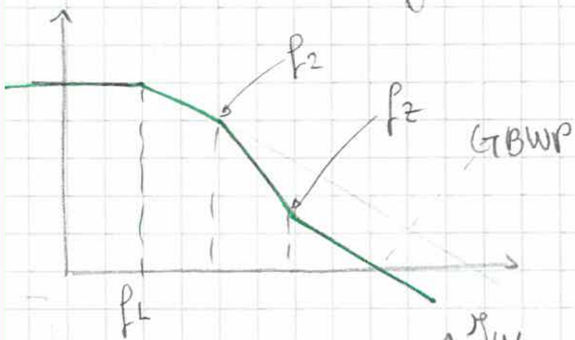


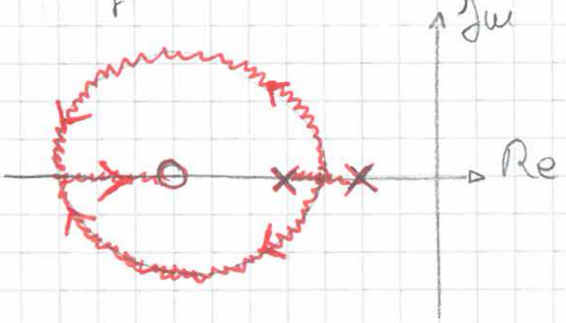



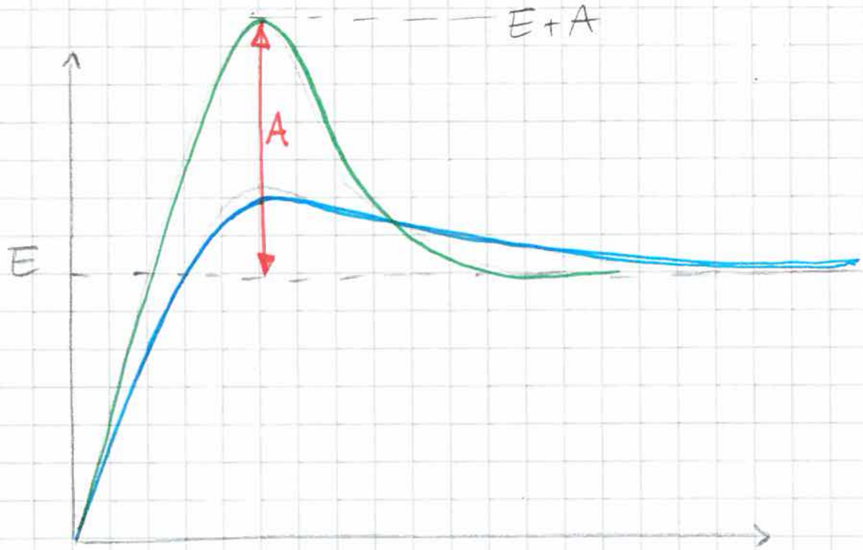

(*1) $f_Z$ approaches $GBWP^*$ → $\tau_Z$ ↘↘ → A ↗↗. $\tau_L = \tau_Z + \frac{\tau_1}{G_0}$ is moderatly slow (*3)

(*2) $f_Z$ approaches $f_L$ → $\tau_Z$ ↗↗ → A ↗↗ and $\tau_L = \tau_Z + \frac{\tau_1}{G_0}$ is increased, therefore (*2) settling time is higher than (*1)

A similar thing happens when the zero is after $f_2$



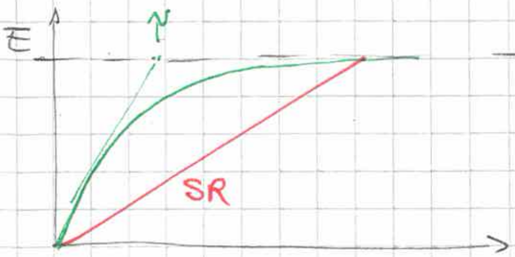



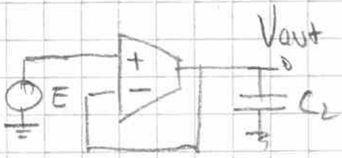

In this case, the issue will be E+A.

Design note: to get fast settling time we accept the trade off with large A to have low $\tau_L$, therefore the design should foresee a $f_z \to GBWP^*$

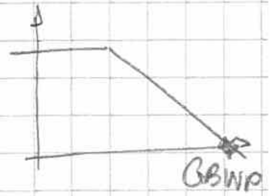
# 10) Slew Rate: impact on settling time, $SR_{INT}$, $SR_{EXT}$, class AB




– Suppose we're driving a load using an OTA buffer:



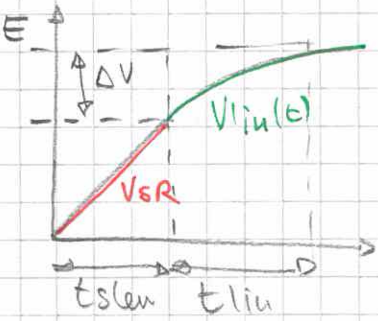

$$\tau = \frac{1}{GBWP}$$




OTA will be limited by $\tau$ (linear component) and the Slew Rate

$$V_{out}(t)\Big|_{lin} = E\left(1 - e^{-t/\tau}\right) \longrightarrow \frac{dV_{out}(t)}{dt} = \frac{E}{\tau}$$

If SR is slower, slopes will be $\frac{E}{\tau} > SR$. This means that we'll have a slew rate section + linear section:




There will be a condition where SR will become faster than the linear limitation set by the GBWP → slew rate becomes linear:

$$V_{lin}(t) = E - \Delta V e^{-t/\tau} \longrightarrow \frac{dV_{lin}(t)}{dt} = \frac{\Delta V}{\tau}$$

Condition is $\frac{dV_{lin}(t)}{dt} = SR \longrightarrow \frac{\Delta V}{\tau} = SR \longrightarrow \Delta V = SR \cdot \tau$

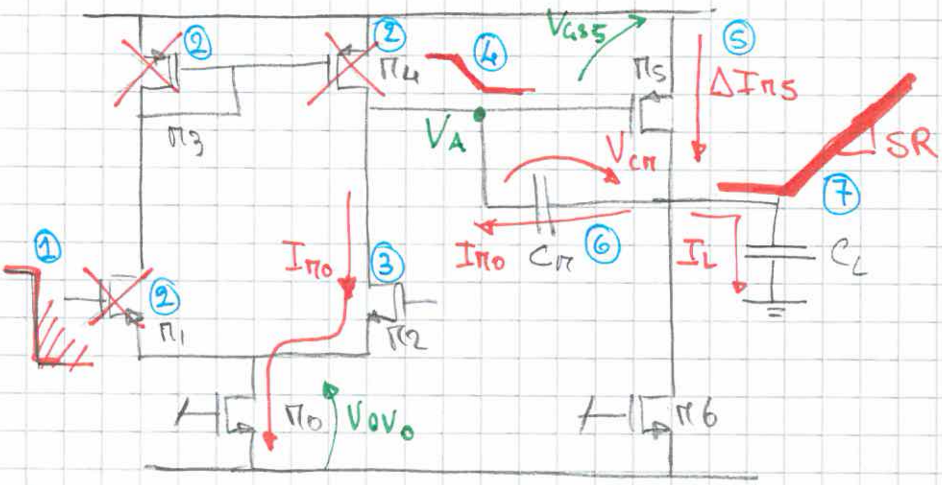
We can now compute $t_{slew} = \frac{E - \Delta V}{SR}$ and the $t_{lin}$ will set when $V_{out}(t)$ reaches 99% of input step $E$:

$$V_{lin}(t) = \cancel{E} - \Delta V e^{-\frac{t_{lin}}{\tau}} = \cancel{E} - \frac{E}{100} \longrightarrow t_{lin} = \tau \ln\left(\frac{\Delta V}{E} \cdot 100\right)$$

Total settling time will be $t_{settling} = t_{slew} + t_{lin}$

# Slew rate limitations of a two stage OTA

## Positive SR




1) A large step is applied
2) $M_1$, $M_3$, $M_4$ shut off because of ①
3) All the generator current flows through ③ (suppose $M_0$ is still in saturation
4) The current $I_{M0}$ is drained from node $V_A$ → $V_A$ decreases in voltage
5) (Don't consider $C_L$ for the moment) → neglect the presence of $C_L$. $V_A$ will generate a $\Delta I_5$ current. A stable condition would be when $M_5$ takes care of all the $I_{M0}$. This translates on its node by:

$$V_{ov_5}' = \sqrt{\frac{I_{5bias} + I_{M0}}{K_5}}$$

Note: $\Delta I_5 = I_{M0}$ but $I_5\big|_{TOT} = I_{5bias} + \Delta I_5$

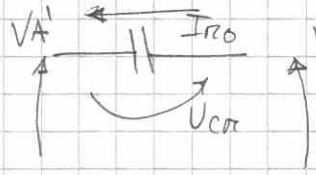
Suppose $K = 100\mu$ $\quad I_{M0} = 5\mu$ $\quad I_{5bias} = 1\mu$ → $V_{ov_5} = 0{,}25V$

Since $V_A\big|_{bias} \simeq \frac{V_{DD}}{2} = 1{,}5V$ → $V_A' = 1{,}25V$

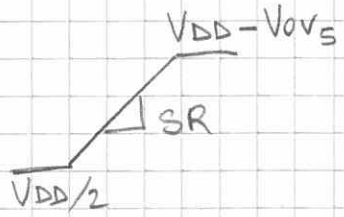
This decrease poses no issue on the saturation of $M_0$, $M_2$.
Note: if $M_2$ goes ohmic, $I_{M0}$ will still flow through node A. $V_A'$ would have to decrease all the way down to $V_{ov_{M0}}$ in order to change $I_{M0}$ current. It does not happen

6) With point 5 $C_M$ sees a constant ($V_A$) voltage on the left and a constant current $I_{M0}$. This will generate a ramp on its right pin that is $SR_{INT} = \frac{I_{M0}}{C_M}$

Situation is now this one:




Vout ⟶ $V_{OUT} = SR_{INT} \cdot t$




Since $V_A'$ is constant, the voltage ramp on $C_\pi$ defined as $SR_{INT}$ will be the same exact ramp on $V_{OUT}$. Therefore, if we define $SR_{EXT}$ as the ramp exhibited on the out, we can say $SR_{EXT} = SR_{INT}$

7) Now include $C_L$ presence. $V_{ov5}$ will increase more and $V_A'$ will decrease even more. At point 5 we will have a new (stable) condition that is $\Delta I_5 = I_{RO} + I_L$

$V_{ov5} = \sqrt{\dfrac{I_{B5} + I_{RO} + I_L}{K_5}}$ The issue here will be when $C_L$ is large enough to generate a too high $V_{ov5}$ that puts $M_O$ out of saturation.

If this does not happen, $M_5$ can perfectly handle $I_{RO}$ and $I_L$, therefore the voltage ramp will be set again by $C_\pi$, leading to $SR_{EXT} = \dfrac{I_{RO}}{C_\pi}$

The only thing is that we now have $I_L = SR_{EXT} \cdot C_L$ so

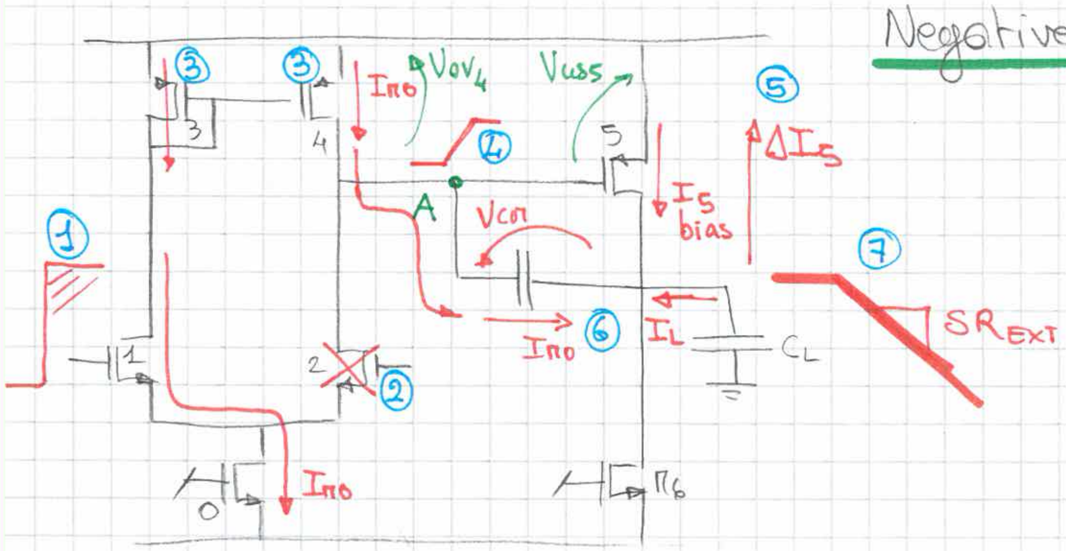
$$\Delta I_5 = I_{RO} + SR_{EXT}\, C_L \underset{SR_E = SR}{=} I_{RO} + SR_{INT}\, C_L$$

Recap:

$$SR_{EXT}\Big|_{\text{WITHOUT } C_L} = SR_{INT} = \frac{I_{RO}}{C_\pi}$$

$$SR_{EXT}\Big|_{\text{WITH } C_L} = SR_{INT} = \frac{I_{RO}}{C_\pi}$$ but keep an eye on $M_O$ saturation

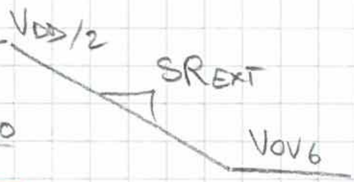
# Negative SR




1) large positive is applied
2) $M_2$ shuts off and all $I_{M0}$ flows through $M_1$
3) $I_{M0}$ gets mirrored on $M_4$
4) $I_{M0}$ flows into node (A) $\rightarrow$ $V_A$ increases leading to a decrease on $V_{GS5}$
5) $V_{GS5}$ $\rightarrow$ $V_{ov_5}$ decreases so it can accept $\Delta I_5$.

Suppose now that $I_{5bias} > \Delta I_5$ (Neglect $C_L$ presence)

6) since $I_{5bias} > I_{M0}$, $M_5$ can handle $I_{M0}$ and $I_{M0}$ can flow through $C_M$
7) Same as before $SR_{EXT} = SR_{INT} = \dfrac{I_{M0}}{C_M}$



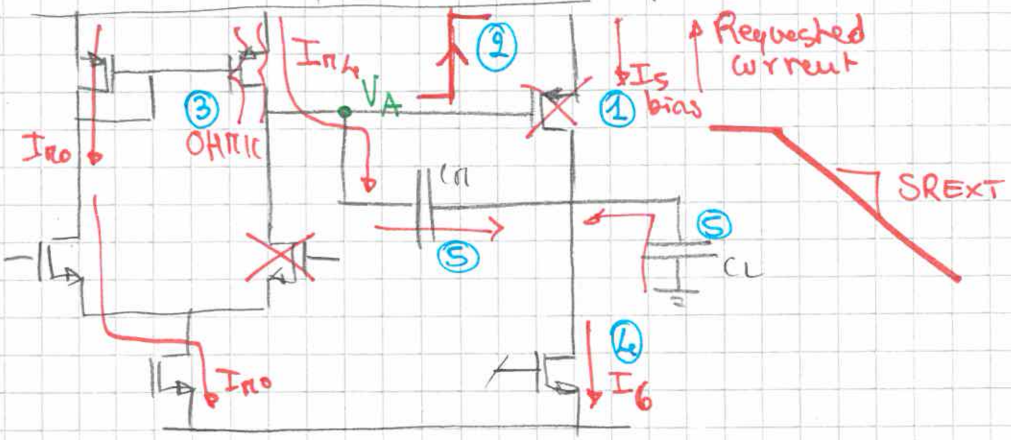

Now consider $C_L$ presence $\rightarrow$ $\Delta I_5 = I_{M0} + I_L$ where

$$I_L = SR_{EXT}\, C_L = SR_{INT}\, C_L = \frac{I_{M0}\, C_L}{C_M}$$

If $I_{5bias} > I_{M0} + C_L = I_{M0}\left(1 + \dfrac{C_L}{C_M}\right)$ then $M_5$ can handle the additional current and nothing changes

$$SR_{EXT} = SR_{INT} = \frac{I_{M0}}{C_M} \quad \text{but} \quad \Delta I_5 = I_{M0} + \frac{I_{M0}}{C_M} C_L$$

Now let's analyze what happens with $I_{Sbias} < I_{M0} + I_L$ :



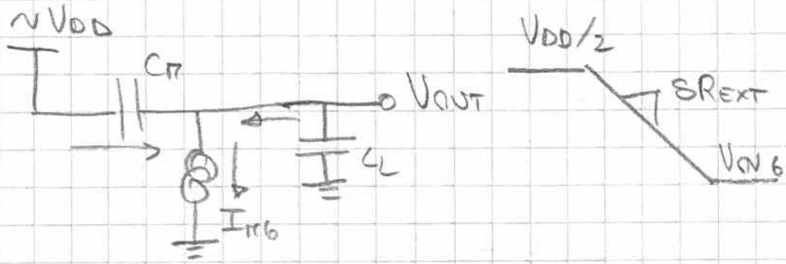

1) Since the requested current $I_{M0}\left(1+\frac{C_L}{C_M}\right) > I_{Sbias}$ is opposite and greater than $I_{Sbias}$, $M_5$ shuts off

2) The $M_5$ shut-off translates to a rapid reduction of $V_{GS5}$ → $V_A$ rises to $V_{DD}$

3) Since $V_A \nearrow\nearrow$ → $V_{DS4} < V_{OV4}$ therefore $M_4$ goes ohmic and its current $I_{M4}$ won't be $I_{M0}$ anymore

4) The only current source remaining is $M_6$

5) The situation is now:




$C_M$ and $C_L$ have one end connected to a fixed voltage and the other tied to $V_{out}$ → $I_{M6}$ sees them "in parallel" this means that out ramp will be set at:

$$SR_{EXT} = \frac{I_6}{C_M + C_L}$$

Note: if $C_L$ wasn't connected $SR_{EXT} = SR_{INT} = \frac{I_6}{C_M}$

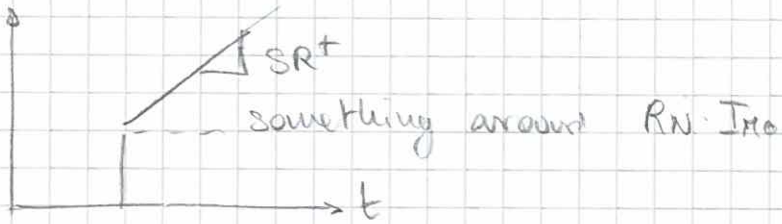
so $I_{M4} = I_6$
→ OHMIC

Recap:

$$\begin{cases} SR^+_{EXT} = SR_{INT} = I_{MO}/C_M \\ SR^-_{EXT} = \begin{cases} \to SR_{INT} = I_{MO}/C_M \\ \to \dfrac{I_6}{C_M + C_L} \quad \text{if} \quad I_5 < I_{MO}\left(1 + \dfrac{C_L}{C_M}\right) \end{cases} \end{cases}$$

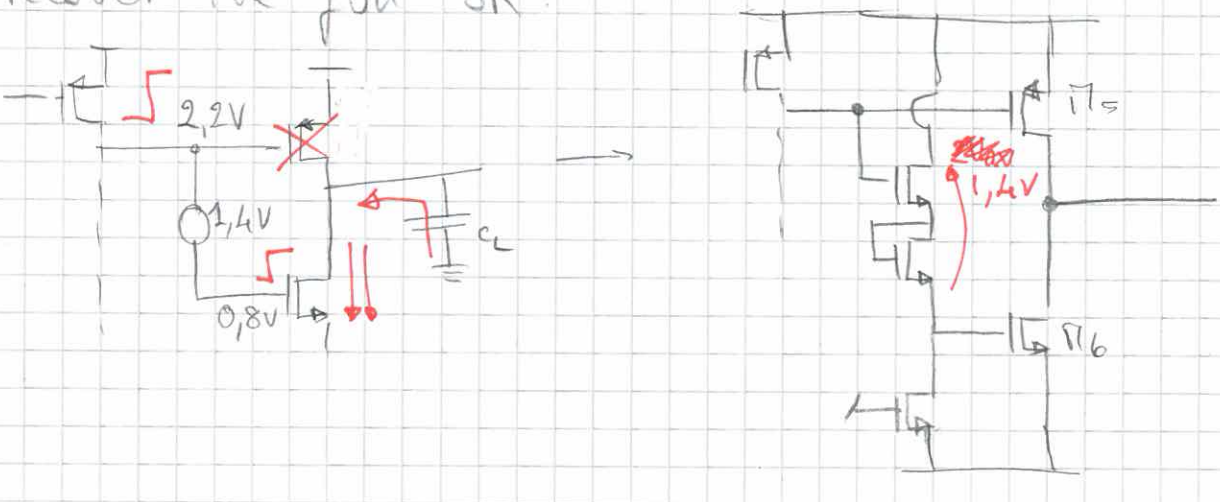
## SR on a two stage compensated with $R_N$

Nothing changes except $R_N$ will show an immediate voltage step on $I_{MO}$:


## Class AB stage

On $SR^-$ we need to make sure that $I_5 > I_{MO}\left(1 + \frac{C_L}{C_M}\right)$ thus burning more power. The issue is that only $M_5$ can move. If we linked $M_6$ on the negative step only, we would recover the full SR:

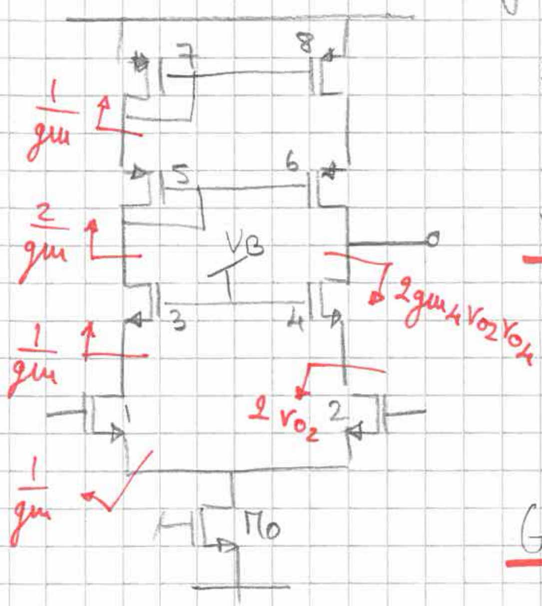



Therefore $SR^-_{EXT} = SR_{INT} \to$ FoM improved because of the lower output current needed

# 11) Telescopic + folded cascode amplifiers

High gain ⟶ 2 stage amp adopted ⟶ 2 high impedance node that are critical to frequency response.

We can use a single stage:




By using cascode stages, we can increase the output impedance

$$R_{OUT} = g_{m8} r_{o8} r_{o6} \parallel R_X \qquad R_X = \frac{2 g_{m4} r_{o4} r_{o2}}{1 + \frac{1}{1}}$$

$G_{loop} = -1$ as always, so

If $r_{o2} = r_{o4} = r_{o8} = r_{o6}$, $g_{m8} = g_{m4} = g_m$

$$G_D = g_{m1} \frac{g_m r_o^2}{2}$$ ⤳ we increased

the usual $g_{m1} r_o$ of a factor $\mu$.

The main limit to frequency response is $C_L$ (other $C_{gs}$ will show poles at $\sim f_T$ so they're way higher)

$$GBWP = g_{m1} \cancel{\frac{g_m r_o^2}{2}} \frac{1}{2\pi C_L \cdot \cancel{\frac{g_m r_o^2}{2}}} = \frac{g_{m1}}{2\pi C_L}$$

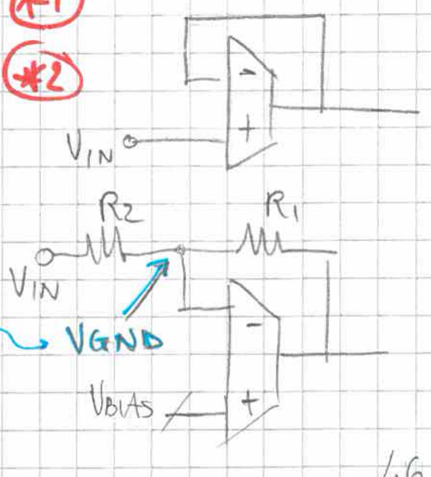
## Voltage dynamics:

$$V_{CM,IN}^{+} = V_B - V_{GS3} + |V_T| \qquad V_{CM,IN}^{-} = V_{GS1} + V_{OV0}$$

$$V_{OUT}^{+} = V_{GS7} + V_{OV6} \qquad V_{OUT}^{-} = V_B - |V_T|$$

$V_B$ serves as reference to set max $V_{CM,IN}$ and min $V_{OUT}$.

- If $V_B$ is high ⟶ $\Delta V_{OUT} \downarrow$ but $\Delta V_{IN_{CM}} \uparrow$ (*1)
- If $V_B$ is low ⟶ $\Delta V_{OUT} \uparrow$ but $\Delta V_{IN_{CM}} \downarrow$ (*2)

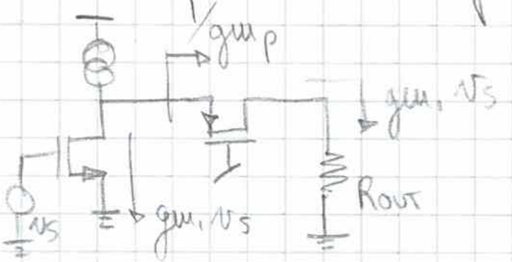



(*1) useful for OTAs used as buffers (large input swing needed)

(*2) Useful for OTAs with virtual ground (input swing is kept as low as possible)

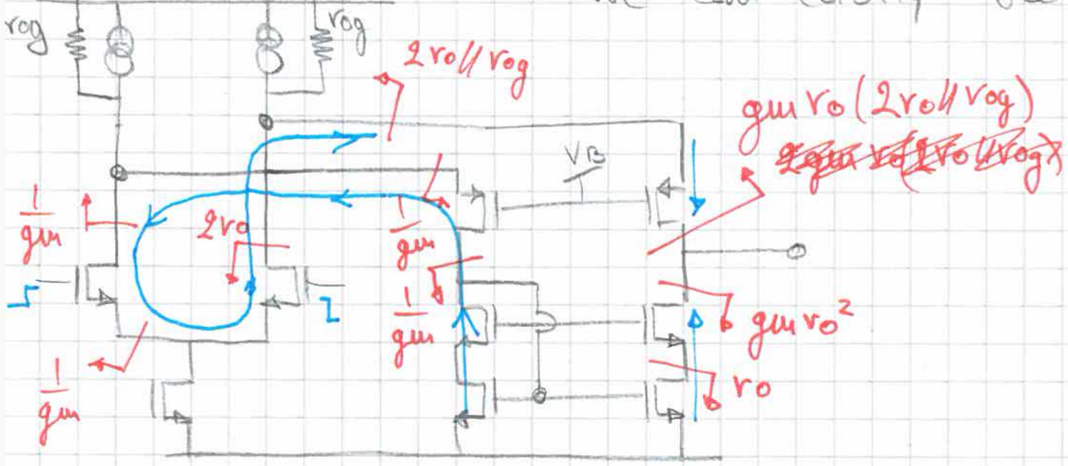
# Folded cascode structure

Piling up transistors can't be done with low $V_{DD}$ ($V_{DD}$ decreases with technology scaling), this means we need to change the structure: use pMOS cascodes:



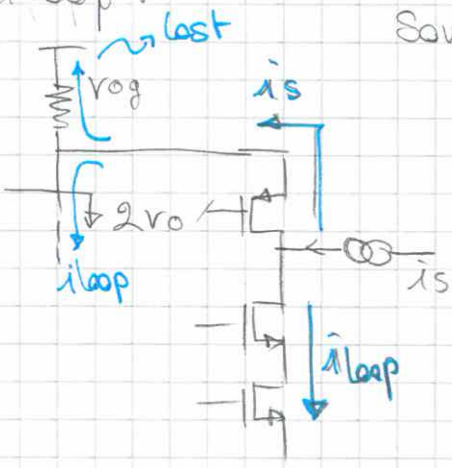

↳ This is the concept we use when we talk about folding

We can easily see that $i_{cc} = g_{m1} v_D$




$$R_{OUT} = R_{DOWN} // \frac{R_{UP}^{(0)}}{1-G_{loop}} \qquad R_{DOWN} = g_m r_o^2 \qquad R_{UP} = g_m r_o (2r_o // r_{og})$$

Gloop:




Some current will be lost on $r_{og}$:

$$G_{loop} = -\frac{r_{og}}{r_{og} + 2r_o}$$

If $r_o \simeq r_{og} \rightarrow G_{loop} = -\frac{1}{3}$

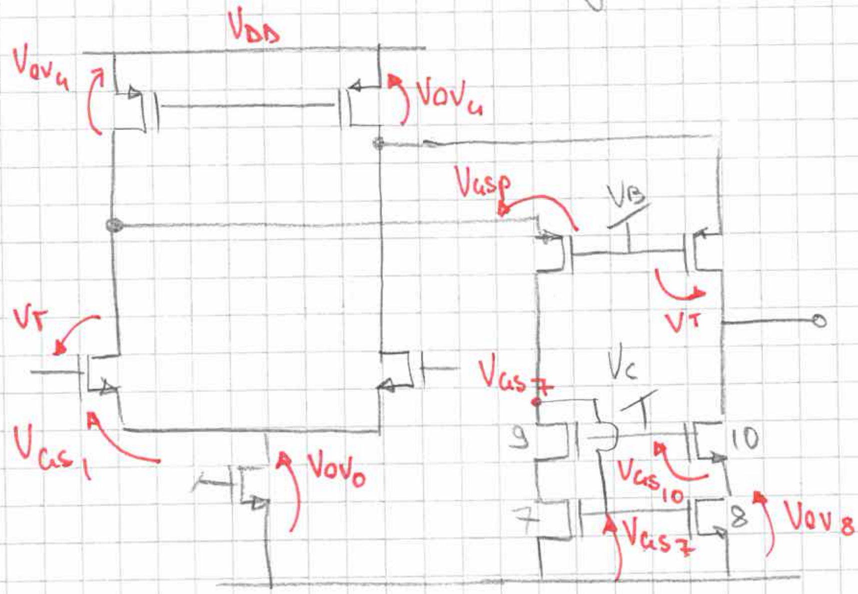
$$R_{up} = \frac{g_m r_o \cdot \frac{2}{3} r_o}{1 + \frac{1}{3}} = g_m \frac{r_o^2}{2}$$

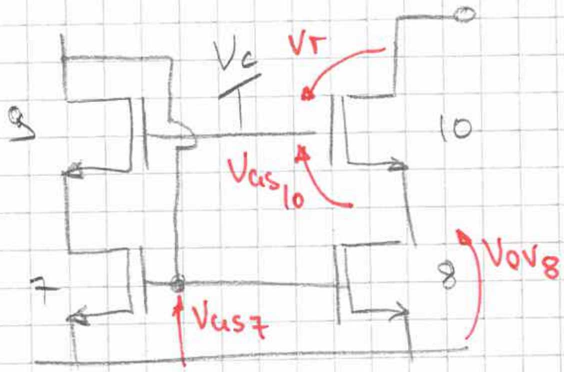
Therefore $R_{OUT} = g_m \frac{r_o^2}{3}$ comparable value with a telescopic OTA

$R_{OUT}|_{FOLDED}$ can be matched to a $R_{OUT}|_{TELE}$ by changing MOSFETS LENGTH

# Folded cascode dynamics

Let's see how the cascode improved voltage dynamics (issue with $V_{DD}$ scaling)




$V_{CM_{IN}}^{+} = V_{DD} - V_{ov_4} + |V_T|$ or $V_B + |V_{GSP}| + |V_T|$ the more stringent of the two ($V_{B\,max} = V_{DD} - V_{ov_4} - |V_{GSP}|$). If $V_B = V_{B\,MAX}$ the two conditions are the same

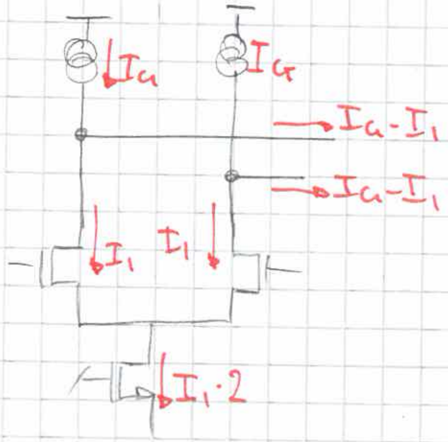
$V_{CM_{IN}}^{-} = V_{GS_1} + V_{ov_0}$

$V_{out}$, without the enhanced mirror, would be limited to $V_{GS_7} + V_{ov_{10}}$ but here we can choose $V_C = V_{ov_8} + V_{GS_{10}}$ so that:

$V_{OUT}^{-} = V_C - V_T$ or $V_{ov_{10}} + V_{ov_8}$ (check and see that the two conditions are the same!)

$V_{OUT}^{+} = V_B + |V_T|$

## Drawback of folded cascode: current consumption




Condition to have a working circuit is $I_G > 2I_1$ because on bias the two branches will see $I_G - I_1$, while on large steps (SR):

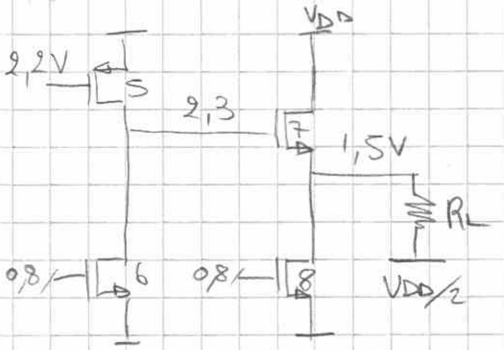
- One branch will have $I_G$
- The other will have $I_G - 2I_1$ (*)

Since with $I_G < 2I_1$ (*) can push MOSFETS in ohmic region, we need to burn more power.

This means we need to burn at least twice the current of a equal telescopic OTA

# 12) Opamp output stages: Class A, Class B, Efficiency + HD

We need to decouple OTA's high out impedance in order to drive resistive loads → Design a buffer stage




Set $V_{ov} = 0{,}2$ for all mosfet.
Previously $V_{G7}$ was set to 1,5V but now we need 1,5V on $V_{OUT}$ so that $I_{R_L}\big|_{bias} = 0$

$$V_{G7} = V_{OUT} + V_{GS7} = 1{,}5 + 0{,}8 = 2{,}3\,V$$

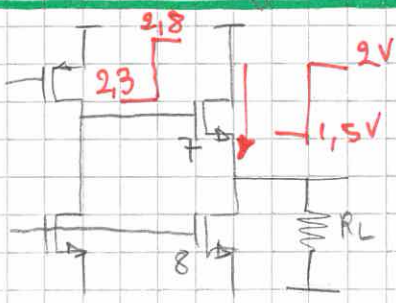
$$\frac{1}{2}\mu_p' C_{ox}' \frac{W_5}{L_5}\left(1 + \frac{V_{DS5} - V_{DSSAT5}}{V_{A5}}\right) = \frac{1}{2}\mu_n' C_{ox}' \frac{W_6}{L_6}\left(1 + \frac{V_{DS6} - V_{DSSAT6}}{V_{A6}}\right)$$

$V_{DS5} = 3V - 2{,}3$ $\quad V_{DSSAT5} = 0{,}2V$ $\quad V_{DS6} = 2{,}3V$ $\quad V_{DSSAT6} = 0{,}2V$

$L_5, L_6$ are set because of differential gain: $L_5 = L_6$

So $$W_6 = W_5 \left(\frac{1 + \frac{0{,}5}{V_{A5}}}{1 + \frac{2{,}1}{V_{A6}}}\right)$$

## Positive/negative swings




Positive swing: $V_{G7}$ can go up to 2,8V ($V_{DD} - V_{ov5}$) → $V_{OUT}$ goes up to 2V if $Gain\big|_{BUF} = 1$ → ideal

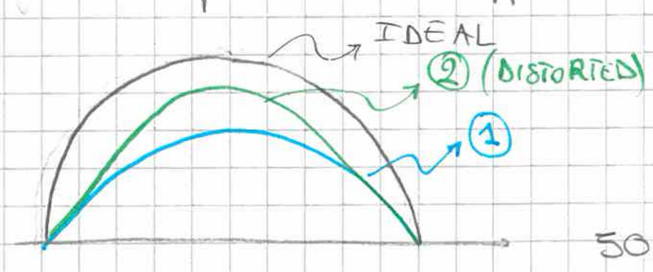
This does not happen for two reasons:

1) • $$G_{BUF} = \frac{R_L // r_{o8}}{R_L // r_{o8} + 1/g_{m7}} \sim \frac{R_L}{R_L + \frac{1}{g_{m7}}} < 1$$

2) • $\frac{1}{g_{m7}}$ is not linear, when $V_{GS}\nearrow\nearrow$, $\frac{1}{g_{m7}} \searrow\searrow$ leading to an output distortion of (distortion of a MOS buffer):

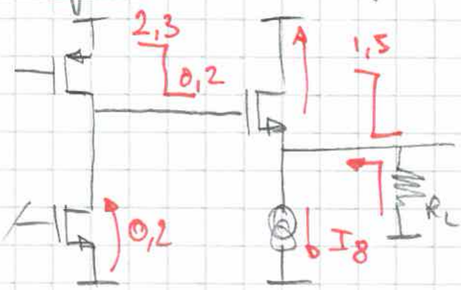
$$HD_2 = \frac{V_{GS}}{2V_{ov}} \frac{1}{1 + g_m R_L}$$

(Not explained in this course!!)



On the positive swing, peak current $I_S = I_8 + \frac{\Delta V_{OUT}}{R_L}$
where $\Delta V_{OUT} = 5V$ and $R_L$ (arbitrary) $= 500\,\Omega$

## Negative swing




Even though $V_{G7} \to V_{OU6}$, $M_7$ will move towards off state. If $I_8 = I_7 = 0,5\,mA$, when $M_7$ shuts off there's only $I_8$ providing current →

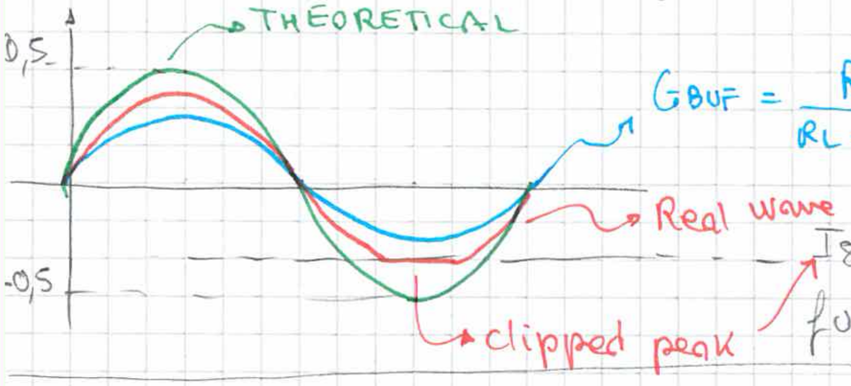
$\Delta V_{OUT}^- = I_8 \cdot R_L = 0,5\,mA \cdot 500\,\Omega = 0,25V \to V_{OUT} = 1,5 - 0,25$

Therefore $V_{G7} = V_{OUT} + V_T = 1,25 + 0,6 = 1,85V$

Since on the positive side we would have 0,5V theoretical swing, we would like to also have that on the negative →

$\Delta V_{OUT}^- = I_8 R_L = 0,5V \to I_8 = \frac{0,5V}{500\,\Omega} = 1mA$

$I_7 = I_8 = 1mA$ current doubled to guarantee 1Vpp swing:



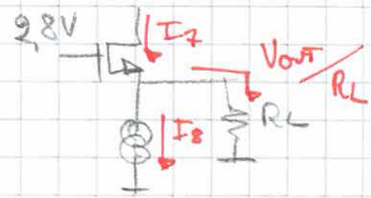

$G_{BUF} = \frac{R_L}{R_L + 1/g_{m7}}$

$I_8 R_L$ if $I_8$ can't guarantee full swing

To verify the peaks: consider max positive peak:

$$I_7\big|_{MAX} = K_7 \left( V_{G7} - \frac{V_{DD}}{2} - \Delta V - V_T \right)^2 = \frac{\Delta V}{R_L} + I_8$$

$K_7 = 12,5\ \frac{mA}{V^2}$, $V_{G7} = 2,8$, $\frac{V_{DD}}{2} = 1,5$, $V_T = 0,6$, $R_L = 500\,\Omega$, $I_8 = 1mA$

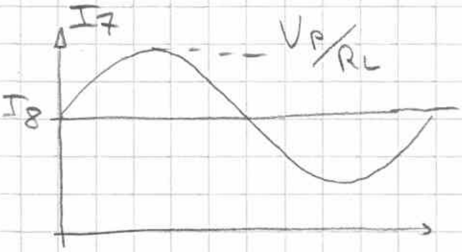



$\Delta V = 0,33V$ ⇝ definitely lower than 0,5 (ideal)

Note: formula includes a squared variable

$M_7$ is in large signal operation → we can't consider it as a linear stage anymore. Positive peaks will be slightly higher and negative peaks smaller

## Class A buffer power efficiency




$$\eta = \frac{P_{Load}}{P_{DISSIPATED}}$$ consider a sinusoidal waveform

$$\eta = \frac{\frac{V_P^2}{2R_L}}{V_{DD}\cdot I_8} = \frac{V_P^2}{2R_L V_{DD} I_8}$$ → estimate this

Since $V_P < \frac{V_{DD}}{2}$ at most and we know that $I_8 R_L = V_P$

$$\eta = \frac{V_P^{\cancel{2}}}{2 V_{DD}\cdot \cancel{V_P}} = \frac{V_{DD}/2}{2\cdot \cancel{V_{DD}}} = \frac{1}{4}$$ max theoretical out efficiency

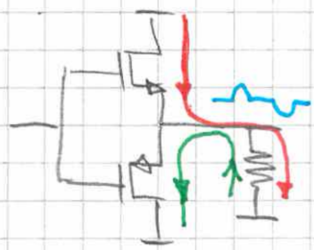
25 % is very low because of the continuous current ($I_7, I_8$) consumption even when no signal is applied

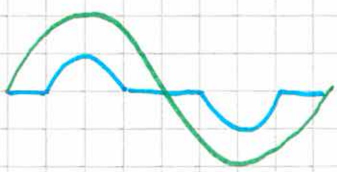
## Class B stages - push - pull

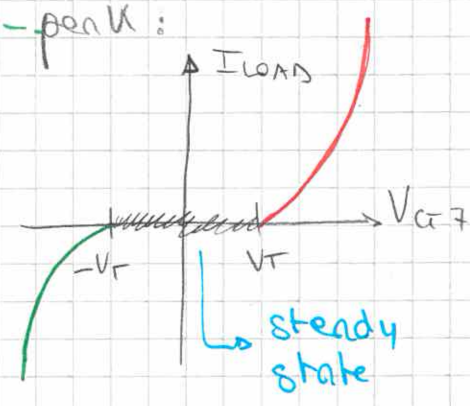
Concept: deliver current only when needed → remove $I_8$


→ M7 can deliver current only when $V_{G7} > \frac{V_{DD}}{2} + V_T$ ] → heavy distortion


Solution: take care of the negative - peak:








We can already see that out wave is symmetric → no HD2, only third harmonics

## Check distortion


$i_{OUT} = i_7 - i_8$

$i_7 = A_0 + A_1 \sin(\omega_0 t + \varphi_1) + A_2 \sin(2\omega_0 t + \varphi_2) + A_3 (3\omega_0 t + \varphi_3)$

$i_8 = i_7\left(t - \frac{T}{2}\right) \longrightarrow i_8$ is the shifted version of $i_7$ by half a period :

$i_8 = A_0 + A_1 \sin\left(\omega_0\left(t - \frac{T}{2} + \varphi_1\right) + A_2 \sin\left(2\omega_0\left(t - \frac{T}{2}\right)\varphi_2\right) + \ldots\right.$

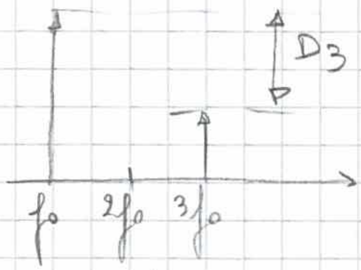
We can see that $\omega_0 \frac{T}{2} = \frac{2\pi}{T} \cdot \frac{T}{2} = \pi \qquad 2\omega_0 \frac{T}{2} = 2\pi \ldots$ so

$i_8 = A_0 - A_1 \sin(\omega_0 t + \varphi_1) + A_2 \sin(2\omega_0 t + \varphi_2) - A_3 \sin(\omega_0 t + \varphi_3)$

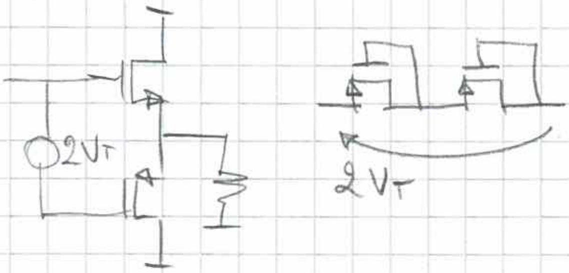
Since $i_{OUT} = i_7 - i_8 =$ even terms cancel out $=$

$i_{OUT} = 2A_1 \sin(\omega_0 t + \varphi_1) + 2A_3 \sin(3\omega_0 t + \varphi_3)$

No HD2 if $M_7, M_8$ have exact voltage-current curves



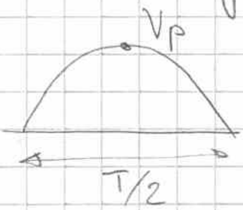

Distortion is produced by the dead zone. To eliminate it, use a voltage shifter of $2V_T \longrightarrow$ Class AB




$\hookrightarrow$ set transdiodes $V_{GS}$ in order to have $2V_T$ shift

# Class B efficiency

$P_L = \frac{V_p^2}{2R_L}$ as always $\quad P_{DISS}^{+} = \frac{\overset{\to V_{REF}}{V_{DD}}}{2} \cdot \overline{I_L}$ where $\overline{I_L}$ is the average dissipated current over a period




Neglect signal distortion $\quad \omega = \frac{2\pi}{T}$

$$\overline{I_L}\Big|_{\frac{T}{2}} = \frac{T}{2\pi} \cdot \frac{2}{T} \cdot \frac{V_p}{R_L} \int_0^{T/2} \sin(\omega t)\, dt \cdot \frac{2\pi}{T} =$$

$$= \frac{1}{\pi} \frac{V_p}{R_L} \int_0^{\pi} \sin\theta\, d\theta = \frac{2}{\pi} \frac{V_p}{R_L} \longrightarrow P_{DISS}^{+} = \frac{V_{DD}}{\cancel{2}} \cdot \frac{\cancel{2}}{\pi} \frac{V_p}{R_L}$$

Same happens for $P_{DISS}^{-}$, so $\overline{I_L}$ is the same for the entire period.

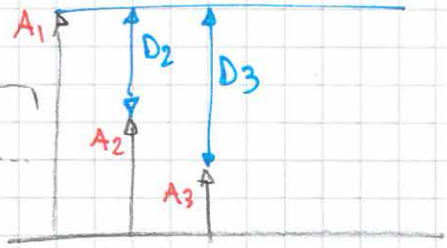
$$\eta = \frac{P_L}{P_{DISS}} = \frac{\frac{V_p^2}{2R_L}}{\frac{2}{\pi} \cdot \frac{V_{DD}}{2} \cdot \frac{V_p}{R_L}} = \frac{\pi}{4} \frac{2V_p^{\cancel{2}}}{\cancel{V_p} V_{DD}} = \frac{\pi}{4} \frac{2V_p}{V_{DD}}$$

We set $V_p < \frac{V_{DD}}{2}$ therefore $\eta_{MAX} = \frac{\pi}{4} \hat{=} 78\%$

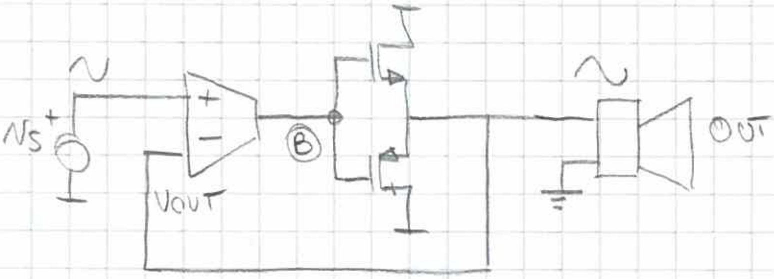
We improved efficiency by a lot

# 13) Distortion and feedback

$$THD = \sqrt{\frac{A_2^2}{A_1^2} + \frac{A_3^2}{A_1^2} + \dots} = \sqrt{D_2^2 + D_3^2 + \dots}$$

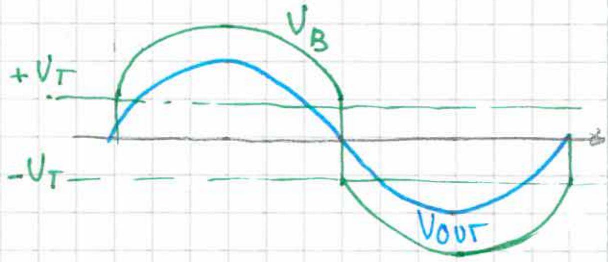



Loop can reduce distortion:

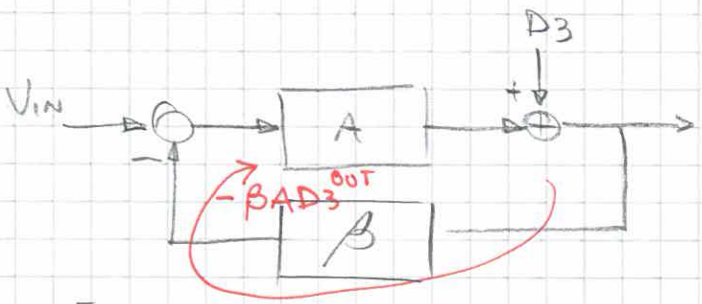



Suppose $G_{loop} = -\infty$.
With this configuration
$V_{OUT} = V_{IN}$

The only way to have $V_{OUT} = V_{IN}$ is that on $V_B$ the voltage is pre distorted:




This can be also seen through feedback theory by adding a distortion:

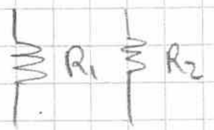



$D_3^{OUT}$ is the distorted out signal with a $D_3$ disturb

$$D_3^{OUT} = D_3 - \beta A D_3^{OUT} \longrightarrow D_3^{OUT} = D_3 \frac{1}{1 + A\beta}$$

We see that out distortion is reduced by $A\beta$ and on input there will be a $-\beta A D_3^{OUT}$ signal (pre-distort)

## (4) Variability and matching of resistors → Pelgrom




Suppose $R_1 = R_2 = R$. In reality, processes will have a variability on these values


$$R = \rho \cdot \frac{L}{\Delta \cdot W} = \frac{\rho}{\Delta} \cdot \frac{L}{W} = R_{\square} \cdot \frac{L}{W} \qquad \frac{\Delta R}{R} = \frac{\Delta R_{\square}}{R_{\square}} + \underbrace{\frac{\Delta L}{L} + \frac{\Delta W}{W}}$$

→ worst case

process is precise = Negligible contribution

$R_{\square}$ = unit square resistance, fixed by technology

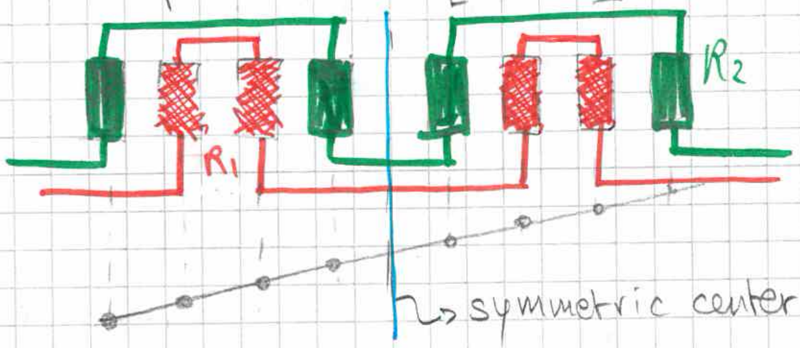
$R_{\square}$ = 110 Ω unsilicided n+ polysilicon

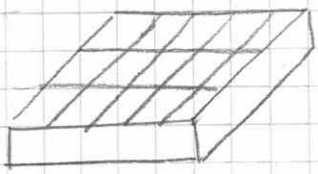
= 10 Ω silicided (doped with metals) n+ polysilicon

Along $R_{\square}$, manufacturers will give a %·μm coefficient used in the variability formula:

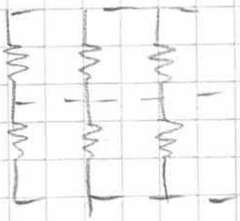
$$\frac{\sigma_{\Delta R}}{R} = \frac{K_{\Delta R/R}}{\sqrt{WL}} \rightsquigarrow \text{statistical spread}, \quad K = [\% \cdot \mu m]$$

Deterministic spread can be reduced

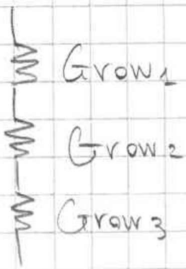


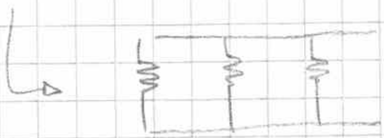
Each $R_{\square}$ has a different value (statistical)

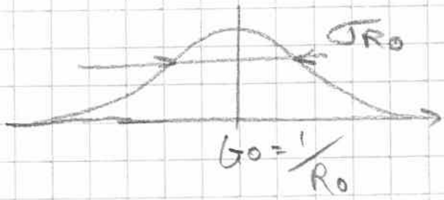


We can divide the resistor in rows and colums of single resistors. $G_0 = 1/R_0$ ↳ average resistance




$$G_{row} = \sum G_i = N\, G_0$$

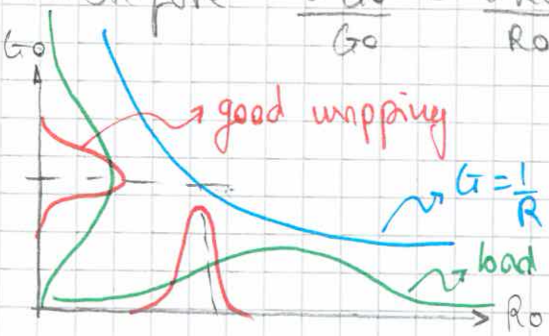





$\sigma^2_{G_{row}} = N\, \sigma^2_{G_{r0}}$ → Now we want to link the conductance σ to resistance σ → relation is not linear because $G = \frac{1}{R}$

Linearize $dG_0 = -\frac{dR_0}{R_0^2}$ $\quad \frac{dG_0}{G_0} = -\frac{dR_0}{R_0^2} R_0 = -\frac{dR_0}{R_0}$ ← *1

Therefore $\frac{\sigma^2_{G_0}}{G_0} = \frac{\sigma^2_{R_0}}{R_0}$ → be careful! This holds for small $\sigma^2$, large $\sigma^2$ will be influenced by the $1/R$ relation




Now that we have computed the variability for a row, we can analyze the columns

$$R_T = \sum_M R_r = M R_r = \frac{M}{N} R_0 \quad \text{↝ intuitive}$$

$R_T$ ↳ TOTAL, $R_r$ ↳ ROW

We know that $\sigma^2_{R_T} = M\sigma^2_{R_r} \rightarrow \frac{\sigma^2_{R_T}}{R_T^2} = \frac{M\sigma^2_{R_r}}{M^2 R_r^2} = \frac{1}{M}\frac{\sigma^2_{R_r}}{R_r^2} \overset{*1}{=}$

$$\frac{1}{M}\frac{\sigma^2_{G_r}}{G_r^2} = \frac{1}{M}\frac{N\sigma^2_{G_0}}{N^2 G_0^2} = \frac{1}{MN}\frac{\sigma^2_{G_0}}{G_0^2} = \frac{1}{MN}\frac{\sigma^2_{R_0}}{R_0^2}$$

small relative spread

So $$\frac{\sigma^2_{R_T}}{R_T^2} = \frac{\sigma^2_{R_0}}{R_0^2}\cdot\frac{1}{MN}$$

Now, suppose that each square has length $\Lambda$, then $N = \frac{L}{\Lambda}$ $\quad M = \frac{W}{\Lambda}$ we can express the relative spread

$$\frac{\sigma^2_{R_T}}{R_T^2} = \underbrace{\frac{\Lambda^2}{WL}\frac{\sigma^2_{R_0}}{R_0^2}}_{K^2} = \frac{K^2}{WL} \qquad \frac{\Delta R_T}{R_T} = \sqrt{\frac{\sigma^2_{R_T}}{R_T^2}} = \frac{K_R}{\sqrt{WL}}$$

Apply this to $R_1, R_2$ : $\Delta R = R_1 - R_2$ $\quad \frac{\sigma_{R_1}}{R_1} = \frac{K}{\sqrt{WL_1}}$ $\quad \frac{\sigma_{R_2}}{R_2} = \frac{K}{\sqrt{WL_2}}$

Suppose $WL_1 = WL_2$ then

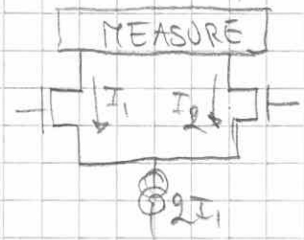
$$\sigma^2(\Delta R) = \sigma^2_{R_1} + \sigma^2_{R_2} = 2\sigma^2_{R_1} = 2\frac{K^2}{WL}R_1^2$$

So $$\frac{\Delta R}{R} = \sqrt{\frac{\sigma^2_{\Delta R}}{R^2}} = \sqrt{2}\frac{K_R}{\sqrt{WL}}$$

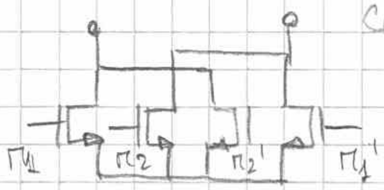
The $K_{\frac{\Delta R}{R}}$ the manufacturer gives us is exactly

$$K_{\frac{\Delta R}{R}} = \sqrt{2}K \quad [\% \,\mu m]$$

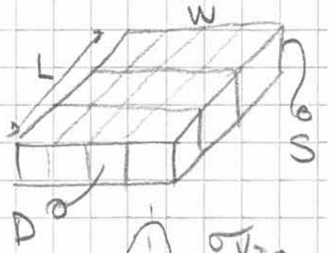
## 15) Variability and matching: $V_T$ relative matching



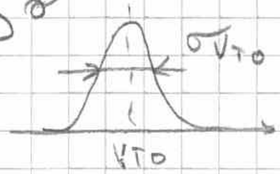

$I_1 \neq I_2$ always because of mosfet $V_T$ variability. We can change the deterministic contribution by using the common centroid technique $\rightarrow$ layout issue




$\rightarrow$ We lay out MOSFets in // on a center axis




Suppose we divide the mosfet in square cells
Each cell has a $V_{T0}$ square spread plus a $\sigma_{V_{T0}}$ contribution (statistical)




$$\overline{V_T} = \frac{\sum V_{Ti}}{N} = \frac{N V_{T0}}{N} = V_{T0}$$

Since $\sigma^2(\sum_N x) = N\sigma_x^2$ and $\sigma^2\left(\frac{x}{b}\right) = \frac{\sigma_x^2}{b^2}$

We can say that $\sigma^2_{\overline{V_T}} = \frac{\sigma^2(\sum_N V_{Ti})}{N^2} = \frac{N\sigma^2_{V_{T0}}}{N^2} = \frac{\sigma^2_{V_{T0}}}{N}$

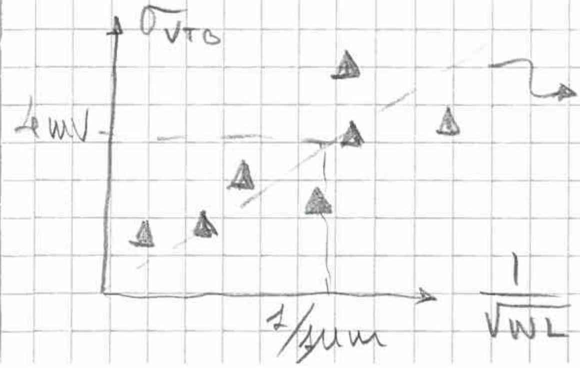
Let's say $A_0 = \frac{WL}{N}$ area of a single cell, then

$$\sigma^2(\overline{V_T}) = \frac{\sigma^2_{V_{T0}} A_0}{WL} \simeq \frac{K^2_{V_T}}{KL} \rightarrow \underline{\sigma_{V_T}} = \underline{\frac{K_{V_T}}{\sqrt{WL}}}$$

Since we're interested on the mismatch between $M_1$, $M_2$ we have:

- mean square $E(\Delta V_T) = V_{T1} - V_{T2} = V_{T0} - V_{T0} = 0$
- $\sigma^2(\Delta V_T) = \sigma^2_{V_{T1}} + \sigma^2_{V_{T2}} = 2\sigma^2_{V_{T0}}$

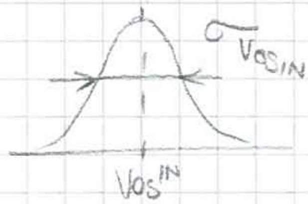
Therefore $\sigma^2(\Delta V_T) = 2\frac{K^2}{WL} \rightarrow \boxed{\sigma_{\Delta V_T} = \sqrt{2}\frac{K_{\Delta V_T / V_T}}{\sqrt{WL}}}$



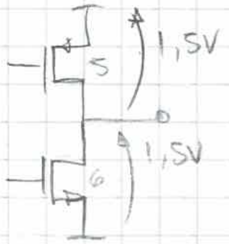

$\rightarrow$ slope set by manufacturing process K

## 16) Offset: deterministic + statistical contribution

We can always model an output offset to an input referred one $V_{OS}^{IN} = \frac{V_{OS}^{OUT}}{A_d}$ differential gain




### Deterministic offset



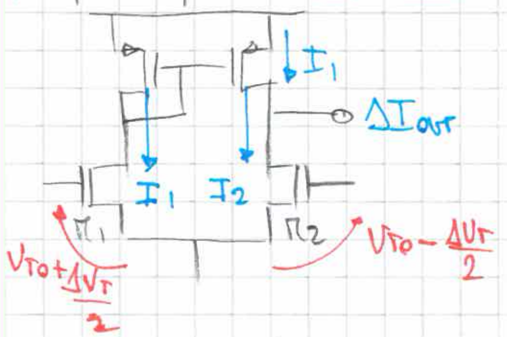

$M_5$ and $M_6$ should be properly sized to have the same $V_{DS}$, thus the proper $\frac{V_{DD}}{2}$ bias at the output $\rightarrow$ NO $V_{OS}$

### Statistical offset

This kind of offset is introduced by the following:

- Input pair $\rightarrow$ $V_T$ mismatch + $K$ mismatch
- Diff mirror $\rightarrow$ $V_T$ mismatch + $K$ mismatch

### Input pair statistical $V_T$ mismatch




Consider a symmetrical mismatch $\pm \frac{\Delta V_T}{2}$ between $M_1, M_2$. Then currents will be;

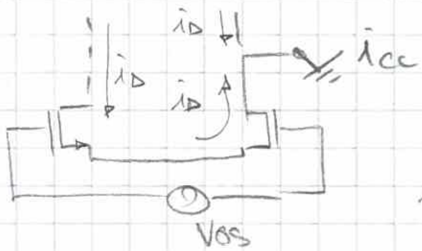
$$I_1 = K_{IN}\left(V_{GS_1} - V_{T0} - \frac{\Delta V_T}{2}\right)^2 \qquad I_2 = K_{IN}\left(V_{GS_2} - V_{T0} + \frac{\Delta V_T}{2}\right)^2$$

$\Delta I = I_2 - I_1$ and $V_{GS} - V_{T0} = V_{OV}$, so

$$\Delta I = K_{IN}\left(V_{OV} - \frac{\Delta V_T}{2}\right)^2 - K_{IN}\left(V_{OV} + \frac{\Delta V_T}{2}\right)^2 =$$

$$= K_{IN}\left(V_{OV}^2 - V_{OV}\Delta V_T + \frac{\Delta V_T^2}{4}\right) - K_{IN}\left(V_{OV}^2 + V_{OV}\Delta V_T + \frac{\Delta V_T}{4}\right)$$

$\Delta I = 2 K_{IN} V_{OV} \Delta V_T$ since $2 K V_{OV} = g_{m_{IN}}$:




$i_{cc} = \frac{g_{m_1}}{2} V_{OS}^{IN} \cdot 2$ then

$$\cancel{g_{m_1}} V_{OS}^{IN} = \frac{\cancel{2 K_{IN} V_{OV}} \Delta V_T}{g_{m_1}} \qquad \boxed{V_{OS}^{IN} = \Delta V_T}$$

## Input pair statistical K mismatch

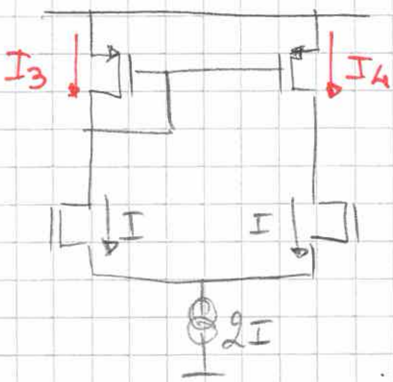
Same reasoning $I_1 = \left(K + \frac{\Delta K}{2}\right) V_{ov}^2$ $\quad I_2 = \left(K - \frac{\Delta K}{2}\right) V_{ov}^2$

$$\Delta I = I_1 - I_2 = \Delta K\, V_{ov}^2 \rightarrow g_{m1} V_{os}^{IN} = \Delta K\, V_{ov}^2$$

$$V_{os}^{IN} = \frac{\Delta K\, V_{ov}^2}{g_{m1}} \cdot \frac{K}{K} = \frac{\Delta K}{K} \frac{\overbrace{V_{ov}^2 K}^{\to I}}{\underbrace{g_{m1}}_{\to \frac{2I}{V_{ov}}}} = \frac{\Delta K}{K} \frac{I}{2I} V_{ov} = \frac{\Delta K}{K} \frac{V_{ov1}}{2}$$

so $V_{os}^{IN} = \frac{\Delta K}{K} \cdot \frac{V_{ov}}{2}$

## Mirror statistical $V_T$ mismatch




$$I_3 = K_\pi \left(V_{GS} - V_{T0} + \frac{\Delta V_T}{2}\right)^2 = K_\pi \left(V_{ov} + \frac{\Delta V_T}{2}\right)^2$$

$$I_4 = K_\pi \left(V_{ov} - \frac{\Delta V_T}{2}\right)^2$$

$$I_3 = K_\pi \left(V_{ov}^2 + V_{ov}\Delta V_T + \frac{\Delta V_T^2}{4}\right)$$

$$I_4 = K_\pi \left(V_{ov}^2 - V_{ov}\Delta V_T + \frac{\Delta V_T^2}{4}\right)$$

$$\Delta I = I_4 - I_3 = \underbrace{K_\pi \cdot 2 \cdot V_{ov}}_{\to g_m} \Delta V_T \quad \text{so}$$

$$g_{m1} V_{os}^{IN} = g_{m\pi} \Delta V_{T\pi} \rightarrow V_{os}^{IN} = \frac{g_{m\pi}}{g_{m1}} \Delta V_{T\pi}$$

$$V_{os}^{IN} = \frac{V_{ov1}}{V_{ov\pi}} \Delta V_{T\pi}$$

## Mirror statistical K mismatch

$$I_3 = \left(K_\pi - \frac{\Delta K}{2}\right) V_{ov\pi}^2 \quad I_4 = \left(K_\pi + \frac{\Delta K}{2}\right) V_{ov\pi}^2 \rightarrow \Delta I = I_4 - I_3 = \Delta K\, V_{ov\pi}^2$$

$$V_{os}^{IN} g_{m1} = \Delta K\, V_{ov\pi}^2 \rightarrow V_{os}^{IN} = \frac{\Delta K}{K} \cdot \frac{K V_{ov}^2}{g_{m1}} = \frac{\Delta K_\pi}{K_\pi} \frac{V_{ovIN}}{2}$$

$$V_{os}^{IN} = \frac{\Delta K_\pi}{K_\pi} \frac{V_{ovIN}}{2}$$

## Summary of mismatches for input referred offset

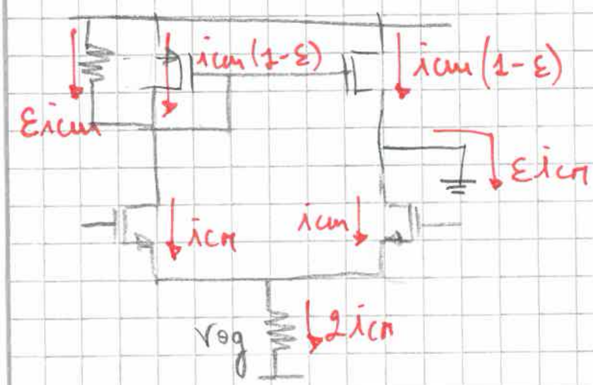
$$V_{OS}\Big|_{\substack{INPUT\\REF}} = \underbrace{\Delta V_{T\,IN} + \frac{\Delta K_{IN}}{K_{IN}}\frac{V_{OV\,IN}}{2}}_{\text{input pair}} + \underbrace{\Delta V_{TM}\frac{V_{OV\,IN}}{V_{OV\,M}} + \frac{\Delta K_M}{K_M}\frac{V_{OV\,IN}}{2}}_{\text{mirror pair}}$$

$$\sigma_{V_{OS}} = \sqrt{\sigma_{V_{T\,IN}}^2 + \sigma_{V_{TM}}^2\left(\frac{V_{OV\,1}}{V_{OV\,M}}\right)^2 + \left[\sigma^2_{\left(\frac{\Delta K_{IN}}{K_{IN}}\right)} + \sigma^2_{\left(\frac{\Delta K_M}{K_M}\right)}\right]\left(\frac{V_{OV\,IN}}{2}\right)^2}$$

## 17) CMRR: deterministic and statistical limits

CMRR deterministic contribution is purely related to the circuit topology. In our case, the only asimmetry we can find is the mirror and input pair impedance

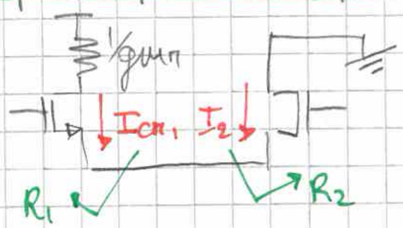
### Mirror deterministic contribution




It's easy to find that some portion of $i_{cm}$ is lost through $r_{oM}$

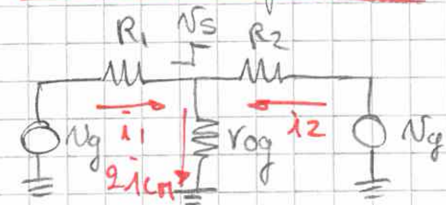
$$\varepsilon = \frac{1/g_{mM}}{\frac{1}{g_{mM}} + r_{oM}} = \frac{1}{1 + g_{mM} r_{oM}}$$

### Input pair impedance mismatch




We supposed that $2 i_{cm}$ always perfectly divides through $M_1$, $M_2$. This is not true as $R_1 \neq R_2$

$$R_1 = \frac{1/g_{mM} + r_{o1}}{1 + g_{m1} r_{o1}} \qquad R_2 = \frac{r_{o1}}{1 + g_{m1} r_{o1}}$$




We can say with minimal error that $v_g \sim v_{cm}$ $v_s \sim v_{cm}$ so

$$i_1 = 2 i_{cm} \frac{R_2}{R_1 + R_2} = \frac{v_s}{r_{og}} \frac{R_2}{R_1 + R_2} \quad \text{and} \quad i_2 = \frac{v_s}{r_{og}} \frac{R_1}{R_1 + R_2}$$

With a perfect mirror, $i_{cc_{OUT}} = i_2 - i_1 = \frac{v_s}{r_{og}} \frac{R_1 - R_2}{R_1 + R_2}$

$$R_1 - R_2 = \frac{1/g_{mM}}{1 + g_{m} r_{o1}} \qquad R_1 + R_2 = \frac{2 r_{o1} + 1/g_{mM}}{1 + g_{m1} r_{o1}}$$

Therefore $\varepsilon \dfrac{v_{cm}}{2 r_{og}} = \dfrac{v_s}{r_{og}} \dfrac{1/g_{m\pi}}{\cancel{1+g_{m\pi} r_{o1}}} \dfrac{\cancel{1+g_{m\pi} r_{o1}}}{2 r_{o1} + 1/g_{m\pi}}$

negligible

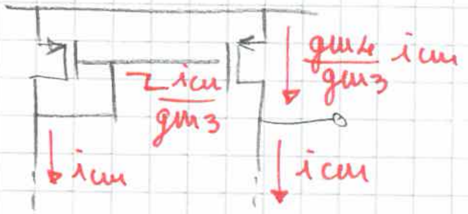
$\varepsilon \dfrac{v_{cm}}{2 r_{og}} = \dfrac{v_s}{r_{og}} \cdot \dfrac{1}{2} \dfrac{1}{g_{m\pi} r_{o1}}$ If $\alpha = \dfrac{v_s}{v_{cm}} \sim 1$ then

$$\varepsilon \simeq \frac{1}{g_{m\pi} r_{o1}}$$

If we also consider mirror mismatch:

$$\varepsilon_{DET} \simeq \frac{1}{g_{m\pi} r_{o1}} + \frac{1}{g_{m\pi} r_{o\pi}}$$

Mirror statistical gm mismatch



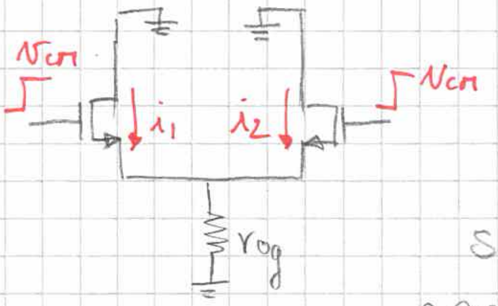

We now consider a difference in gm (caused by $V_T$ or $K$ statistical variability).

$$i_{OUT} = i_{cm}\left(\frac{g_{m4}}{g_{m3}} - 1\right) = i_{cm} \frac{g_{m4} - g_{m3}}{g_{m3}} \triangleq i_{cm} \frac{\Delta g_{m\pi}}{g_{m\pi}}$$

Since $\varepsilon i_{cm} = i_{cm} \dfrac{\Delta g_{m\pi}}{g_{m\pi}} \longrightarrow \varepsilon_{\substack{STAT \\ MIRROR}} = \dfrac{\Delta g_{m\pi}}{g_{m\pi}}$

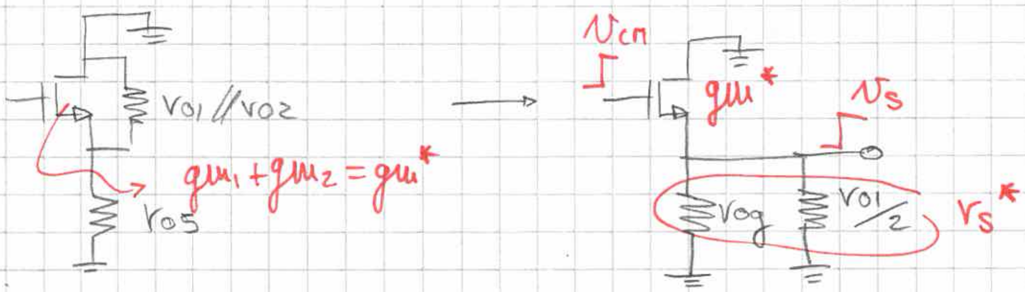
# Input pair statistical mismatch for CMRR

Since we already estimated the mirror mismatch, put everything to ground:




As usual $i_{cc_{OUT}} = i_2 - i_1$

To estimate $g_{m_1}, g_{m_2}$ mismatches we could think of folding the structure in half (since $v_{cm}$ is applied on both gates)




We can compute $v_s = v_{cm} \dfrac{r_s^*}{r_s^* + 1/g_m^*}$, now, since $v_{cs}$ it's the same, it's easy to see that:

$i_1 = g_{m_1}(v_{cm} - v_s) \qquad i_2 = g_{m_2}(v_{cm} - v_s)$

$$i_2 - i_1 = (g_{m_2} - g_{m_1})(v_{cm} - v_s) = \Delta g_m \, v_{cm} \left(1 - \frac{r_s^*}{r_s^* + 1/g_m^*}\right)$$

Let's compare it with typical CMRR current

$$\varepsilon \frac{v_{cm}}{2 r_{o9}} = \Delta g_m \, v_{cm} \frac{1/g_m^*}{r_s^* + \cancel{1/g_m^*}} \quad \text{negligible}$$

where $g_m^* = g_{m_1} + g_{m_2} = 2 g_m$

$r_s^* = r_{o9} // r_{o1}/2$

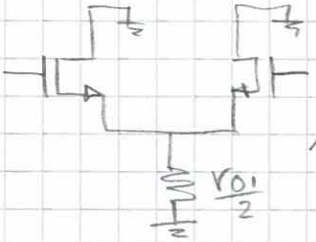
$$\varepsilon \frac{v_{cm}}{2 r_{o9}} = \frac{\Delta g_m}{2 g_m} \cdot \frac{r_{o9} + r_{o1}/2}{r_{o9} \, r_{o1}/2} = \frac{v_{cm}}{2 r_{o9}} \cdot \frac{\Delta g_m}{g_m} \left(1 + \frac{2 r_{o9}}{r_{o1}}\right)$$

$$\varepsilon = \frac{\Delta g_{m,IN}}{g_{m\,IN}} \left(1 + \frac{2 r_{o5}}{r_{o1}}\right)$$

What's the meaning of this?

$$CMRR = \frac{2\, g_{m1}\, r_{og}}{\varepsilon} = \frac{2\, g_{m1}\, r_{og}}{\frac{\Delta g_{m1}}{g_{m1}}\left(1+\frac{2 r_{og}}{r_{o1}}\right)} \overset{r_{og}\to\infty}{=} \frac{g_{m1}^2\, r_{o1}}{\Delta g_{m1}}$$

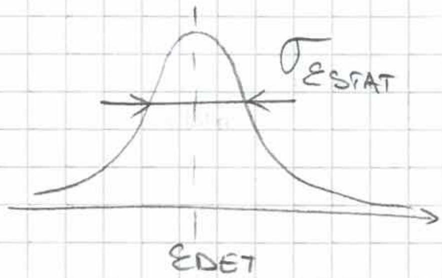
We see that even though we have a perfect tail generator ($r_{og} \to \infty$) we still have a higher limit for the CMRR set by $r_{o1}$. This means:




This tells us that having a great $r_{og}$ may be not enough to have a CMRR high enough

## Recap

$$\varepsilon_{STAT} = \frac{\Delta g_m}{g_m}\bigg|_{M} + \frac{\Delta g_m}{g_m}\bigg|_{IN}\left(1+\frac{2 r_{og}}{r_{o1}}\right)$$




$$\varepsilon_{TOT} = \varepsilon_{DET} + \varepsilon_{STAT}$$

$$\sigma_{STAT} = \sqrt{\sigma^2_{\frac{\Delta g_{mM}}{g_{mM}}} + \sigma^2_{\frac{\Delta g_{m,IN}}{g_{m,IN}}}\left(1+\frac{2 r_{og}}{r_{o1}}\right)^2}$$

estimate $\sigma_{\frac{\Delta g_m}{g_m}}$

$$\frac{\Delta g_{mM}}{g_m} = \frac{1}{g_m}\frac{\partial g_m}{\partial V_T} + \frac{1}{g_m}\frac{\partial g_m}{\partial K} =$$

$dV_T = d(V_{ov} - V_T) = -\Delta V_T$

$$= \frac{1}{\cancel{2K V_{ov}}}\cancel{2V_{ov}}\, dK + \frac{}{\cancel{2K} V_{ov}}\cdot \cancel{2K}\, dV_T$$

$$= \frac{\Delta K}{K} - \frac{\Delta V_T}{V_{ov}}$$

$$\sigma^2\left(\frac{\Delta g_m}{g_m}\right) = \sigma^2\left(\frac{\Delta K}{K}\right) + \frac{\sigma^2(\Delta V_T)}{V_{ov}^2}$$

$\to$ statistically independent but in reality they are correlated

(considered uncorrelated for simplicity)

## 18) Noise: PSD, thermal noise on resistors, MOSFETs

We found out that gain/BW tradeoff is independent from the current $\left( g_m r_o = \frac{2V_A'}{V_{ov}} \text{ and } f_T = \frac{\mu V_{ov}}{2\pi L^2} \right)$. What sets the current? Noise. For this discussion, consider gaussian noise:

$$\rho(\bar{v}) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{\bar{v}^2}{2\sigma^2}}$$ $\rho(\bar{v})$ is the probability of the amplitude

~68% of samples will fall within $\pm\sigma$.

We can describe noise as a superposition of orthogonal harmonics (like signals). Consider just two:

$$x(t)\big|_{NOISE} = A \sin(\omega_1 t + \varphi_1) + B \sin(\omega_2 t + \varphi_2)$$

- Mean value is zero
- Mean square value is

$$\langle x(t)^2 \rangle = \langle A^2 \sin^2(\,) + B^2 \sin(\,) + 2AB \sin(\,)\sin(\,) \rangle$$

Therefore:

$$\langle x(t)^2 \rangle = \langle A^2 \sin^2(\,) \rangle + \langle B^2 \sin(\,) \rangle + \underbrace{\langle 2AB\, \cancel{\sin(\,)} \sin(\,) \rangle}_{\text{orthogonal}}$$

$$= \frac{A^2}{2} + \frac{B^2}{2} \quad \rightarrow \text{Recall } A^2 \cdot \frac{1}{T}\int_0^T \sin^2\left(\frac{2\pi t}{T}\right) dt = \frac{A^2}{2}$$

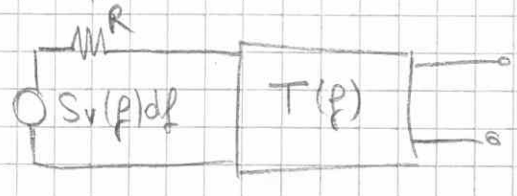
Since mean value is zero: $\langle x(t)^2 \rangle = \sigma_{xn}^2$

If we consider a set of sinusoids (different freq/amplitude)

$$\sigma^2 = \sum_i \sigma_i^2 = \int_0^{+\infty} S_n(f)\,df \quad \text{where } S_n(f) \text{ is the PSD}$$

For a small frequency BW $\Delta f$ we can say $S_n(f_0)\Delta f = \Delta\sigma_{f_0}^2$

Suppose that a resistor is generating noise through a filtering network, the mean square value contribution will be




$S_v(f)\,|T(j\omega)|^2\,df$

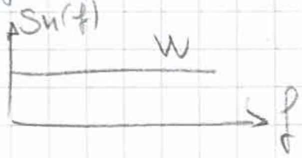
We can say the same for every component inside a linear network, leading to:

$$S_{OUT}(f)\,df = S_{n1}(f)\,|T_1(j\omega)|^2 df + S_{n2}(f)\,|T_2(j\omega)|^2 df + \dots$$

$$\sigma_{OUT} = \sqrt{\int_0^{+\infty} S_{OUT}(f)\,df}$$

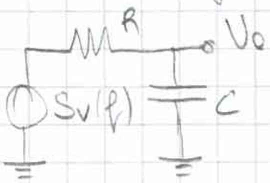
## Thermal noise in resistors

Charge movement and collision with ions is fast ($< 1\,ps$). This means that the spikes generated are short → short signal (impulse) means broad spectrum → approx to flat




$S_n(f) = W$ W stands for white

To estimate W value we can use an energetic argument by filtering the noise with a capacitor




$$V_0(s) = V_i(s)\,\frac{1}{1+sRC}$$

Since $S_v(f) = W$ and we said $\langle V_0^2\rangle = \sigma_0^2 = W\int_0^{+\infty} |T(j\omega)|^2 d\omega$

$$\int_0^{+\infty} |T(f)|^2 df = \int_0^{+\infty} \frac{1}{1+(2\pi f\tau)^2}\,df \cdot \frac{2\pi\tau}{2\pi\tau} = \frac{1}{2\pi\tau}\Big[\arctan(2\pi f\tau)\Big]_0^{+\infty} =$$

$$\frac{1}{2\pi\tau}\cdot\frac{\pi}{2} = \frac{1}{4\tau}$$ Therefore $$\sigma_0^2 = \frac{1}{4\tau}\cdot W$$

If we express this in frequency cutoff $\frac{1}{4\tau} = \frac{\pi}{2} BW_{-3dB}$

So the ENBW = equivalent noise BW is $\frac{\pi}{2}$ higher the pole cutoff $f_{CUT} = \frac{1}{2\pi\tau}$

It's higher because it accounts for the additional integrated noise over $f_{CUT}$

Energy stored in the capacitor is

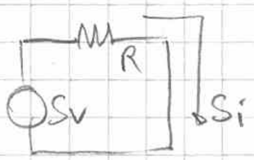
$$\frac{1}{2} C \langle v_o^2 \rangle = \frac{1}{2} K_B T \rightarrow \frac{1}{2} \text{ for a single degre of freedom}$$

according to Boltzmann:

$$\cancel{\frac{1}{2}} C \cdot \frac{W}{4\tau} = \cancel{\frac{1}{2}} K_B T \qquad \frac{\cancel{C} \cdot W}{4 R \cancel{C}} = K_B T \rightarrow \underline{W = 4 K_B T R}$$

$$W = S_v(f)\big|_R = \left(\frac{V}{\sqrt{Hz}}\right)^2 \text{ or } \left(\frac{A}{\sqrt{Hz}}\right)^2 \text{ if current referred}$$

$S_i(f)\big|_R$ :



$$\rightarrow S_i(f) = 4 K_B T R \cdot \frac{1}{R^2} \quad |T(j\omega)|^2$$

$$\underline{S_i(f)\big|_R = \frac{4 K_B T}{R}}$$

Same happens for the resistive of MOSFETs in ohmic region:

$$R_{ch} = \frac{1}{g_{mo}} \quad \text{where} \quad g_{mo} = K(V_{GS} - V_T) = 2K V_{ov}$$

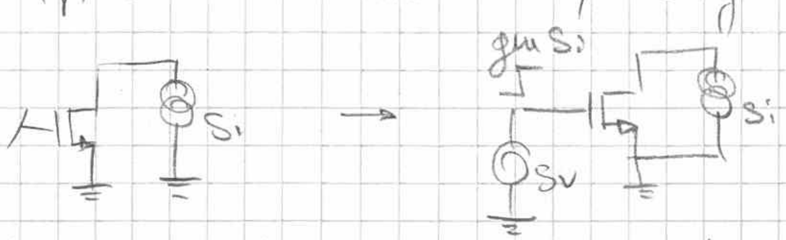
So $S_v(f)\big|_{MOS} = 4KT g_m \rightarrow$ ohmic

For MOSFETs in saturation region, the resistive channel is shorter and less uniform $\rightarrow$ compensate with factor $\gamma$

$\underline{S_i(f)\big|_{MOS}}$:
- $\rightarrow \underline{4KT\, 1\, g_m} \rightarrow$ <u>OHMIC</u>
- $\rightarrow \underline{4KT \frac{2}{3} g_m} \rightarrow$ <u>SAT</u>
- $\rightarrow \underline{4KT\, 2\, g_m} \rightarrow$ <u>SAT (short channel L)</u>

The factor is expressed using $\gamma = 1, \frac{2}{3}, 2$

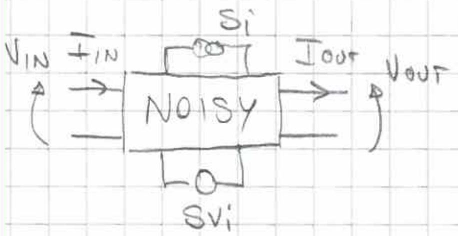
$S_v(f)$ is also available by using the following:



$$\underline{S_v(f)\big|_{MOSFET} = \frac{4KT\gamma}{g_m}}$$

Note: see next question for a better procedure for $S_v$

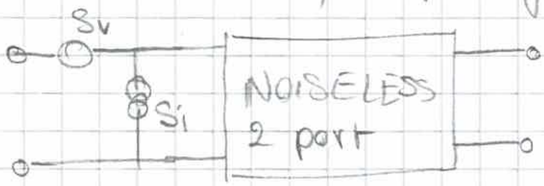
# 19) Input referred noise sources (2 port) + Diff stage noise



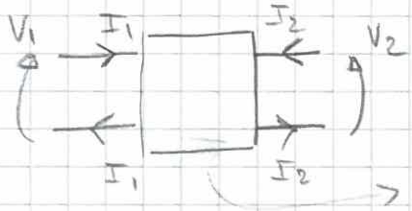

$$\rightarrow \left(\frac{S}{N}\right)^2_{OUT} = \frac{S_{OUT}^2}{\left[S_{I_1}(f) + S_{I_2} + \dots\right] df}$$

$S_{OUT}^2$ → signal transfer

We can model the noisy network as a two port noiseless network + noisy input generators (input referred noise)




$S_V$, $S_i$ are independent on in/out impedances.

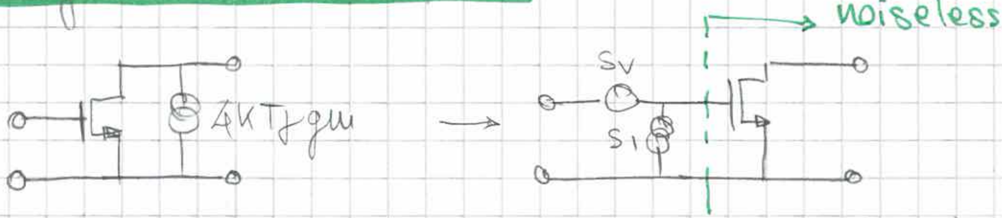



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

→ Definition of two port network

A two port network can be represented with the Z matrix

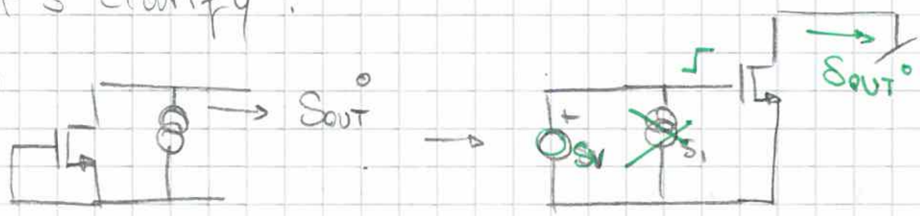
## Single MOSFET $S_V$, $S_I$




Its easy to see that
- short input → $S_V$ → ①
- open input → $S_I$ → ②

Let's clarify:

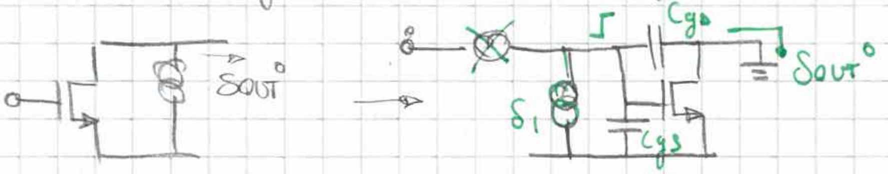
①




↳ superposition of effects: $S_I$ is short

$S_{OUT}^0 = 4KT\gamma g_m$ $\quad S_{OUT}^0 = g_m^2 S_{V_{IN}} \rightarrow S_{V_{IN}} = \dfrac{4KT g_m}{g_m^2}$

②

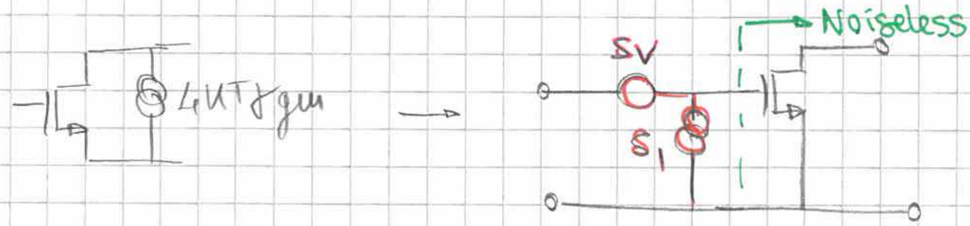



$S_{OUT}^0 = 4KT\gamma g_m$ $\quad S_{OUT}^0 = S_I \left|\dfrac{1}{j\omega(C_{gd}+C_{gs})}\right|^2 \cdot g_m^2 = S_I\, g_m^2 \dfrac{1}{\omega^2 C_{ox}^2}$

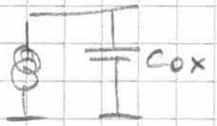
$$4KT\gamma \cancel{g_m} = S_1 g_m^{\cancel{2}} \frac{1}{\omega^2 C_{ox}^2} \rightarrow \underline{S_1 = 4KT\gamma g_m \left(\frac{\omega}{\omega_T}\right)^2}$$

Where $\omega_T$ is the cutoff $\omega_T = \frac{g_m}{C_{ox}}$

## Recap on input referred noise for a MOSFET

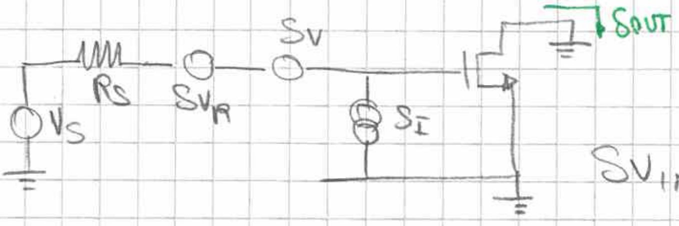



$$\underline{S_V\Big|_{IN} = \frac{4KT\gamma}{g_m}} \qquad \underline{S_I\Big|_{IN} = 4KTg_m\left(\frac{\omega}{\omega_T}\right)^2}$$

Note: $\left(\frac{\omega}{\omega_T}\right)$ is a rising factor because since 


$Z_{IN} = \frac{1}{sC_{ox}}$ is decreasing with frequency, to get a flat output spectrum ($4KT\,g_m$), $S_1$ has to compensate by increasing its power amplitude.

## Example of application

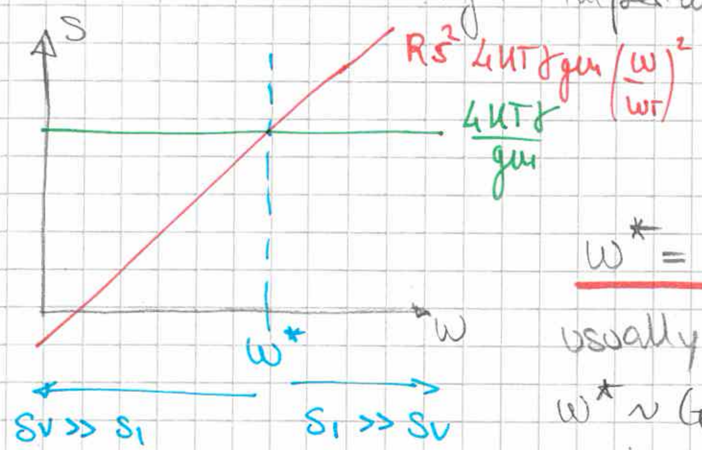



$S_{OUT} = g_m^2 S_{V_{IN}}$ where

$$S_{V_{IN}} = S_{V_R} + S_V + R^2 S_1$$

$$S_{V_R} = 4KTR_S \qquad S_V = \frac{4KT\gamma}{g_m} \qquad S_1 = 4KT\,g_m\left(\frac{\omega}{\omega_T}\right)^2$$

Be careful: → low impedance $R_S$ → $S_V$ is relevant
→ high impedance $R_S$ → $S_1$ is relevant

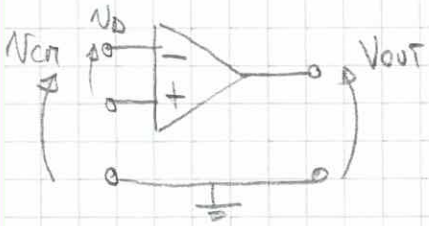



$$\frac{\cancel{4KT\gamma}}{g_m} = R_S^2 \cancel{4KT\gamma}\, g_m \left(\frac{\omega}{\omega_T}\right)^2$$

$\underline{\omega^* = \omega_T / g_m R_S}$ but since $\omega_T$ is usually very high and $R_S$ very low $\omega^* \sim GHz$ → we can almost always neglect $S_1$ contribution (for $\sim MHz$)

# Input referred noise for a differential stage

Important: is the differential stage a 2 port network?




$\rightarrow$ It's not, but it can be if we don't consider $V_{cm}$. This means that CMRR $\rightarrow \infty$. Let's be more specific

$V_{out} = A^+ v^+ - A^- v^-$ $\quad V_D = V^+ - V^-$ $\quad V_{cm} = \frac{V^+ + V^-}{2}$ so

$V^+ = V_{cm} + \frac{V_D}{2} \qquad V^- = V_{cm} - \frac{V_D}{2}$

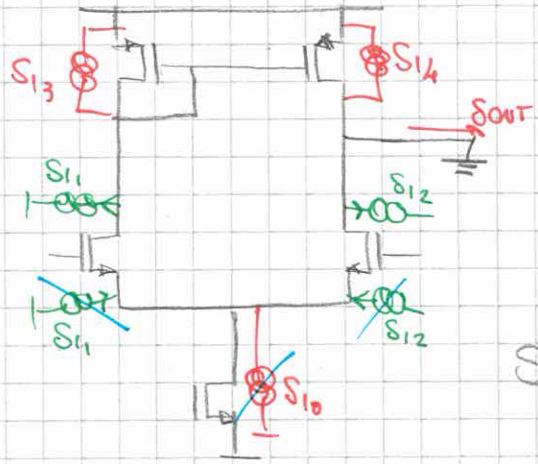
$$V_{out} = A^+\left(V_{cm} + \frac{V_D}{2}\right) - A^-\left(V_{cm} - \frac{V_D}{2}\right) = V_{cm}\left(A^+ - A^-\right) + V_D\left(\frac{A^+ + A^-}{2}\right)$$

- If $A^+ = A^-$ $\quad V_{cm}(A^+ - A^-) = 0$ therefore:

$V_{out} = V_D\left(\frac{A^+ + A^-}{2}\right) = A \cdot V_D \rightarrow$ it's a two port network

In reality CMRR $\sim 100\,dB$ to make these assumptions

# Output noise current of a differential stage



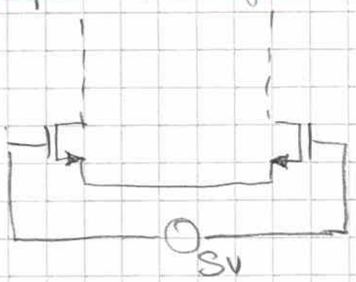

Important! CMRR → ∞

Note: all the generators connected to the source give a common mode contribution → No output noise current

$S_{OUT}^0 = S_{I1} + S_{I2} + S_{I3} + S_{I4} = 8KT\gamma g_{m_{IN}} + 8KT\gamma g_{m_\pi}$

$$S_{OUT}^0 = 8KT\gamma\left(g_{m_{IN}} + g_{m_\pi}\right)$$

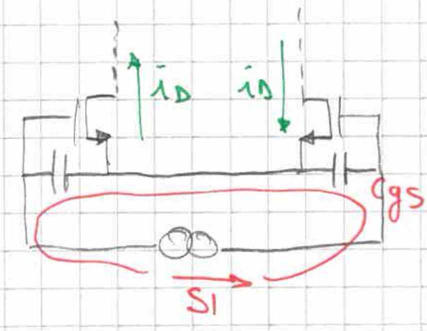
## Input voltage referred noise / current referred noise




$S_{OUT}^0 = S_V \cdot g_{m_{IN}}^2$ ⇝ differential transfer function

$S_V\big|_{MOS} = \left(\frac{1}{g_{m_{IN}}}\right)^2 8KT\gamma\left(g_{m_{IN}} + g_{m_\pi}\right) =$

$$S_V\big|_{MOS} = \frac{8KT\gamma}{g_{m_{IN}}}\left(1 + \frac{V_{OV_{IN}}}{V_{OV_\pi}}\right)$$




We can see that $i_D = g_m V_{gs} = g_m \frac{i_n}{sC_{gs}}$

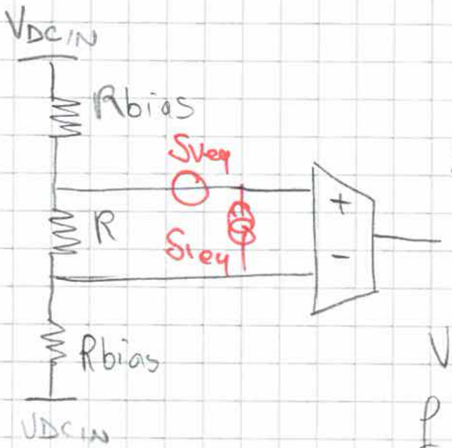
$V_{gs}$ is the voltage on $C_{gs}$

$i_{n_{OUT}} = 2 g_m \frac{i_n}{sC_{gs}} \to S_{OUT}^0 = \frac{4 g_m^2}{\omega^2 C_{gs}^2} S_I$

by setting $\omega_T = \frac{g_m}{C_{ox}} \sim \frac{g_m}{C_{gs}}$ with minimum error we can say $S_{OUT}^0 = 4 S_I \left(\frac{\omega}{\omega_T}\right)^2$ → if we compare $S_I$, $S_V$:

$S_{V_{IN}} \cdot g_{m_{IN}}^2 = 4 S_I \left(\frac{\omega}{\omega_T}\right)^2 \to S_{I_{eq}} = S_{V_{eq}} \frac{g_{m_{IN}}^2}{4} \left(\frac{\omega}{\omega_T}\right)^2$

## Current, voltage noise comparison

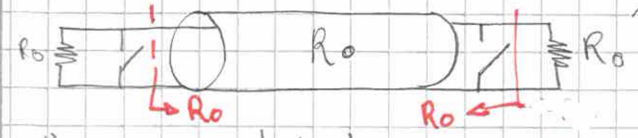



$$S = S_{veq} + S_{ieq} R_S^2$$

$$\hookrightarrow S_{veq} \frac{g_{m_{IN}}^2}{4} \left(\frac{\omega}{\omega_T}\right)^2$$

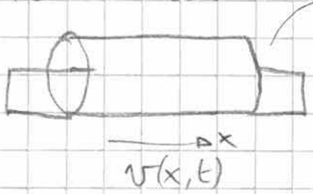
Voltage noise is dominant up to $f = \dfrac{2 f_T}{g_{m_{IN}} R_S}$ so, unless for very high frequency (or very high impedances) $\rightarrow S_I$ negligible

# 19) Noise models: Nyquist experiment for PSD




$\rightarrow \phi_{12} = \phi_{21}$ fluxes are equal
Same temperature $T$

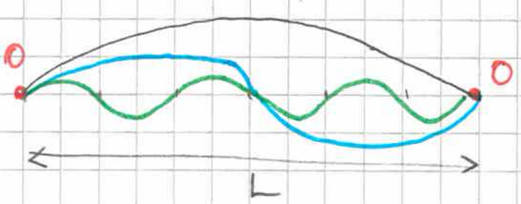
Use a matched coax cable. For $t = -\infty$ system will be at equilibrium. We set up matched impedances $\rightarrow$ no reflections take place therefore the same energy (electric/magnetic field) is stored in the cable.
At $t = 0^+$ we short the two ends so we separate the $R_0$ from the coax. Some e.m. waves will be trapped inside. Also, the energy $\mathcal{E}_0$ of the resistor will be twice inside the cable.




$\rightarrow$ e.m. system with the following equations

$$\frac{\partial v(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial v(x,t)}{\partial t^2} \qquad \frac{\partial i(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial i(x,t)}{\partial t^2}$$

Since the system has boundaries, surviving modes will be:



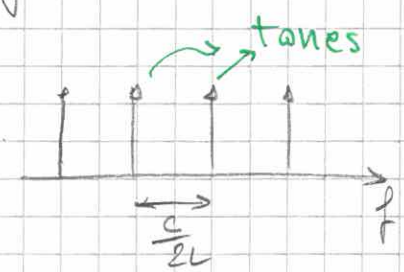

first mode is $L = \frac{\lambda_1}{2}$ using the dispersion relationship $f_1 \cdot \lambda_1 = c$

$$f_1 = \frac{c}{2L}$$

We find $\infty$ modes trapped with a wavelength that is a multiple of the lenght of the coax:

$$f_K = \frac{c}{2L} \cdot K$$

every tone is spaced by $\frac{c}{2L}$

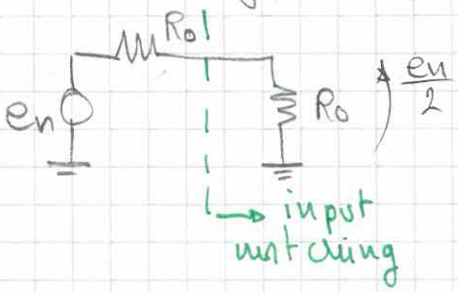



If we select a $\Delta f$ bandwidth, can count the number of modes falling inside $\#\text{modes} = \frac{\Delta f}{c/2L}$

Each mode has electrical/magnetic degrees of freedom
$\rightarrow \frac{KT}{2} \cdot 2 = KT$ We can say that the average energy trapped in a single interval is $\mathcal{E}_{\Delta f} = KT \cdot \frac{\Delta f}{c} \cdot 2L$

$$\mathcal{E}_{\Delta f} = \underbrace{KT}_{\text{energy per single mode}} \cdot \underbrace{\frac{\Delta f}{c} \cdot 2L}_{\text{number of modes}}$$

We can now connect the trapped energy with the input energy supplied. Consider a sinusoidal generator $e_n$:




Power delivered to coax is

$$P = \left(\frac{e_n}{2} \cdot \frac{1}{\sqrt{2}}\right)^2 \cdot \frac{1}{R_o} = \left(\frac{e_n}{2}\right)^2 \cdot \frac{1}{2R_o} \quad \left[\frac{\text{joule}}{\text{seconds}}\right]$$


$\rightarrow$ $e_n$ is the peak voltage of gen

We now need to compute the energy, therefore we need to compute how much time it takes to travel:

$T = \frac{L}{c}$ where $T$ = transit time $L$ = length of coax

$c$ = speed of light

$$E_{TOT}\Big|_{SUPPLIED} = \underbrace{2}_{\text{double contribution}} \cdot \underbrace{T}_{\text{transit}} \cdot \underbrace{P}_{\text{power}} = \cancel{2} \cdot \frac{L}{c} \left(\frac{e_n}{2}\right)^2 \cdot \frac{1}{\cancel{2} R_o}$$

Since $e_n$ is the voltage noise generator, we link it to the mean square value

$$\frac{e_n^2}{2} = S_v \Delta f$$


$$\langle e_n(t) \rangle^2 = \frac{e_n^2}{2}$$

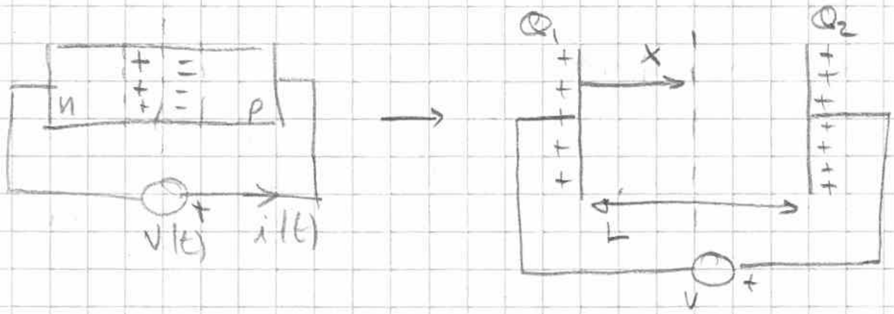
Therefore we "linked" the power of a $\Delta f$ in a single sinusoid. We can now write

$$E_{TOT}\Big|_{\Delta f} = \frac{L}{c}\left(\frac{e_n}{2}\right)^2 \cdot \frac{1}{R_o} = \frac{L}{c} \frac{e_n^2}{2} \cdot \frac{1}{2} \cdot \frac{1}{R_o} = \frac{S_v \Delta f}{2} \cdot \frac{L}{c} \cdot \frac{1}{R_o}$$

This must be equal to the stored $\mathcal{E}_{\Delta f}$:

$$\frac{\cancel{L}}{\cancel{c}} \cdot \frac{1}{R_o} \cdot \frac{S_v \cancel{\Delta f}}{2} = KT \cdot \frac{\cancel{\Delta f}}{\cancel{c}} \cdot 2\cancel{L} \rightarrow S_v = 4KTR_o$$

# 20) Shot noise in PN junctions + weak inversion MOSFETS




The electron moving inside the PN junction behaves like in a capacitor with plane plates:

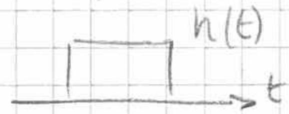
$$\underbrace{Q_2 = \frac{x}{L} \cdot q}_{>0} \qquad \underbrace{Q_1 = \left(\frac{L-x}{L}\right) q}_{>0} \qquad Q_1 + Q_2 = q$$ ⤳ charge of the electron

The transit of the electron generates a current

$$\left|\frac{dQ_2}{dt}\right| = \left|\frac{dQ_1}{dt}\right| = \frac{q}{L}\frac{dx}{dt} = \frac{q}{L}\underbrace{v(t)}_{\text{carrier velocity}} = I_{CARRIER}(t) \quad \left[\frac{C}{s}\right]$$

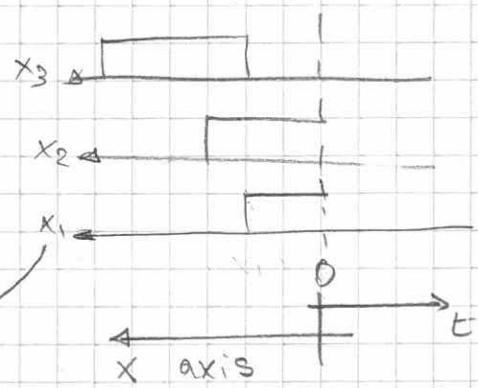
The average flow of electrons is $\lambda = \frac{\#\text{ electrons}}{s} = I/q$

Therefore in a time interval $dt$ the probability that an electron starts to flow is $\lambda dt$




Suppose the carriers travel through the junction with saturation velocity. The current contribution will have a rectangular shape. This means that, for a single $e^-$:

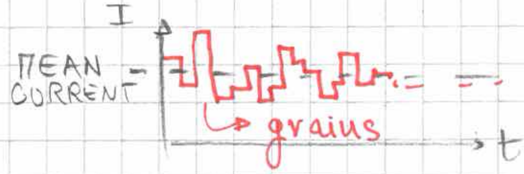
$$i(t) = q\,h(t) \quad \text{where} \quad \int_0^{+\infty} h(t)\,dt = 1$$




We can say that total current will be a sum of elementary grains (pulses):

$i(t) = q\,h(x_1) + q\,h(x_2) + \text{----}$

↳ arbitrary axis to take into account the shape during the integration.

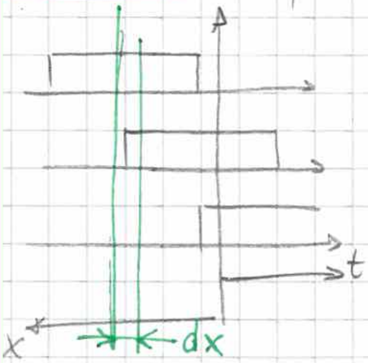


We now compute the mean value and the mean square

$$\langle i(t)\rangle = \langle q h(x_1) + q h(x_2) + \dots\rangle = \int_0^{+\infty} q\, h(x)\cdot \underbrace{\lambda\, dx}_{} = q\lambda \int_0^{+\infty} h(x)dx = q\lambda$$

continuous

probability of a pulse to start in a $dx$




It makes sense, the average current is given by the rate per unit time multiplied by the carrier charge $\langle i(t)\rangle = q\lambda$

(*1)

$$\langle i^2(t)\rangle = \langle (q h_1 + q h_2 + \dots)^2\rangle$$

$$= \langle \underbrace{(q h_1)^2 + (q h_2)^2 + (q h_3)^2}_{\text{single squared contributions}} + \underbrace{2q^2 h_1 h_2 + 2q^2 h_2 h_3 + 2q^2 h_1 h_3 + \dots}_{\text{biproducts}}\rangle$$

Pass from discrete to continuous

$$\langle i^2(t)\rangle = q^2\int_0^{+\infty} h^2(x)\lambda\, dx + q^2 \iint_0^{+\infty} h_1(x) h_2(y)\lambda^2 dx\, dy$$

$$= q^2\int_0^{+\infty} h^2(x)\lambda dx + q^2\int_0^{+\infty} h_1(x)\lambda dx \int_0^{+\infty} h_2(y)\lambda dy \qquad \int_0^{+\infty} h(x)dx = 1$$

$$= q^2\lambda\int_0^{+\infty} h^2(x)\,dx + q^2\lambda^2 = \langle i^2(t)\rangle$$ from statistical theory:

(*1)

$$\sigma_i^2 = \langle i^2(t)\rangle - \langle i(t)\rangle^2 = q^2\lambda\int_0^{+\infty} h^2(x)dx + q^2\lambda^2 - (q\lambda)^2$$

Therefore $\sigma_i^2 = \underbrace{q^2\lambda}_{qI}\int_{-\infty}^{+\infty} h^2(x)\,dx = qI\int_{-\infty}^{+\infty}|H(f)|^2 df$

parseval

$-\infty$ instead of zero because (*2) $h^2(x)=0$ for $x<0$

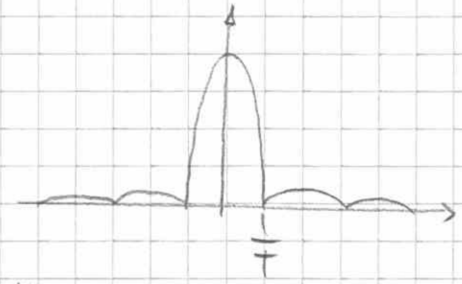
Since $\sigma_i^2 = \int_0^{+\infty} S_I(f)\,df \longrightarrow S_I(f) = 2qI\,|H(f)|^2$

(*3)

Factor 2 is given by (*2) and (*3) because $H(f)$ is symmetric thus giving $\int_{-\infty}^{+\infty} = 2\int_0^{+\infty}$

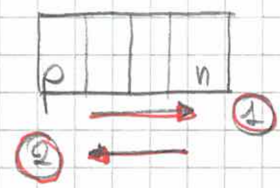
$$S_I(f) = 2qI\,|H(f)|^2$$

where, for a rect it is $|H(f)| = \left|\dfrac{\sin 2\pi f/T}{2\pi f/T}\right|$



Since $T$ is very short (few ps) we can say that $\frac{1}{T}$ will be placed at $\sim 100$ GHz so we can basically say that $S_I$ is $\sim$ white unless some special cases

## Shot noise on a PN junction



$$I = I_s e^{\frac{qV}{KT}} - I_s$$

$I_s e^{\frac{qV}{KT}}$ → p→n contribution ① (diffusion)

$I_s$ → n→p contribution ② (minority carriers)

$I_{DIFFUSION} = I + I_s$ $\quad I_{REVERSE} = -I_s$ Since $I_D$ and $I_R$ are independent on each other, they sum up in spectrum

$$S_I = 2q(I + I_s) + 2q I_s \quad \text{where}$$

- Reverse bias $I_{DIFF} = 0 \rightarrow S_I = 2qI_s$
- Forward bias $I_{DIFF} \gg I_{REV} \rightarrow S_I = 2qI$
- Zero bias $I = 0 \rightarrow S_I = 4qI_s$

$\frac{I_s}{V_{TH}}$

Note: at zero bias $g_{D0} = \left.\dfrac{\partial I}{\partial V}\right|_{V=0} = \left[\dfrac{qI_s}{KT}e^{\frac{qV}{KT}}\right]_{V=0} = \dfrac{qI_s}{KT}$

So $I_s = g_{D0} \cdot \dfrac{KT}{q} \rightarrow S_I = 4qI_s = 4KTg_{D0}$

This is very familiar to a resistor white noise. In fact, we can apply the same reasoning of the Nyquist experiment:

## Weak inversion MOSFET shot noise

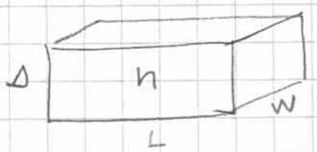
$$I_{DRAIN}\Big|_{DIFF} = I_0\, e^{q\frac{V_{GS}-V_T}{nKT}} \longrightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{q}{nKT} I_0\, e^{q\frac{V_{GS}-V_T}{nKT}} = \frac{q\, I_D}{nKT}$$

$$n\, g_m \frac{KT}{q} = I_D \longrightarrow S_I\Big|_{MOS} = 2qI_D = 2KT\, n\, g_m$$

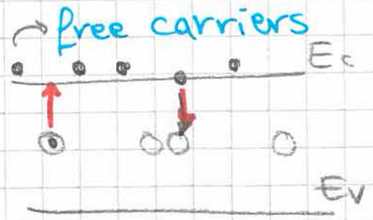
We can adjust the formula to $4KT\gamma g_m$:

$S_I$
- $\gamma = \frac{2}{3}$ saturation (2 for short channel)
- $\gamma = 1$ ohmic
- $\gamma = \frac{n}{2}$ weak inversion (shot noise)

## 21) Trapping noise in a resistor




$$I = G \cdot V = q \mu n \frac{W\Delta}{L} \cdot V$$



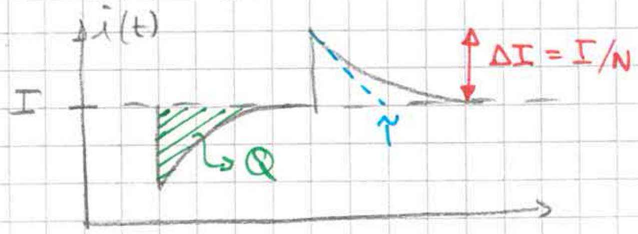

$N = \#\, e^-$ contributing to current flow $= n \cdot \Delta \cdot W \cdot L$ so
↳ free carriers ↳ $\# e^- / cm^3$

$I = q \cdot \mu \cdot \frac{N}{L^2} V$ we can see that $I \propto N$ for a given voltage, so if some electrons are trapped inside the material, I will decrease → $\Delta I / I = \Delta N / N$ $\quad \Delta N = |N - \overbrace{(N \pm 1)}^{N'}|$

→ single electron gets captured/emitted →

If $\Delta N = 1$ the current variation for a capture/release event will be $\Delta I = \frac{I}{N}$. At steady state, the emission/capture processes are in a dynamic equilibrium (if captures ↗↗ then emissions ↗↗ to return to steady state and viceversa)

Transients recover with a time constant $\tau$




→ Waveforms are

$$\underline{i(t) = \Delta I \cdot e^{-t/\tau}}$$

$$i(t) = \overset{\text{area}}{Q}\, h(t) = (\Delta I \cdot \tau) \frac{1}{\tau} e^{-t/\tau} \quad \text{where} \int_0^{+\infty} \frac{1}{\tau} e^{-t/\tau} dt = 1$$

Area of the pulse is $\underline{Q = \frac{I}{N} \cdot \tau}$

→ Just for capture OR emission events

Knowing that $\langle i(t) \rangle = \langle Q\, h_1(t) + Q\, h_2(t) + \dots \rangle$ we can recover the same shot noise reasoning and end up with $\underline{S_I = 2 q^2 \lambda |H(\omega)|^2}$

At equilibrium the rate of emission and capture will be the same → $\lambda_e = \lambda_c = \lambda$ and $H(\omega)$ has a LPF like transfer function, therefore

$$S_I = 2 Q^2 \lambda \frac{1}{(1 + \omega^2 \tau^2)} = 2 \left(\frac{I}{N}\right)^2 \tau^2 \cdot \lambda \cdot \frac{1}{1 + \omega^2 \tau^2}$$

We need to estimate $\lambda$, since $\lambda$ is a rate of events, we can say $\lambda = \frac{1}{\tau} N_T \cdot \beta$ where

$N_T$ = number of traps in the volume

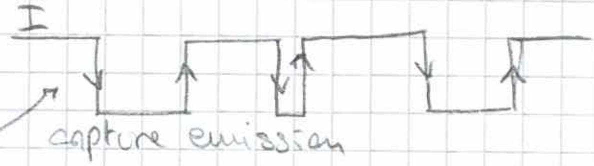
$\beta$ = proportionality factor that adjusts $N_T/\tau$

$$S_I = 2 \cdot 2 \left(\frac{I}{N}\right)^2 \frac{N_T \beta}{\tau} \tau^2 \frac{1}{1+\omega^2\tau^2}$$

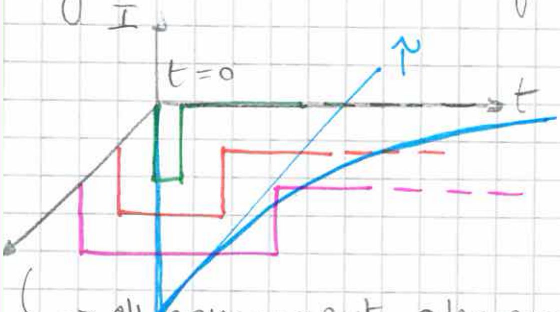
↳ double because of capture + emission contribution

$$S_I = 4\left(\frac{I}{N}\right)^2 \beta N_T \frac{\tau}{1+\omega^2\tau^2}$$ → Random Telegraph Noise

Measures show a binary change:

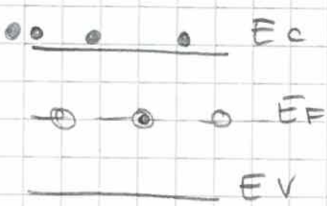



Recovery time should be a step (not exponential) because a single electron can't generate an exp current variation, but if we look to all concurrent events, we discover that the sum of all events will merge into one exponential transient → 1st order model works well!




↳ all concurrent shapes

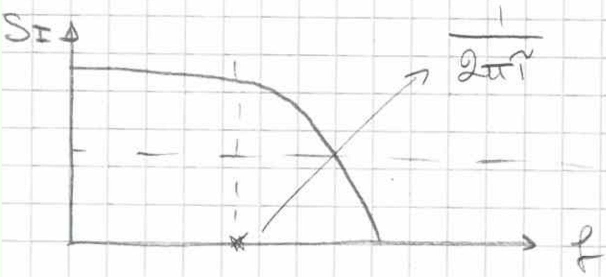
If we limit ourselves to just a single layer of traps placed at Fermi level → $\beta = 1/4$ for $E_F$




For a more rigorous derivation, it is

$$\tau = \frac{\tau_e \tau_c}{\tau_e + \tau_c} \qquad \beta = \frac{\tau_e \tau_c}{(\tau_e+\tau_c)^2} \rightsquigarrow \frac{1}{4} \text{ at } E_{FERMI}$$

Spectrum will be a lorentian shape:



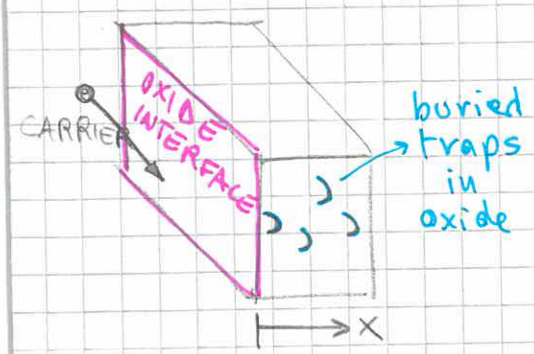

$4kT/R$

for $\beta = 1/4$

$$S_I = N_T \left(\frac{I}{N}\right)^2 \frac{\tau}{1+\omega^2\tau^2}$$

# 22) McWorther model for $1/f$ + Tsividis formula

For now we considered only a set of traps with a single time constant. But there can be defects with a different $\tau$. In fact, McWorther pointed out that carriers flowing in the MOSFET channel can also be trapped in deeper layers of the oxide.




We expect $S_I$ to be a superposition of many lorentian shapes with various $\tau$. We said

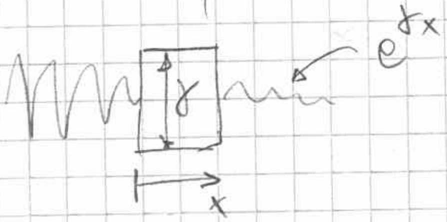
$$S_I = N_T \left(\frac{I}{N}\right)^2 \frac{\tau}{1+\omega^2\tau^2}$$ but now

$N_T$ → Traps at Fermi level

We need to take into account the trapping centers with

$g(\tau)\,d\tau$ = fraction of $N_T$ trapping centers at fermi level with a time constant between $\tau$ and $\tau + d\tau$

From quantum mechanics, we know that:




→ time to tunnel will be $\tau = \tau_0 e^{\gamma x}$

the deeper the oxide, the higher $\tau$

$g(\tau)$ represents the distribution of trap numbers for a given $\tau$, therefore:

$$\underbrace{dN_T(\tau)}_{\text{* elementary trap centers}} = \underbrace{N_T}_{\text{total traps}} \underbrace{g(\tau)\,d\tau}_{\text{portion of traps in a } d\tau}$$

→ We need to link the distribution $g(\tau)$ to the spatial dimension $\longmapsto x$

The spectrum changes now form to:

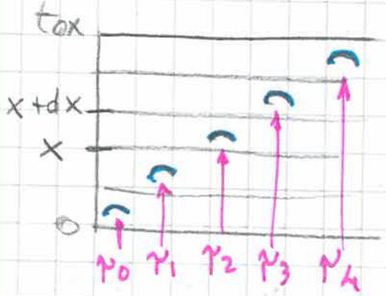
$$S_I = N_T \left(\frac{I}{N}\right)^2 \int_{\tau_{min}}^{\tau_{MAX}} \frac{\tau}{1+\omega^2\tau^2}\, g(\tau)\,d\tau$$

We consider that traps are spread on the whole oxide thickness.

So the whole $N_T$ will be spread between $x=0$ and $x=t_{ox}$

The number of traps contained in a $dx$ segment will be:

$N_T \frac{dx}{t_{ox}} = N_T\, g(\tau)\, d\tau$ → clarify this




If the traps are evenly distributed (*1) through the whole tox:

$\tau_1 = \tau_0 e^{\gamma x_1}$ $\tau_2 = \tau_0 e^{\gamma x_2}$ $\tau_3 = \tau_0 e^{\gamma x_3}$

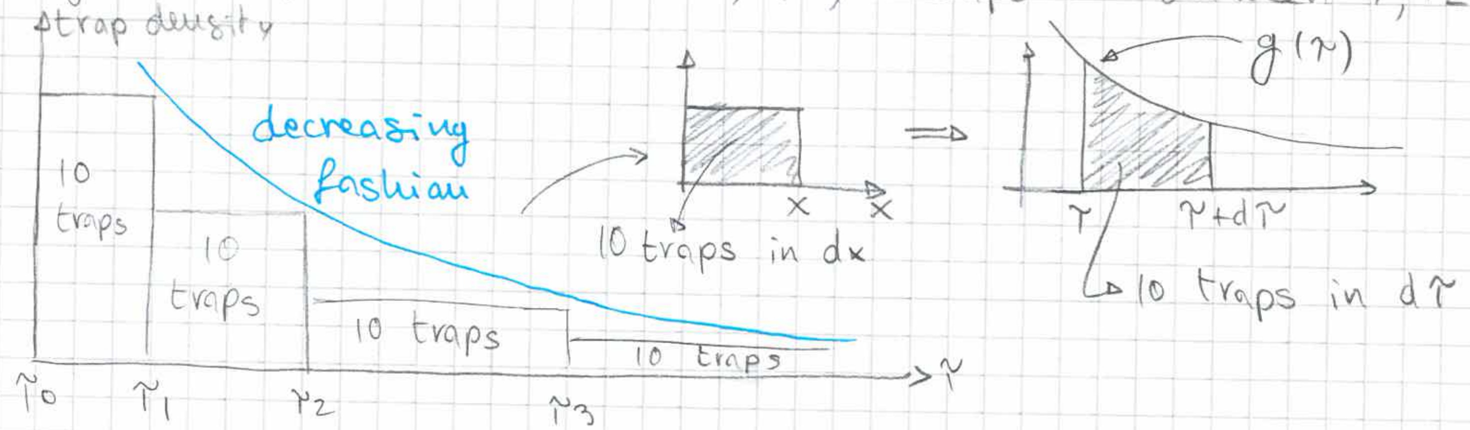
$\tau_0$ ↳ $\tau$ for $x=0$

If (*1) then $x_2 = 2x_1$ and $\tau_2 = \tau_0 e^{2\gamma x_1}$ so

$\frac{\tau_2}{\tau_1} = \frac{e^{2\gamma x_1}}{e^{\gamma x_1}} = e^{\gamma x_1}$ → Same result for $\frac{\tau_3}{\tau_2}$ and so on

Therefore the same portion of traps will be mapped to an exponentially increasing portion of $\tau$.

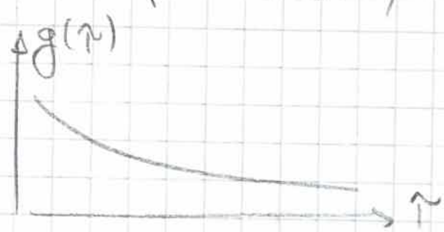
e.g. 10 traps are between $\tau_0, \tau_1$; 10 traps are between $\tau_1, \tau_2$




We confirmed $\cancel{N_T} \frac{dx}{t_{ox}} = \cancel{N_T}\, g(\tau)\, d\tau$ → $g(\tau) = \frac{dx}{d\tau} \cdot \frac{1}{t_{ox}}$

Since $\tau = \tau_0 e^{\gamma x}$ $d\tau = \gamma \underbrace{\tau_0 e^{\gamma x}}_{\tau} dx = \gamma \cdot \tau\, dx$ → $\frac{dx}{d\tau} = \frac{1}{\gamma \tau}$

Therefore $g(\tau) = \frac{1}{\gamma \tau t_{ox}}$ we can clearly see that the trap density for a given $\tau$ decreases (hyperbolic form)



Using this result in the integral:

$$S_I(f) = \frac{N_T}{\gamma t_{ox}} \left(\frac{I}{N}\right)^2 \int_{\tau_{min}}^{\tau_{max}} \frac{d\tau}{1+\omega^2\tau^2}$$

$\tau_{MAX}$ = traps deeper into oxide

$\tau_{MIN}$ = traps on the interface

$$= \frac{N_T}{\gamma t_{ox}\, \omega} \left(\frac{I}{N}\right)^2 \left[\operatorname{arctg}(\omega \tau_{MAX}) - \operatorname{arctg}(\omega \tau_{MIN})\right]$$

$\tau_{MAX}$ = longest time scale measurable by the experiment

$\tau_{MIN}$ = shortest time constant that can be measured by instruments

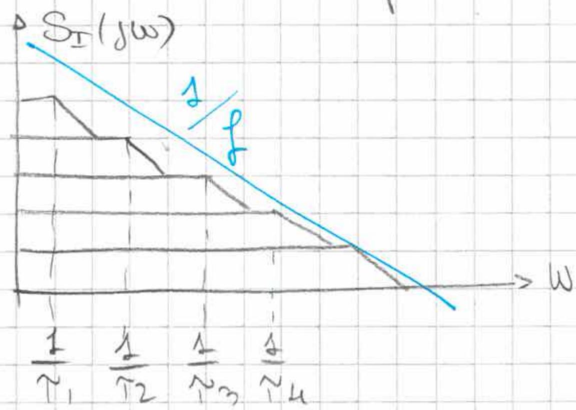
The experimental observation window corresponds to

$$\underbrace{\frac{1}{\tau_{MAX}} < \omega < \frac{1}{\tau_{MIN}}}_{\text{measurable frequencies}} \longrightarrow \omega\tau_{MAX} \gg 1 \quad \omega\tau_{MIN} \ll 1 \longrightarrow \text{so}$$

$$\left[\operatorname{arctg}(\omega\tau_{MAX}) - \operatorname{arctg}(\omega\tau_{MIN})\right] \cong \frac{\pi}{2} - 0$$ Therefore

$$S_I(f) = \frac{N_T}{\gamma t_{ox}} \left(\frac{I}{N}\right)^2 \cdot \frac{1}{\omega} \cdot \frac{\pi}{2} = \frac{N_T}{4\gamma t_{ox}} \left(\frac{I}{N}\right)^2 \frac{1}{f}$$

Experimental measures saw $1/f$ relationship. Now we have a link between $1/f$ and the superposition of Lorentian shapes:



Let's rewrite the formula to be more meaningful to designers:

$I = K V_{ov}^2$ MOSFET current

$N = \dfrac{C'_{ox} WL}{q}(V_G - V_T)$ # carriers in the conductive channel

$N_T = n_T \cdot WL\, t_{ox}$ $n_T$ = average trap density per unit volume within the oxide

So:

$$S_I = \frac{n_T \cancel{W} L \cancel{t_{ox}}}{4 \cancel{t_{ox}} \gamma} \cdot q^2 \left( \frac{K \cancel{V_{ov}^2} \cdot I}{C_{ox}^2 (WL)^{\cancel{2}} \cancel{V_{ov}^2}} \right) = \frac{n_T}{4\gamma} q^2 \cdot \frac{1}{2} \frac{\mu \cancel{C'_{ox}} \cancel{W}}{C'^{\cancel{2}}_{ox} \cancel{W} L^2} =$$

$$= \frac{n_T q^2 \mu}{8 \gamma C'_{ox}} \cdot \frac{I}{L^2} \cdot \frac{1}{f} = K_I \cdot \frac{I}{L^2} \cdot \frac{1}{f} = S_I(f)$$

Refer this to a voltage input noise:

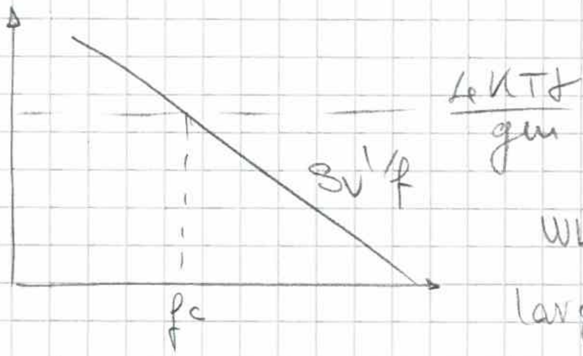
$S_V g_m^2 = S_I \rightarrow S_V = \dfrac{S_I}{g_m^2}$ where $g_m = 2 K V_{ov}$

$$S_V = \frac{K_I^{1/f}}{(2K \cancel{V_{ov}})^2} \frac{K \cancel{V_{ov}^2}}{L^2} \cdot \frac{1}{f} = \frac{K_I^{1/f}}{4 \cdot \frac{1}{2} \mu C'_{ox} \frac{W}{L} \cdot L^2} \cdot \frac{1}{f} =$$

$$= \frac{K_I^{1/f}}{2\mu} \cdot \frac{1}{C'_{ox} WL} \cdot \frac{1}{f} = \frac{K_V^{1/f}}{C'_{ox} WL} \cdot \frac{1}{f}$$

$$\boxed{S_I^{1/f} = K_I^{1/f} \cdot \frac{I}{L^2} \cdot \frac{1}{f}} \qquad \boxed{S_V^{1/f} = \frac{K_V^{1/f}}{C'_{ox} WL} \cdot \frac{1}{f}}$$

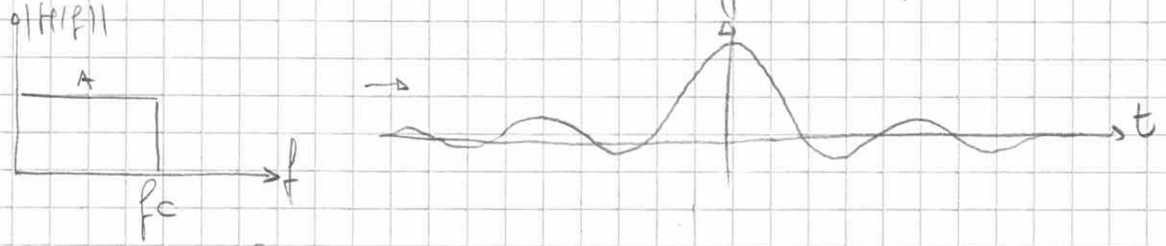
The latter is the Tsividis formula




To lower $I_{BIAS}$ means worsening thermal noise. We could increase WL. This makes sense because the larger area makes the statistical variations and captures/emission count less with respect to the noise. Notice the similarity with Pelgrom's formula

# 23) Introduction to filters: ideal. Limits. Group delay/distortion

Analog filters select harmonics or change the phase relations between harmonics. Ideal filters would select (brickwall) and attenuate with a rectangle shape:




A rect in frequency domain corresponds to a sinc in time domain → this means that in order to be able to have a brickwall filter, we would need to see the future. Therefore these can't be implemented.

Ideally the requirements for a filter are:

- Flat response in the band-pass $|H(j\omega)|_{BP} = A$
- No phase distortion between harmonics at the output.

These translate to:

$$X_{IN}(t) = A\sin(\omega_1 t) + B\sin(\omega_2 t) + C\sin(\omega_3 t)$$

Filter cuts harmonics after $\omega_2$, $|H(j\omega)|_{BP} = G$ so:

$$X_{OUT}(t) = AG\sin\omega_1(t-\tau) + BG\sin(\omega_2(t-\tau))$$

$$= AG\sin(\omega_1 t - \omega_1\tau) + BG\sin(\omega_2 t - \omega_2\tau)$$

We can see that to have constant delay applied to all harmonics we need to apply a proportional phase shift:

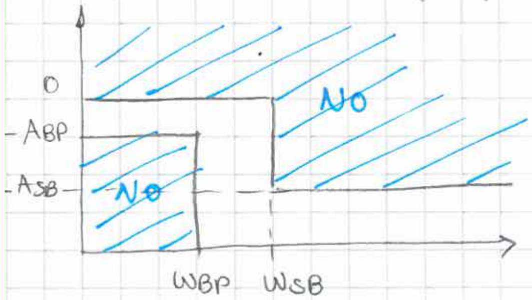
$\varphi_1 = -\omega_1\tau \quad \varphi_2 = -\omega_2\tau \rightarrow \varphi = -\omega\tau$

From that, we can define the group delay as

$\tau = -\dfrac{\varphi}{\omega} \qquad \tau_{GD} = -\dfrac{d\varphi(\omega)}{d\omega}$ → If the phase of the filter is linear, the group delay is constant ($\varphi = -\omega\tau$). If not, we have a max percentage allowable on group delay

## 24) LPF Mask, $\varepsilon_{BP}$, $\varepsilon_{SB}$, $K$, $K_E$, $n$, Butterworth, Cheby...

Since we don't have ideal filters, we have to select the limits to properly size the real analog filter → masks!




$W_{BP}$ = band-pass frequency

$W_{SB}$ = stop-band frequency

$A_{BP}$ = maximum attenuation allowable in band

$A_{SB}$ = minimum attenuation required in stop-band

### Filter families

All-poles : Butterworth, chebyshev type 1, bessel

poles + zeros: chebyshev type 2, Cauer, generalized elliptic

→ out of band ripple

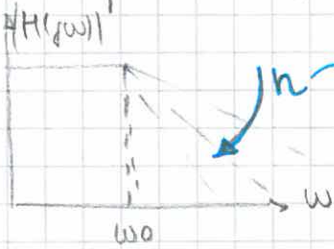
→ in/out ripple

independent in band out of band ripple

### LPF mask - Butterworth

For a particular butterworth function, we know from literature that

$$|H(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}$$




n order of the filter

For $\omega \leq W_{BP}$ we have $A_{BP} = 1dB$ so $\frac{1}{|D(\omega)|} \geq \frac{1}{|A_{BP}|}$

Therefore $|D(\omega)|\Big|_{\omega \leq W_{BP}} \leq A_{BP}$ $\quad 1+\left(\frac{W_{BP}}{\omega_0}\right)^{2n} \leq (A_{BP})^2$

$\left(\frac{W_{BP}}{\omega_0}\right) \leq \sqrt{A_{BP}^2-1} = \varepsilon_{BP}$ defined for a 1st order filter

For a higher order : $\left(\frac{W_{BP}}{\omega_0}\right)^n \leq \varepsilon_{BP} \rightarrow \frac{W_{BP}}{\omega_0} \leq \varepsilon_{BP}^{1/n}$

If $A_{BP}$ is expressed in dB → $\varepsilon_{BP} = \sqrt{10^{\frac{A_{BP}}{10}} - 1}$

We can do the same reasoning for the stopband and obtain (for a 1st order butterworth)

$$\left(\frac{\omega_{SB}}{\omega_0}\right) > \sqrt{A_{SB}^2 - 1} = \varepsilon_{SB} \qquad \text{for a higher order} \quad \frac{\omega_{BP}}{\omega_0} = \varepsilon_{SB}^{1/n}$$

We can now define two coefficients:

$K_\varepsilon = \frac{\varepsilon_{BP}}{\varepsilon_{SB}}$ called discrimination coefficient

$K = \frac{\omega_{BP}}{\omega_{SB}}$ called selectivity coefficient

Since $\omega_{SB} > \omega_{BP}$ and $\varepsilon_{SB} > \varepsilon_{BP}$ $\rightarrow$ $K, K_\varepsilon$ are always $< 1$

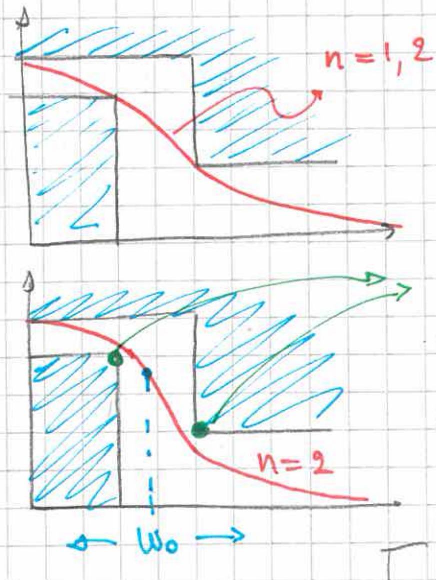
For a n order filter we can write

$$\frac{\omega_{BP}}{\omega_{SB}} \leq \left(\frac{\varepsilon_{BP}}{\varepsilon_{SB}}\right)^{1/n} \rightarrow K \leq K_\varepsilon^{1/n}$$

we can drop the exponent and find the filter order for Butterworth:

$$\underbrace{\ln K}_{<0} \leq \underbrace{\frac{1}{n}}_{>0} \underbrace{\ln K_\varepsilon}_{<0} \rightarrow n \geq \frac{\ln K_\varepsilon}{\ln K} \quad \text{because } K, K_\varepsilon < 1$$

Once we set the filter order, we can move $\Omega_0$ so that we match requirements, for example:




→ The n that touches both $A_{BP}$, $A_{SB}$ is n = 1,2. The real implementable filter needs to be $n \geq 2$

n = 2 doesn't touch both points. We therefore need to move $\omega_0$ so that we meet some given requirements ($A_{SB}$, $A_{SB}$ + margin and so on). To do this, we do:

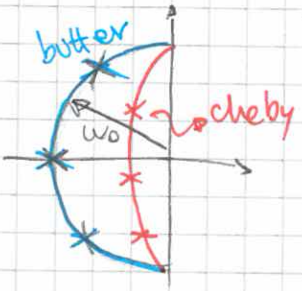
choose the wanted $\omega_0$ ←

$$\begin{cases} 1 + \left(\frac{\omega_{BP}}{\omega_0}\right)^{1/n} = A_{BP} \pm \text{margin} \\ 1 + \left(\frac{\omega_{SB}}{\omega_0}\right)^{1/n} = A_{SB} \pm \text{margin} \end{cases}$$

For a Chebyshev it is $n \geq \frac{Ch^{-1}(k_\varepsilon^{-1})}{Ch^{-1}(k^{-1})}$

## Type of filters

**Butterworth**: requirement of maximum flatness in passband

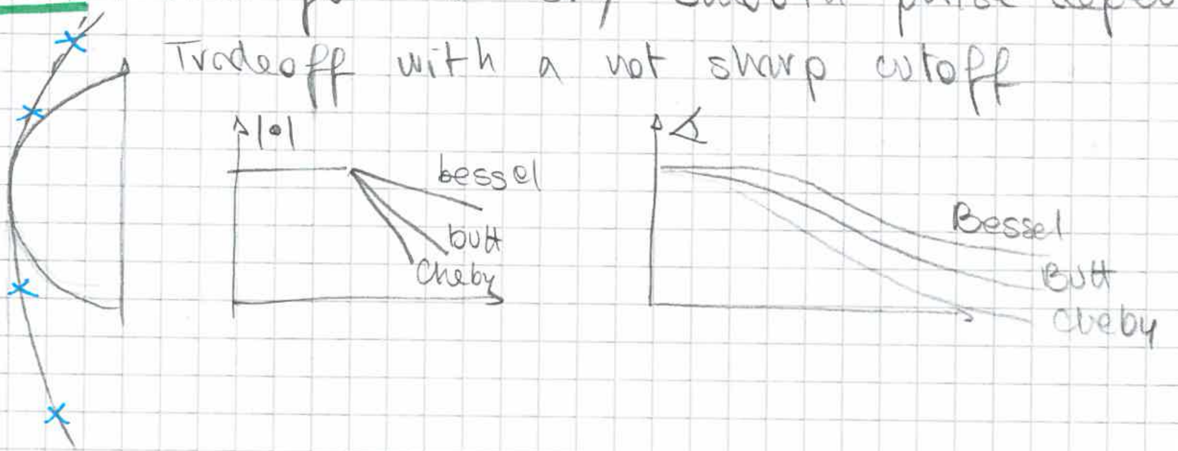



The result is that all poles stay on the circle and are either complex-conj or complex-conj + one real pole (for odd order filters)

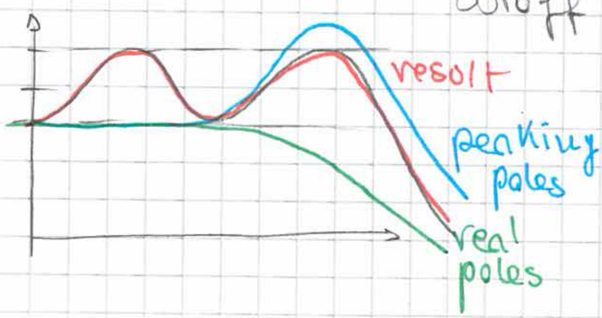
pole pairs will have $s^2 + s\frac{\omega_0}{Q} + \omega_0^2 \quad Q = \frac{1}{2\zeta}$

**Bessel**: used for a very smooth phase dependance in

Tradeoff with a not sharp cutoff




**Chebyshev type 1**: Used to have even more sharpness in cutoff but allowing in-band ripple




It's the result of a sum of real poles + peaking poles → generation of in-band ripple
Still all-poles filter

**Chebyshev type 2**: zeros in the function allow for a flat in-band response but generating ripples in the out-of-band

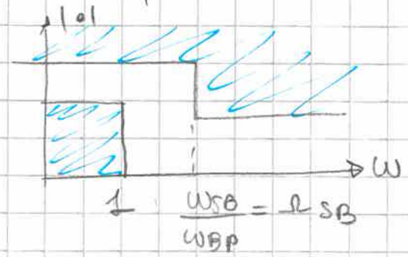
**Cauer** steeper cut than cheby type 2 but now we have in-band + out-of-band ripple. Also $A_{SB}$, $A_{BP}$ can't be set independently

**Generalized elliptic**: Steeper than cauer with independent $A_{BP}$, $A_{SB}$
Issue: worst phase response

# 25) Mapping: HP to LP, BP-to LP transformations

For a LPF, we can map the frequency axis to a normalized frequency axis, where $\omega_{BP} = 1$ rad/s

To emphasize the two masks, use $\Omega_{BP} = 1$ rad, therefore




and $\Omega_{SB} = \dfrac{\omega_{SB}}{\omega_{BP}}$

The normalization procedure is helpful for designing all types of filters (LP, HP, BP, ...). The process is:

[filter mask specs] → [Normalized reference mask] → [Network transfer function T(s)] →

→ [circuit implementation] → [power/noise distortion trimming] → filter is complete

Having the normalized parameters, we can apply a kind of backward transformation to obtain the wanted response (from the normalized filters)

| | | |
|---|---|---|
| $LP(j\omega) \to LP(j\Omega)$ | $\hat{s} = s/\omega_{BP}$ | → intuitive |
| $HP(j\omega) \to LP(j\Omega)$ | $\hat{s} = \omega_{BP}/s$ | |
| $BP(j\omega) \to LP(j\Omega)$ | $\hat{s} = \dfrac{Q(s^2+\omega_0^2)}{s\,\omega_0}$ | → see later |

Notice that the transformations make sense when the arriving filter is compared with the normalized Lowpass

These transformations can now be applied to the normalized LP tf. Example: consider a 1st order Butterworth:

$$T(\hat{s})\Big|_{LP} = \frac{1}{1+\hat{s}} \underset{\substack{\uparrow \\ \text{NORM} \\ \Omega_0}}{=} \frac{1}{1+\dfrac{\hat{s}}{\Omega_0}}$$

To get to a HP we apply $\hat{s} = \omega_{BP}/s$ so

$$T(s)_{HPF} = \frac{1}{1 + \frac{1}{\Omega_0} \cdot \frac{\omega_{BP}}{s}} = \frac{s\,\Omega_0/\omega_{BP}}{1 + \frac{s\,\Omega_0}{\omega_{BP}}}$$

To get to a BP instead:

$$T(s)_{BPF} = \frac{1}{1 + \frac{Q}{\Omega_0}\frac{(s^2+\omega_0^2)}{s\,\omega_0}}$$ if we call $Q_p = \frac{Q}{\Omega_0}$ we will end up with the classic BPF response:

$$= \frac{s\,\omega_0/Q_p}{\frac{s^2}{\omega_0^2} + s\frac{\omega_0}{Q_p} + 1}$$

Example: 3rd order Butterworth:

$$H(\hat{s}) = \frac{1}{(p+1)(p^2+p+1)}$$ $\Omega_0$ – 3dB cut at 1 rad/s

Expansion of the freq axis $p = \frac{\hat{s}}{\Omega_0}$

$$H(\hat{s}) = \frac{1}{\left(\frac{\hat{s}}{\Omega_0}+1\right)\left(\left(\frac{\hat{s}}{\Omega_0}\right)^2 + \frac{\hat{s}}{\Omega_0} + 1\right)} = \frac{\Omega_0^3}{(\hat{s}+\Omega_0)\left[\hat{s}^2 + \Omega_0\hat{s} + \Omega_0^2\right]}$$

Finally, we need another shift $\hat{s} = s/\omega_{BP}$

$$H(s) = \frac{\Omega_0^3}{\left(\frac{s}{\omega_{BP}} + \Omega_0\right)\left(\frac{s^2}{\omega_{BP}^2} + \Omega_0\frac{s}{\omega_{BP}} + \Omega_0^2\right)} =$$

$$= \frac{(\Omega_0\,\omega_{BP})^3}{(s+\Omega_0\,\omega_{BP})\left(s^2 + (\Omega_0\,\omega_{BP})s + (\Omega_0\,\omega_{BP})^2\right)}$$

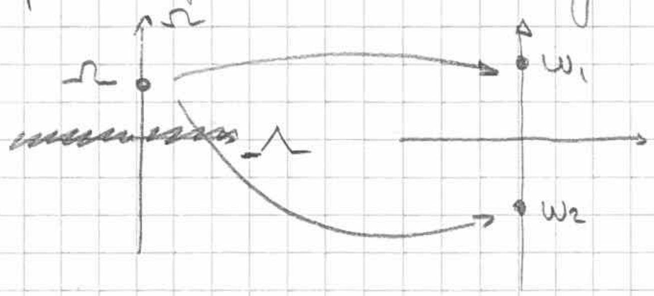
# BPF transformation derivation $LP \rightarrow BP$

A normalized BP can be derived from a LP with $\hat{s} = \hat{p} + \frac{1}{\hat{p}}$ where $\hat{s} = \Lambda + j\Omega$ $\hat{p} = \alpha + j\omega$ are two complex variables, so :

$-\hat{p}\hat{s} + \hat{p}^2 + 1 = 0$ $\hat{p} = \frac{\hat{s} \pm \sqrt{\hat{s}^2 - 4}}{2}$ → develop this

$$\alpha + j\omega = \frac{\Lambda + j\Omega}{2} \pm \frac{\sqrt{\Lambda^2 - \Omega^2 + 2j\Lambda\Omega - 4}}{2}$$

Since were interested in transformations using $\omega$ and $\Omega$, focus just on the imaginary axis → $\Lambda = 0$ :




$\alpha = 0$ and

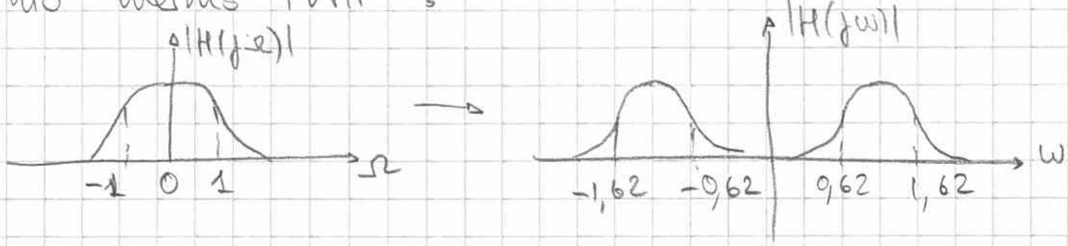
$$j\omega = j\frac{\Omega}{2} \pm j\frac{\sqrt{\Omega^2 + 4}}{2}$$

$$\omega = \frac{\Omega}{2} \pm \sqrt{1 + \left(\frac{\Omega}{2}\right)^2}$$

We therefore conclude that a single $\Omega$ point maps on two points in the $\omega$ axis. It is:

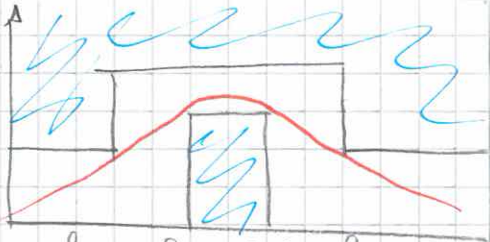
- $\Omega = 0 \rightarrow \omega_{1,2} = \pm 1$
- $\Omega = 1 \rightarrow \omega_{1,2} = \frac{1}{2} \pm \sqrt{1 + \frac{1}{4}} = \begin{cases} +1,62 \\ -0,62 \end{cases}$

This means that :



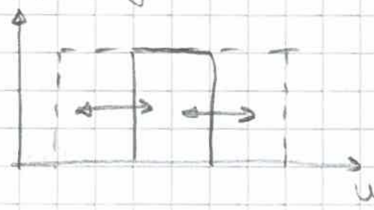

So we can clearly see that for $\Omega = 0 \div 1$ we map a LPF to a BPF. We can set the usual values for the LPF mask and then retransform everything

$$f_0 = \sqrt{f_{BP}^{+} f_{BP}^{-}} \qquad Q = \frac{f_0}{BW} \qquad BW = f_{BP}^{+} - f_{BP}^{-}$$




See that we have a Q factor.

We need to connect that to the mapping, so we can use $\hat{s} = Q\left[\hat{p} + \frac{1}{\hat{p}}\right]$





As usual we apply the transformation needed to set $\omega_0$ as the center BPF frequency:

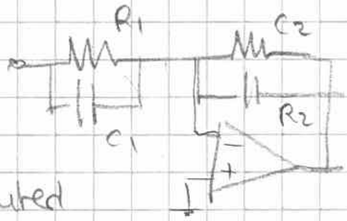
$$s = \omega_0 \hat{p} \rightarrow \hat{p} = \frac{s}{\omega_0} \rightarrow \hat{s} = Q\left[\hat{p} + \frac{1}{\hat{p}}\right] = Q\left(\frac{s^2 + \omega_0^2}{\omega_0 s}\right)$$

This is indeed the transformation listed in the previous table.

# 26) Active cells: SK cell. Sizing + sensitivity

To build the circuits for the wanted transfer function we need active cells so that we can cascade them easily:

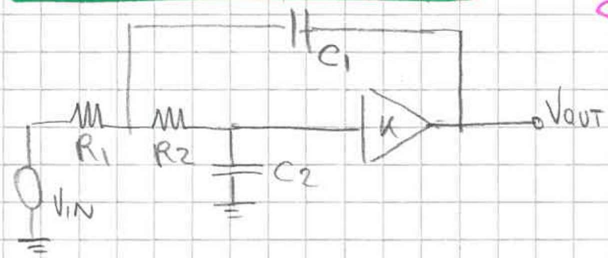
$$V_{out}(s) = T_1 \cdot T_2 \cdot T \dots V_{IN}(s)$$

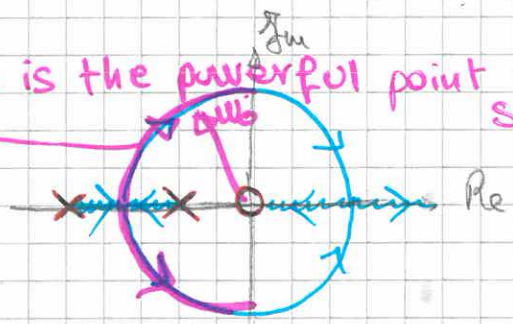

$$T(s) = \frac{1 + sR_2C_2}{1 + sR_1C_1}$$

Pole + zero can be implemented using a single opamp, but how about a complex pole pair?

## Sallen - Key cell

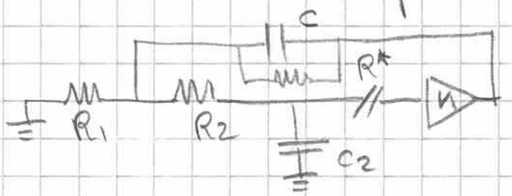



This is the powerful point of the SK cell!!




Even though Gloop is positive, the root locus shows a stable and unstable region. If $K \gg \to$ unstable.

We'll solve the poles by deriving $G_{loop}(s) - 1 = 0$:




$$G_{loop}(s) = \frac{a_1 s + 1}{b_2 s^2 + b_1 s + 1} \cdot \frac{R_1}{R_1 + R^*} \cdot K$$

To bypass the zero in the origin, use $R^*$ and then $R^* \to +\infty$

$$b_2 = C C_2 (R_1 /\!/ R^*) R_2 \xrightarrow[\infty]{} C C_2 R_1 R_2$$

$$b_1 = C(R_1 /\!/ R^*) + C_2(R_2 + R_1 /\!/ R^*) \xrightarrow[\infty]{} C R_1 + C_2(R_1 + R_2)$$

$$a_1 = C R^* \longrightarrow C R^* s + 1 \xrightarrow[\infty]{} C R^* \cdot s$$

$$G_{loop}(s) = K \frac{s R_1 C}{s^2 C C_2 R_1 R_2 + s[C R_1 + C_2(R_1 + R_2)] + 1}$$

Compute now $G_{loop}(s) - 1 = 0 \to$

$$s^2 R_1 R_2 C C_2 + s\left[(1-K) C R_1 + C_2(R_1 + R_2)\right] + 1 = 0$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C C_2}} \qquad Q = \frac{\sqrt{R_1 R_2 C C_2}}{(1-K) C R_1 + C_2(R_1 + R_2)}$$

To exploit the dependance of $Q$ on $R, C$, rewrite $Q$:

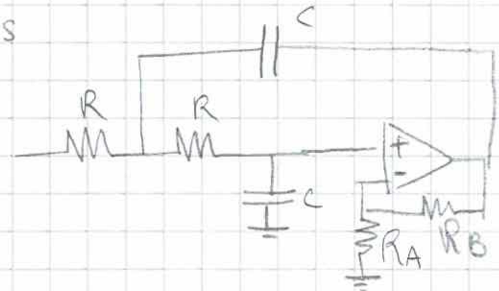
$$Q = \frac{\sqrt{\frac{R_2}{R_1}\frac{C_2}{C_1}}}{(1-K) + \left(1+\frac{R_2}{R_1}\right)\frac{C_2}{C_1}} \quad \text{while} \quad \omega_0 = \frac{1}{\sqrt{R_1 R_2 C C_2}}$$

$\omega_0$ depends on absolute $R, C$ values (and mismatches!)

$Q$ depends on relative mismatch between $R_2, R_1$ / $C, C_2$

If we size $R, C$ areas to be big, variability lowers with area $\left(\frac{1}{\sqrt{WL}}\right)$ so filter $\omega_0, Q$ will change less

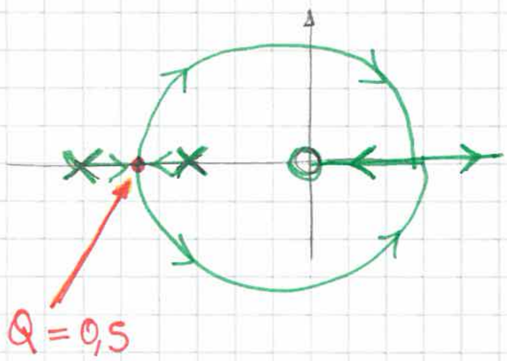
SK with equal values of components

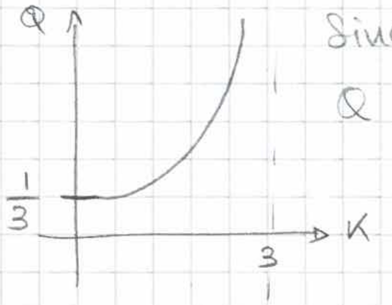



$K = \frac{R_B}{R_A} + 1 \qquad R_1 = R_2 = R \qquad C = C_2 = C$

$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{1}{3-K}$$

If $K \nearrow\nearrow$, poles become complex conjugate as they take off from the root locus. For $Q > 0{,}5$ we have comp. conjugates




Since $K = 1 \to +\infty$

$Q$ goes from $\frac{1}{3}$ to $+\infty$

As we can see, to get high $Q$ we need $K \sim 3$, but

$$\frac{dQ}{dK} = \frac{1}{(3-K)^2} \to \frac{dQ}{Q} = \underbrace{\frac{1}{(3-K)^2}\frac{K}{Q}}\frac{dK}{K}$$

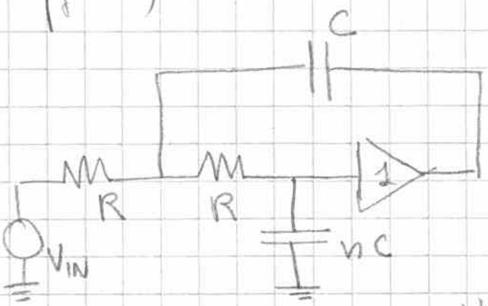
express as a function of $Q$

$$Q = \frac{1}{3-K} \to K = 3 - \frac{1}{Q} \quad \text{therefore}$$

$$\frac{1}{\left(\cancel{3}-\cancel{3}-\frac{1}{Q}\right)^2}\cdot\frac{1}{Q}\cdot\left(3-\frac{1}{Q}\right) = \left(3-\frac{1}{Q}\right)Q \to \frac{dQ}{Q} = \left(3-\frac{1}{Q}\right)Q\,\frac{dK}{K}$$

The last formula shows that for a mismatch of K, the relative variability of Q increases as Q increases, this makes sense if we look the high slope in the Q/K plot (previous page).

To bypass this, we see what happens if we fix K (buffer):




$$\omega_0 = \frac{1}{RC\sqrt{n}} \qquad Q = \frac{\sqrt{n}}{2}$$

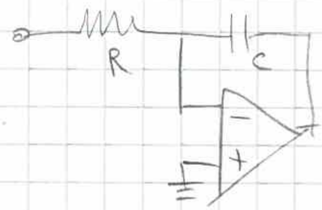
We can derive $\frac{dQ}{Q} = \frac{1}{2}\frac{dn}{n}$

We can see that now relative variability of Q doesn't change.

However, to get high Q $n \gg 1 \rightarrow nC$ occupies a large area

See additional notes to see why SK cell is so powerful!

# 97) Universal cell: block diagram + Tow Thomas



$$\frac{V_{out}(s)}{V_{IN}(s)} = \frac{1}{sRC}$$

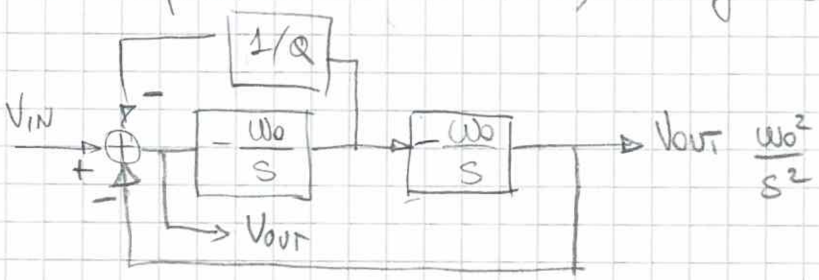
From previous courses we found out that this is unstable (by itself).

Let's do some magic: HPF design

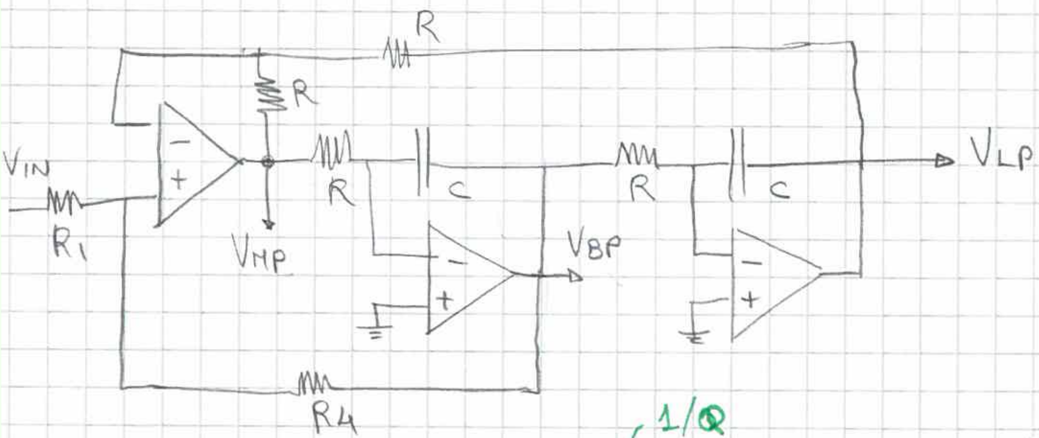
$$T(s) = \gamma \frac{s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \qquad \frac{V_{out}}{V_{IN}} = \frac{\gamma}{1 + \frac{\omega_0}{sQ} + \frac{\omega_0^2}{s^2}}$$

$$V_{OUT}\left(1 + \frac{\omega_0}{sQ} + \frac{\omega_0^2}{s^2}\right) = \gamma V_{IN} \rightarrow V_{OUT} = \gamma V_{IN} - V_{OUT} \cdot \frac{1}{Q} \cdot \frac{\omega_0}{s} - V_{OUT} \frac{\omega_0}{s}\frac{\omega_0}{s}$$

We exploited the T(s) by using single integrators



This can be implemented in a circuit:



$$V_{HP} = V_{IN} \underbrace{\frac{2 \cdot R_4}{R_1 + R_4}}_{\gamma} + V_{BP} \overbrace{\frac{2R_1}{R_1 + R_4}}^{1/Q} - V_{LP}$$

Using superposition of $V_{LP}$, $V_{BP}$, $V_{HP}$

$$\gamma = \frac{2R_4}{R_1 + R_4} \qquad Q = \frac{1 + \frac{R_4}{R_1}}{2} \longrightarrow \gamma = \frac{R_4}{R_1} \cdot \frac{1}{Q}$$

If Q is set, then $\gamma$ will be linked to Q with $R_4/R_1$

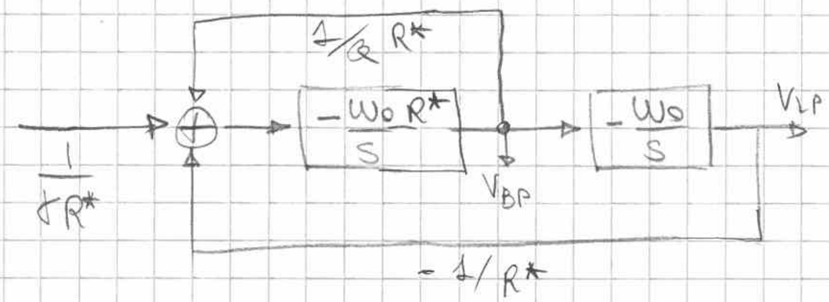
The procedure is helpful to build whatever function:

$$T(s) = \frac{s^2 + \frac{\omega_z}{Q_z}s + \omega_z^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} = \text{split} = \frac{s^2}{(\quad)} + \frac{s\,\omega_z/Q}{(\quad)} + \frac{\omega_z^2}{(\quad)}$$

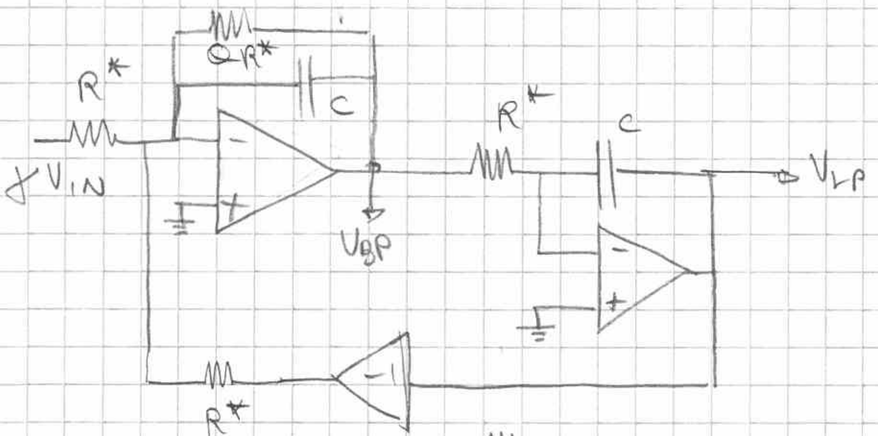
We can always use the general cell and then proceed with a weighted sum to get the wanted output

## Tow Thomas

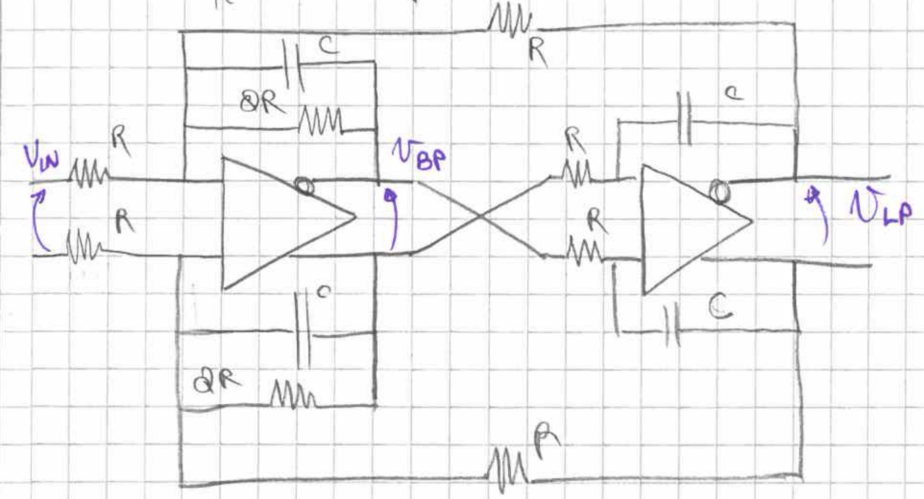
It is possible to simplify the system even more by exploiting the sum of currents at integrator inputs. We switch from voltage to current by using an arbitrary resistor $R^*$




However, the $V_{HP}$ node will be missing therefore the circuit can implement BPF and LPF only




Notice that we still need a $-1$ gain. This is not a problem for fully differential opamps

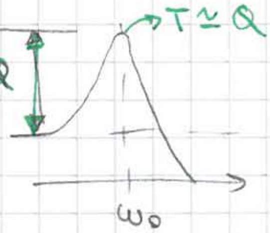


# Filter variability

The more filters we cascade to get the proper (high number of poles/zeros) transfer function, the higher we will be subjected to variability of components.

For example consider $Q = \left(\frac{R_4}{R_1} + 1\right)\frac{1}{2} \longrightarrow \frac{dQ}{Q} = 1 \frac{d(R_4/R_1)}{R_4/R_1}$

Q is sensitive to the ratio of resistor, which is better than the absolute value of the two




In $\omega_0$ we clearly see that $\frac{\Delta T}{T} = \frac{\Delta Q}{Q}$

The relative T variation is directly connected to the relative Q variation. If we consider the absolute T variation

$$\Delta T = T \frac{\Delta Q}{Q} = Q \frac{\Delta Q}{Q} = Q \frac{d(R_4/R_1)}{(R_4/R_1)}$$

$$T|_{\omega=\omega_0} \simeq Q$$

We see that the absolute variation of T increases for an increasing Q given a fixed $\frac{d(R_4/R_1)}{R_4 R_1}$

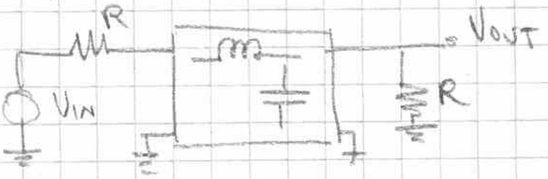
If $\frac{\Delta R}{R} = \frac{K_R'}{\sqrt{WL}}\frac{1}{\sqrt{2}}$ then $\frac{d R_4/R_1}{R_4/R_1} = \frac{dR_4}{R_4} + \frac{dR_1}{R_1} = \frac{K_R'}{\sqrt{2W_4L_4}} + \frac{K_R'}{\sqrt{2W_1L_1}}$

where $K_R' = K_R\sqrt{2}$ given by manufacturer

We see that for a reduced variation we need large area of silicon. Small changes on a large area will be more negligible with respect to a small area resistor

# 28) Ladder Networks: Orchard theorem, implementation. Flow graph. Denormalization

Consider a lossless network (L, C are ideal) in a doubly terminated network:




At the peaks of the transfer function, the reactive elements resonate in a way that the impedance seen $Z_{IN}$ will be R.
The power delivered to the load will then be maximum and its derivative will be nil:

$$P_L = \frac{V_{IN}^2}{2R} |T(j\omega, x_0)|^2 \quad \text{reactive elements}$$

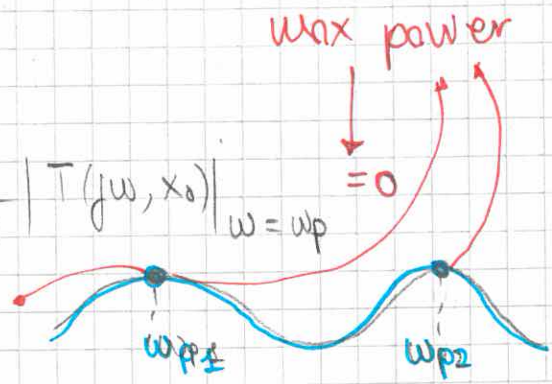
This means that no matter what, the reactive elements on the peaks won't give any contribution → their variability will be negligible.

Exploit the derivative of the power:

$$\left.\frac{\partial P_L}{\partial \omega}\right|_{\omega=\omega_{peak}} = \frac{V_{IN}^2}{2R} \frac{\partial}{\partial \omega} \left.|T(j\omega, x_0)|^2\right|_{\omega=\omega_p} =$$

$$= \frac{V_{IN}^2}{2R} \cdot 2|T(j\omega_p, x_0)| \cdot \frac{\partial}{\partial \omega} \left.|T(j\omega, x_0)|\right|_{\omega=\omega_p}$$

Therefore $\frac{\partial}{\partial \omega} \left.|T(j\omega, x_0)|\right|_{\omega=\omega_p} = 0$



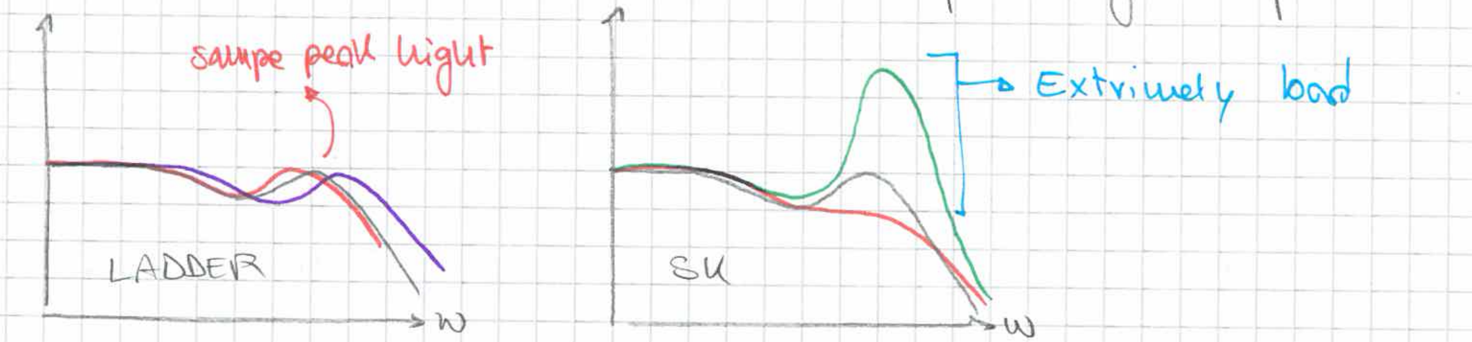

If power is maximum, the transfer function is on a peak.

The transfer function depends also on reactive elements X, but when the $P_L = max$ only R remains, therefore also the derivative on reactive elements is nil:

$$\left.\frac{\partial P_L}{\partial x}\right|_{x=x_0} = \dots \frac{\partial}{\partial x} \left.|T(j\omega, x)|\right|_{\substack{\omega=\omega_{peak} \\ x=x_0}} = 0$$

The last condition is a very powerful result and it's called **Orchard Theorem**.

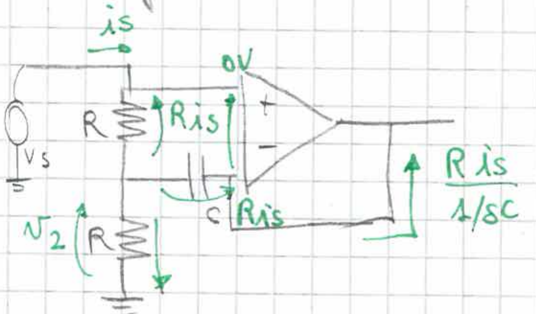
Example: III order cheby implemented with ladder v.s. SK cell. Consider a 10% variation of a single capacitor:




See that peaks move slightly ($w_0$ changes), but the peak hight won't be affected because $P_L\big|_{max} = \frac{V_{IN}^2}{2R}$

## 29) Implementation with gyrators

The issue here is to implement the filter without the use of inductors.



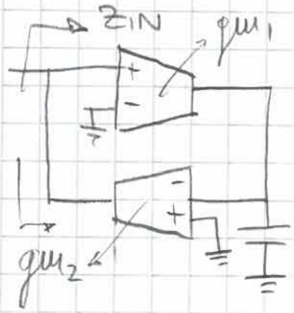

$v_2 = (is + sCRis)R = isR + sCR^2 is$

$v_s = v_2 + v_1 = 2isR + sCR^2 is$

$Z_{IN} = 2R + sCR^2 = R_{eq} + s L_{eq}$

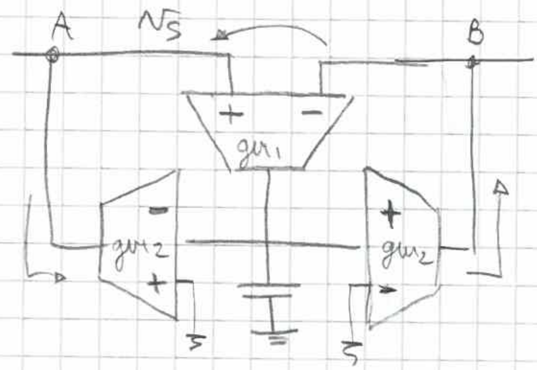
$R_{eq} = 2R \quad L_{eq} = CR^2$

Other kinds of gyrators:


$Z_{IN} = \frac{sC}{gm_1 gm_2}$

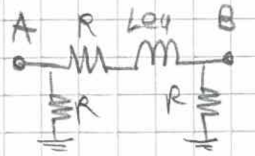
$L_{eq} = \frac{C}{gm_1 gm_2}$





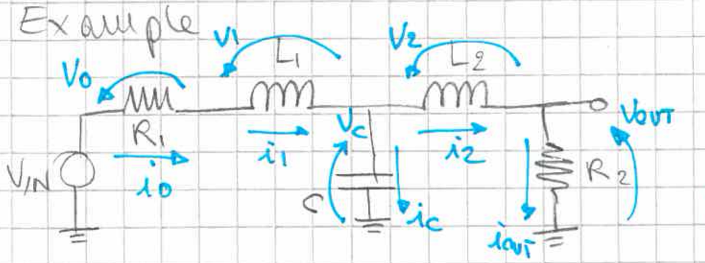
$Z_{IN} = s\frac{C}{gm_1 gm_2}$

Issues with gyrators: finite OTA $R_{OUT}$, mismatches on $g_m$, noise (we use active elements), distortion, BW is limited because of the GBWP of OTAs, voltage swing is limited, common voltage is limited for floating inductors. The real inductor model will be something like



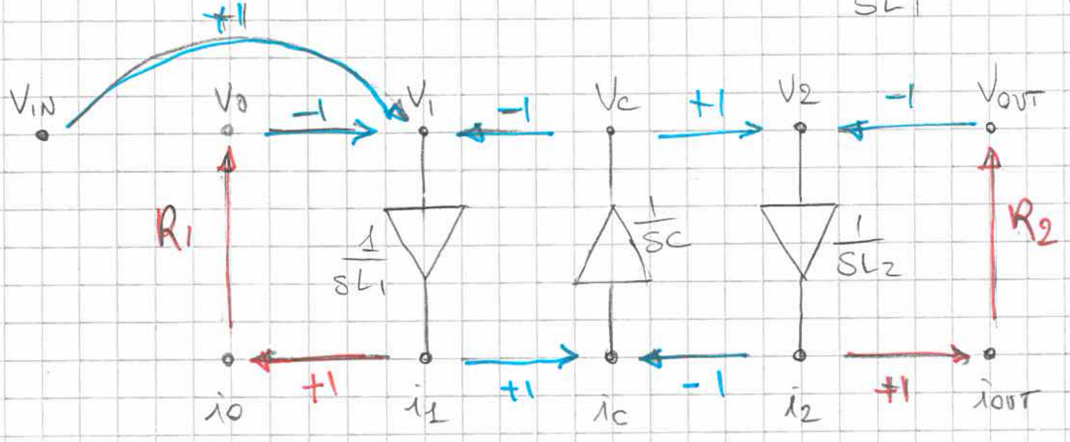

28 part II) Flowgraph procedure

Example



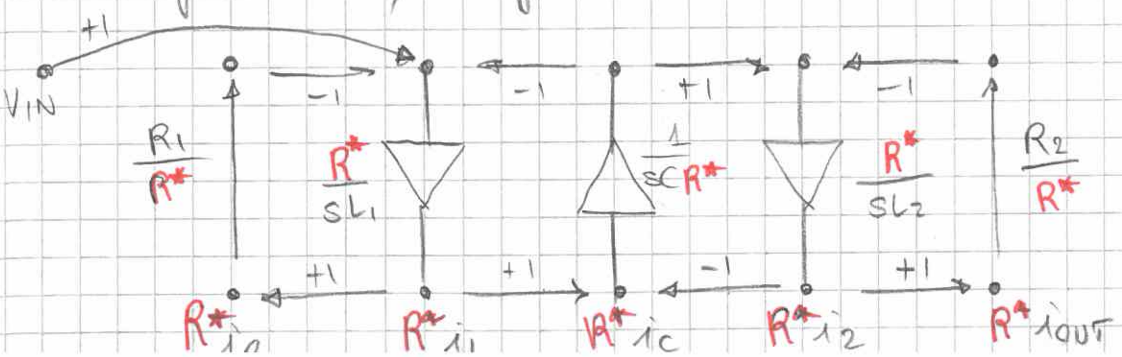

1) highlight all currents and voltages

2) Exploit the integration link between reactive elements $I_1 = \frac{V_1}{sL_1}$ $I_2 = \frac{V_2}{sL_2}$ $V_c = \frac{I_c}{sC}$

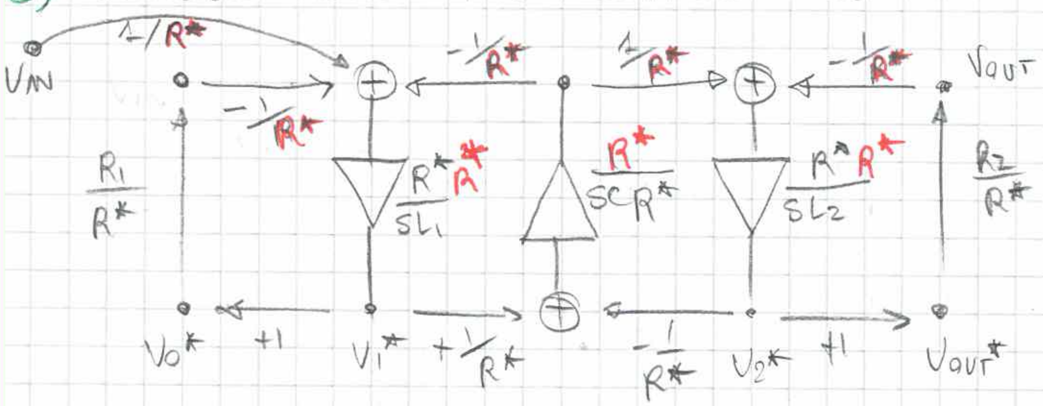



3) Then, starting from the integrator input, link all the variables by looking at all the voltage/current relations. Then, finish connecting the rest of the nodes, note he $i_0 \cdot R_1 = V_0$ and $i_{OUT} \cdot R_2 = V_{OUT}$

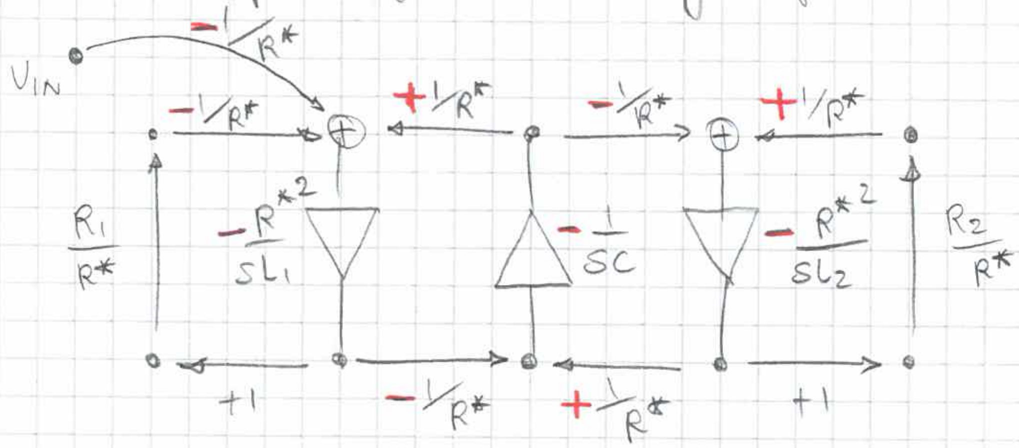
4) It's more convenient to sum currents instead of voltages (less opamps used) change $i_0, i_c, i_1, i_2, i_{out}$ into voltages and adjust everything



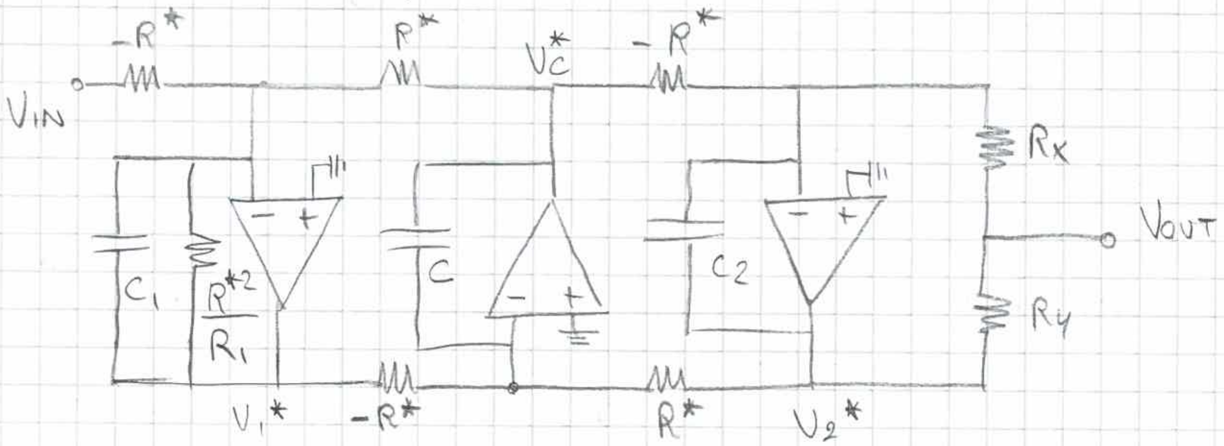
5) Continue with the current sum




6) Final step: adjust the sign for inverting integrators




We can now implement the real circuit

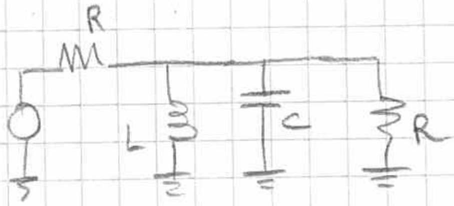



$$R_x + R_y = \frac{R^{*2}}{R_2} \qquad \frac{R_y}{R_x + R_y} = \frac{R_2}{R^*} \quad \Bigg] \rightarrow \quad \underline{R_x = \frac{R^{*2}}{R_2} - R^*} \quad \underline{R_y = R^*}$$

$$\underline{C_1 = L_1 / R^{*2}} \qquad \underline{C_2 = L_2 / R^{*2}}$$

# Denormalization procedure

Ladder networks are usually listed for 1 rad/s.
To shift the filter cutoff we need to scale components so that we can have the proper $w_0$ and a realistic capacitor size. For example:




$$\longrightarrow T(s) = \frac{1}{2} \frac{s \frac{w_0}{Q}}{\left(s^2 + s\frac{w_0}{Q} + w_0^2\right)}$$

$$w_0 = \frac{1}{\sqrt{LC}} \qquad Q = w_0 \frac{R}{2} \cdot C$$

To shift from 1 rad/s to $w_0$ we need

$$w_0' = N \frac{1}{\sqrt{C^\circ L^\circ}} \longrightarrow w_0' = \frac{1}{\sqrt{\frac{C^\circ}{N}\frac{L^\circ}{N}}} \qquad \underline{C' = \frac{C^\circ}{N}} \qquad \underline{L' = \frac{L^\circ}{N}}$$

$$Q' = \cancel{N} w_0 \cdot \frac{R}{2} \cdot \frac{C}{\cancel{N}} = Q^\circ \rightsquigarrow Q \text{ doesn't change}$$

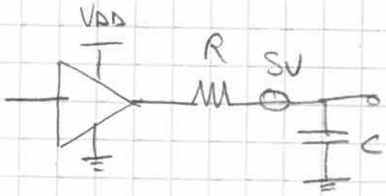
To change resistor size we need to: keep the same $Q$ without changing $w_0$, so

$$\underline{R' = M R^\circ} \longrightarrow Q' = \underbrace{w_0}_{\text{unchanged}} \cdot \frac{\overbrace{R \cdot \cancel{M}}^{R'}}{2} \cdot \frac{C^\circ}{\cancel{M}}$$

we need to divide $C$ by $M$ so that $Q' = Q^\circ$

If $\underline{C'' = \frac{C^\circ}{N \cdot M}}$ then $\underline{L'' = \frac{M}{N} L^\circ}$ so $w_0$ doesn't change with the new $R$

## 30) Dynamic Range: ENOB, guidelines to reduce noise floor




$$\rightsquigarrow \left(\frac{S}{N}\right)^2 = \frac{\frac{A^2}{2}}{\cancel{4}KT\cancel{R}\cdot\frac{1}{\cancel{4}RC}} = \frac{\frac{A^2}{2}}{\frac{KT}{C}}$$

This result is obtained using a noiseless opamp, if

$A = \frac{V_{DD}}{2} \rightarrow$ half range $\quad (SNR)^2 = \dfrac{V_{DD}^2/8}{\frac{KT}{C}}$

With noisy opamps $\rightarrow (SNR)^2 = \dfrac{V_{DD}^2/8}{\frac{KT}{C}(1+F)}$ Where F is an additional noise factor that degrades the SNR because of active elements

More in general, if signal amplitude is lower:

$$SNR = \sqrt{\frac{\alpha^2 V_{DD}^2/8}{\frac{KT}{C}(1+F)}} = DR \text{ dynamic range}$$

For a given frequency cutoff: If $C \nearrow\nearrow$, $R \searrow\searrow \rightarrow S_V \searrow\searrow$

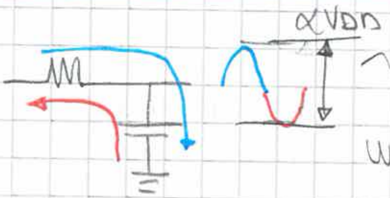
Higher capacitor values minimize the noise.

If the system is connected to an ADC:

$$2^n \geq DR \qquad n \geq \log_2 DR = \frac{\log_{10} DR}{\log_{10} 2} \qquad n \geq \frac{DR|_{dB}}{6dB}$$

$$DR_{dB} = 20\, n \log 2 = 6{,}02dB \cdot n$$

## Noise - power dissipation link




$\rightsquigarrow$ We're charging, discharging the capacitor with a $\alpha V_{DD}$ signal

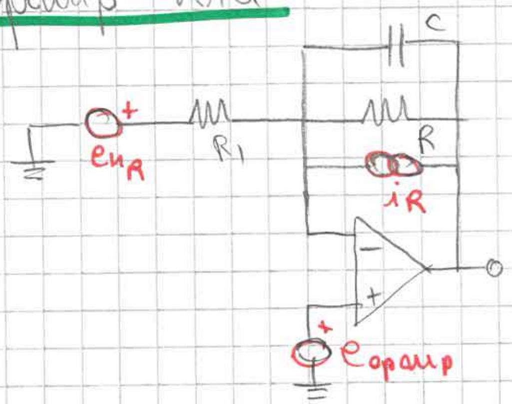
$$Q = C\Delta V = \alpha V_{DD} \cdot C \rightsquigarrow \text{asked charge}$$

We need to calculate the energy dissipated per cycle.

We're taking charge from power supply $V_{DD}$

$$E_{DISS} = \underbrace{\alpha V_{DD} \cdot C}_{Q} \cdot \underset{\text{supply}}{\underset{\uparrow}{V_{DD}}} \rightsquigarrow \text{for a complete cycle} \qquad P_{DISS} = \frac{\alpha V_{DD}^2 C}{T_S} = \alpha V_{DD}^2 C f_S$$

# Opamp noise




We want to calculate all the contributions to the output noise

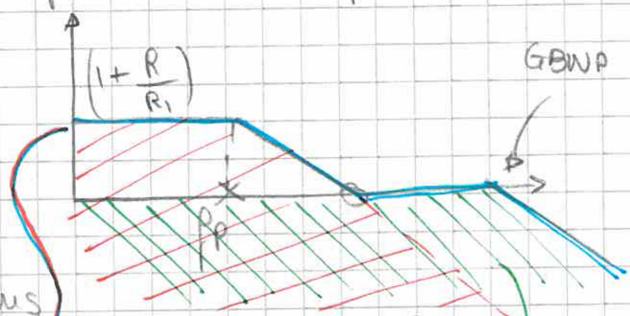
$$G = -\frac{R}{R_1} \qquad \omega_0 = \frac{1}{RC}$$

- **$R_1$** : $S_{OUT} = 4KTR_1 G^2 \left| \frac{1}{1+S/\omega_0} \right|^2 = 4KTR_1 G^2 \cdot \frac{\omega_0}{4}$

- **$R$** : $S_{OUT} = 4KTR \cdot \frac{\omega_0}{4}$

- **opamp** : noise gen at $V^+$ sees a different transfer function:

$$T(s) = 1 + \frac{R}{R_1} \frac{1+s(R/\!/R_1)C}{1+sRC}$$




$S_{out}$ can be seen as a superposition of two contributions

$$S_{OUT}\Big|_{OPAMP} = \underbrace{S_V \cdot (1+G)^2 \cdot \frac{\omega_0}{4}}_{\text{RED AREA}} + \underbrace{S_V \cdot 1^2 \cdot \frac{\omega_0}{4}}_{\text{GREEN AREA}}$$

This is however just an approximation (total noise will be lower → look at tables to find the correct estimate)

- Lastly, input noise will lead to $S_{OUT} = S_{IN} \cdot G^2 \frac{\omega_0}{4}$

We finally can compare the various sources to input noise:

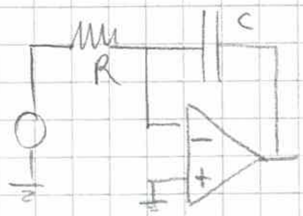
$$\langle V_n^2\rangle = S_{IN}\, G^2 \frac{\omega_0}{4}\left[1+\underbrace{\frac{4kTR_1}{S_{IN}}}_{1}+\underbrace{\frac{4kTR}{S_{IN}G^2}}_{2}+\underbrace{\frac{S_{VA}}{S_{IN}}\frac{(1+G)^2}{G^2}}_{3}+\underbrace{\frac{S_{VA}}{S_{IN}}\frac{1}{G^2}\frac{\omega_u}{\omega_0}}_{4}\right]$$

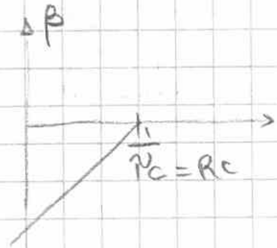
1) Use low $R_1$ values as it directly compares with input noise
2) Increase G? Not bad because a higher gain increases input noise as well as input signal
3) G does not have any influence $\left(\frac{1+G^2}{G^2}\sim 1\right) \rightarrow$ Just select a low noise opamp so that $S_{VA} \ll S_{IN}$
4) Gain reduces this contribution. $\omega_u = \frac{GBWP}{2\pi}$ so ~~↓~~ $\omega_u$ would lower noise. However $2\pi\, GBWP = \omega_u$ so it's definitely a bad idea.

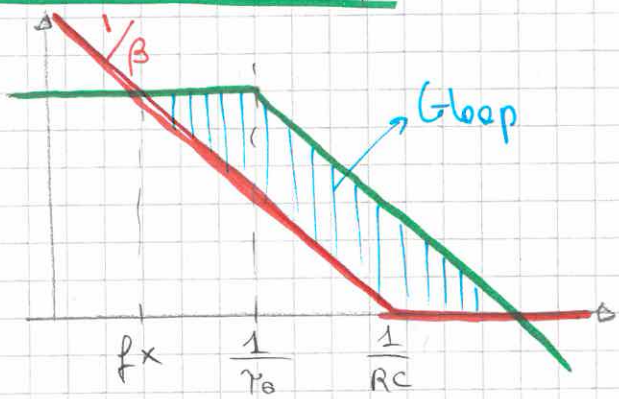
Instead, we could add a LPF at the output that cuts at $\omega_0$ with the small contribution of $\frac{kT}{C}$

Note: $S_{VA} = \frac{8kT\gamma}{g_m}\left(\frac{3}{2}\right) \rightarrow$ Lower $S_{VA}$ means higher $g_m$ therefore higher power dissipation (bias current ↗↗)

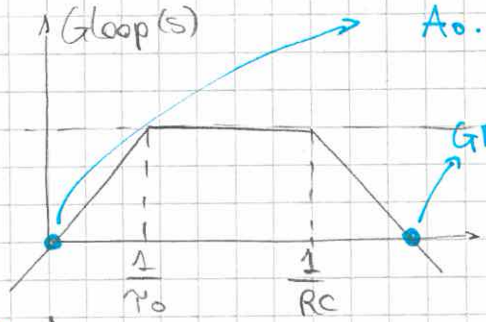
# 31) Impact of opamp non-idealities on $\omega_0$ and $Q$ for biquad cells



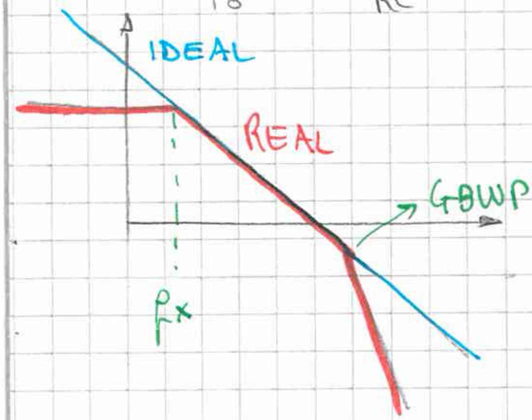




$$\frac{1}{\beta} = 1 + \frac{\omega_0}{s} \qquad |A(\omega)| = \frac{A_0}{1 + s\tau_0}$$




$$A_0 \cdot f_x = 1 \cdot \frac{1}{2\pi RC} \longrightarrow f_x = \frac{1}{2\pi A_0 RC}$$

Gloop is > 1 in the range $f_x$ and $\frac{1}{2\pi RC}$




For an ideal integrator: $\rightarrow \boxed{-\frac{\omega_0}{s}} \rightarrow$

With the real one we get:

$$\rightarrow \boxed{\frac{-A_0}{\left(1+\frac{s}{\omega_x}\right)\left(1+\frac{s}{\omega_0}\right)}} \rightarrow$$

$$\omega_x = \frac{1}{A_0 RC} \qquad \omega_0 = 2\pi GBWP$$

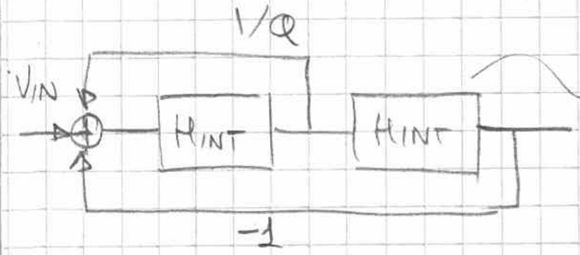
$A_0 RC = \omega_0$

Let's use this in the biquad cell.

$$T(s)\Big|_{BIQUAD} = \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \rightarrow \text{highlight the integrators}$$

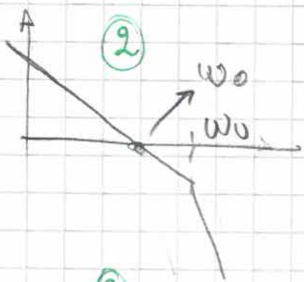
$$= \frac{\left(-\frac{\omega_0}{s}\right)^2}{\left(-\frac{\omega_0}{s}\right)^2 - \frac{1}{Q}\left(-\frac{\omega_0}{s}\right) + 1} = \frac{H_{INT}^2(s)}{H_{INT}^2(s) - \frac{1}{Q} H_{INT}(s) + 1}$$

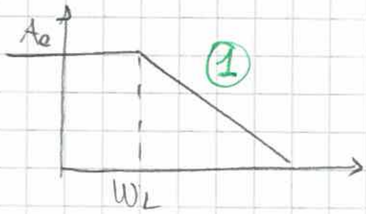
With the usual topology:




Where now $H_{INT}(s)$ takes into account real integrators

The full transfer function is difficult to analyze, so split the analysis with the two poles:




$$\underline{H_{INT}(s)}^{②} = \frac{-A_0}{1+\frac{s}{\omega_L}} = \underline{\frac{1}{\frac{1}{A_0}+\frac{s}{\omega_0}}} \qquad \omega_L = \frac{1}{A_0 RC}$$

$$\underline{H_{INT}(s)}^{①} = -\underline{\frac{\omega_0}{s\left(1+\frac{s}{\omega_0}\right)}}$$

$$T'(s)^{②} = \frac{\frac{1}{\left(\frac{1}{A_0}+\frac{s}{\omega_0}\right)^2}}{\frac{1}{\left(\frac{1}{A_0}+\frac{s}{\omega_0}\right)^2}+\frac{1}{Q\left(\frac{1}{A_0}+\frac{s}{\omega_0}\right)}+1} = \frac{1}{\left(\frac{1}{A_0}+\frac{s}{\omega_0}\right)^2+\frac{1}{Q}\left(\frac{1}{A_0}+\frac{s}{\omega_0}\right)+1} =$$

$$= \frac{\omega_0^2}{s^2+s\,\omega_0\left(\frac{1}{Q}+\frac{2}{A_0}\right)+\omega_0^2\left(1+\frac{1}{QA_0}+\frac{1}{A_0^2}\right)}$$

$$\boxed{\omega_0' = \omega_0\sqrt{1+\frac{1}{A_0 Q}+\frac{1}{A_0^2}}}$$

→ for $\underline{A_0 \to \infty} \to \underline{\omega_0' = \omega_0}$

→ for $\underline{\text{finite } A_0} \to \underline{\text{minor } \omega_0 \text{ shift}}$

$\frac{\omega_0'}{Q'} = \omega_0\left(\frac{1}{Q}+\frac{2}{A_0}\right)$ approx to $\omega_0' \sim \omega_0$, then $\frac{1}{Q'} \simeq \frac{1}{Q}+\frac{2}{A_0}$

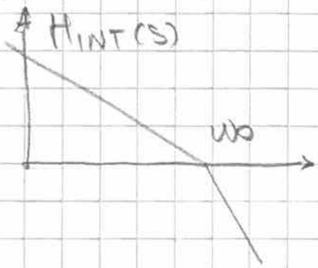
$$\frac{1}{Q'}-\frac{1}{Q} = \frac{2}{A_0} \qquad \frac{Q-Q'}{QQ'} = \frac{2}{A_0} \qquad \frac{Q'-Q}{Q'} = -\frac{2}{A_0}Q$$

$$\frac{Q'-Q}{Q} \simeq \frac{\Delta Q}{Q} \quad \text{so} \quad \boxed{\frac{\Delta Q}{Q} \simeq -\frac{2}{A_0}Q}$$

We see Q factor will be quenched because of the opamp finite DC gain

We can allow a % of Q error that is $\varepsilon_{ERR}\% = \left|\frac{2Q}{A_0}\right|$ thus deriving the minimum $A_0$ requirement

This type of discussion can be made for every active cell filter.




Consider now $H_{INT}' = -\frac{\omega_0}{s}\cdot\frac{1}{1+\frac{s}{\omega_U}}$

$$T(s) = \frac{\frac{\omega_0^2}{s^2\left(1+\frac{s}{\omega_U}\right)^2}}{\frac{\omega_0^2}{s^2\left(1+\frac{s}{\omega_U}\right)^2}+\frac{\omega_0}{Q\,s\left(1+\frac{s}{\omega_U}\right)}+1} = \frac{\omega_0^2}{s^2\left(1+\frac{s}{\omega_U}\right)+\frac{\omega_0}{Q}s\left(1+\frac{s}{\omega_U}\right)+\omega_0^2}$$

$$= \frac{1}{\frac{s^2}{\omega_0^2}\left(1+\frac{s}{\omega_U}\right)^2+s\frac{1}{\omega_0 Q}\left(1+\frac{s}{\omega_U}\right)+1}$$

We get the poly

$$\left(1+\frac{s}{\omega_U}\right)^2\left[\frac{s^2}{\omega_0}+\frac{s}{\omega_0 Q\left(1+\frac{s}{\omega_U}\right)}+\frac{1}{\left(1+\frac{s}{\omega_U}\right)^2}\right] \doteq 0$$

It's a 4-th order polinomial. However if GBWP is large enough $\omega_0 \ll \omega_U$, we can assume that $\frac{s}{\omega_U} \ll 1$ (we're interested in peak reduction that is around $\omega_0$, so $\frac{s}{\omega_U} \simeq \frac{\omega_0}{\omega_U} \ll 1$), therefore we can approx:

$$\frac{1}{\left(1+\frac{s}{\omega_U}\right)^2} \simeq \left(1-\frac{s}{\omega_U}\right)^2 \quad \text{and} \quad \frac{1}{\left(1+\frac{s}{\omega_U}\right)} \simeq \left(1-\frac{s}{\omega_U}\right)$$

This way the polynomial is reduced to:

$$\frac{s^2}{\omega_0^2}+\frac{s}{Q\omega_0}\left(1-\frac{s}{\omega_U}\right)+\left(1-\frac{s}{\omega_U}\right)^2=0$$

$$\frac{s^2}{\omega_0}\left[1-\frac{\omega_0}{Q\omega_U}+\left(\frac{\omega_0}{\omega_U}\right)^2\right]+\frac{s}{\omega_0}\left(\frac{1}{Q}-2\frac{\omega_0}{\omega_U}\right)+1=0$$

$$s^2+s\frac{\omega_0}{1-\frac{\omega_0}{Q\omega_U}+\left(\frac{\omega_0}{\omega_U}\right)^2}\cdot\frac{\left(1-2\frac{\omega_0}{\omega_U}Q\right)}{Q}+\frac{\omega_0^2}{1-\frac{\omega_0}{\omega_U}+\left(\frac{\omega_0}{\omega_U}\right)^2}=0$$

We can finally derive $\omega_0'$, $Q'$:

$$\omega_0' = \frac{\omega_0}{\sqrt{1 - \frac{\omega_0}{Q\omega_u} + \cancel{\left(\frac{\omega_0}{\omega_u}\right)^2}}} \rightarrow \text{slightly larger than } \omega_0$$

$\omega_0 \ll \omega_u$

We then have $\omega_0 \left[\frac{1}{Q} - \frac{2\omega_0}{\omega_u}\right] \rightarrow \frac{1}{Q'} = \frac{1}{Q} - \frac{2\omega_0}{\omega_u}$

$$\frac{1}{Q'} - \frac{1}{Q} = -\frac{2\omega_0}{\omega_u} \qquad \frac{Q'-Q}{QQ'} \simeq \frac{\Delta Q}{Q} = \frac{2\omega_0}{\omega_u}$$

## Recap

Finite DC gain → minor $\omega_0$ shift (slightly larger)

→ $\frac{\Delta Q}{Q} = -\frac{2}{A_0} Q$

Finite BW → minor $\omega_0$ shift (slightly larger)

→ $\frac{\Delta Q}{Q} = \frac{2\omega_0}{\omega_u}$

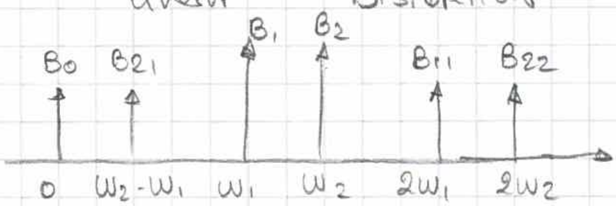
We see that for the finite BW limit, $Q$ is enhanced.
We therefore have two effects, droop because of finite DC and enhancement because of the finite BW.
We need to take into account all the contributions:

$$\frac{\Delta Q}{Q} \simeq Q\left[2\frac{\omega_0}{\omega_u} + 2\frac{\omega_0}{\omega_{p2}} + \dots - \frac{2}{A_0}\right]$$

## Distortion

$x_{IN}(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$

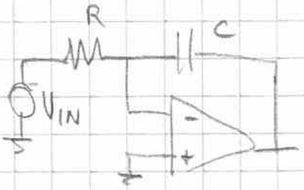

$$y(t) = \underbrace{\alpha_1 x_{IN}(t)}_{\text{linear}} + \underbrace{\alpha_2 x_{IN}^2(t)}_{\text{DISTORTION}}$$



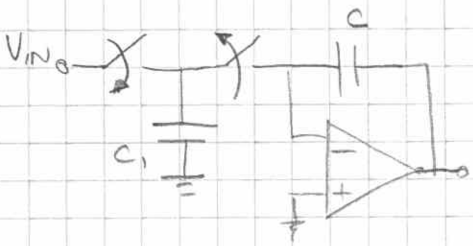

Even though we're filtering our input harmonics, filter distortion can generate other harmonics, this process is called spectral regrowth

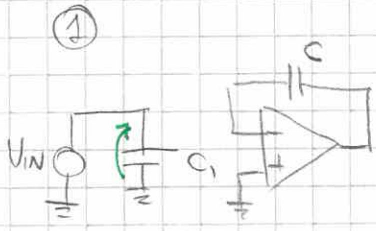
## 32) SW cap filters: Motivations, concepts, implementation + stray insensitive topologies

Audio range is BW ~ 10 kHz → with 1pF capacitors we would need ~10 MΩ → even with $R_\square = 1k\Omega/\square$ the area needed would be too large for implementation. Solution → SW capacitors

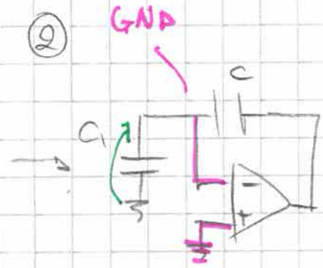



$V_{OUT} = -\frac{V_{IN}\, t}{RC}$ with DC input




① 

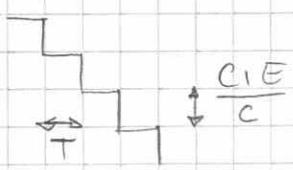

$Q = V_{IN} \cdot C_1$

② 


1) $V_{in}$ charges $C_1$ → $Q_{C_1} = V_{IN} \cdot C_1$ → sampling of $V_{IN}$

2) $C_1$ is between gnd and virtual gnd → charge is transferred on C → $\Delta Q_C = C_1 V_{IN} = C \cdot V_{OUT}$ $\Delta V_c = \frac{C_1}{C} V_{IN}$

Assuming $C, C_1$ ideal, we'll see a staircase at the output: because we're continuously applying new charge to C




→ Slope $\frac{\Delta V_c}{T}$ will be $\frac{C_1 E}{C T} = \frac{\Delta V_c}{T}$ if

Linear analog integrator ramp is $\frac{\Delta V_{OUT}}{\Delta t} = -\frac{V_{IN}}{RC}$ while it is $\frac{\Delta V_{OUT}}{\Delta t} = -\frac{V_{IN}\, C_1}{T C}$ → We see a similarity by posing

$$R_{eq} = \frac{T}{C_1} = \frac{1}{f_s \cdot C_1} \rightarrow T = 10\mu s,\ C_1 = 1pF \rightarrow R_{eq} = 10\,M\Omega$$

It's convenient to use switched caps

The result can be found in an alternative way by saying:

$Q_1 = C_1 V_{IN}$ $i_{average} = \frac{C_1 V_{IN}}{T}$ in a period T

$i_{IN} = \frac{C_1 V_{IN}}{T}$ the same happens for a resistor $i_{IN} = \frac{V_{IN}}{R}$

Thus deriving $R_{eq} = \frac{T}{C_1}$

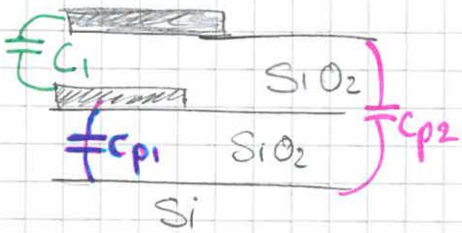

The resulting unity gain frequency is $\omega_0 = \frac{1}{R_{eq} \cdot C}$

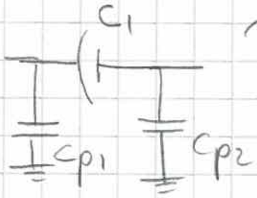
Of course (aliasing) $f_{CLOCK} \gg BW$ of signal

At the same time $f_{CLOCK} \ll GBWP$ of the amplifier in order to provide a good virtual ground and limiting nonlinearity

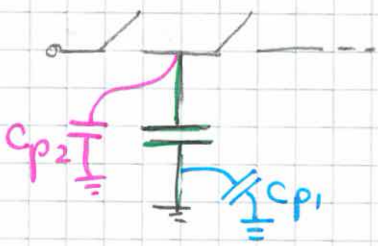
## Stray insensitive topologies

Integrated capacitors suffer from (ironically) parasitic capacitances:




We need to take care of this



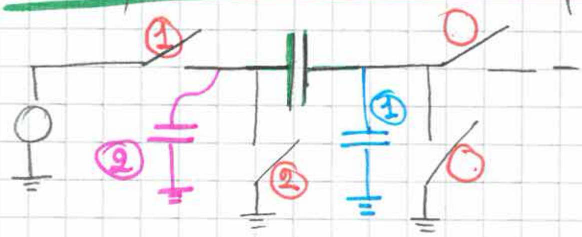

Since $C_{p1} > C_{p2}$ (look at the picture), it's better to short it to gnd

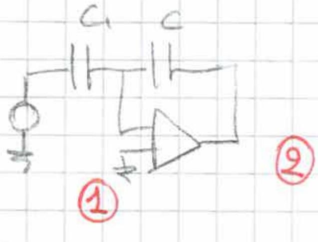
Now only $C_{p2}$ generates issues

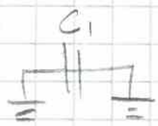
$R_{eq} = \frac{T}{C_1 + C_{p2}}$ We can't control $C_{p2}$ value

Can we eliminate it?

## Stray insensitive configuration



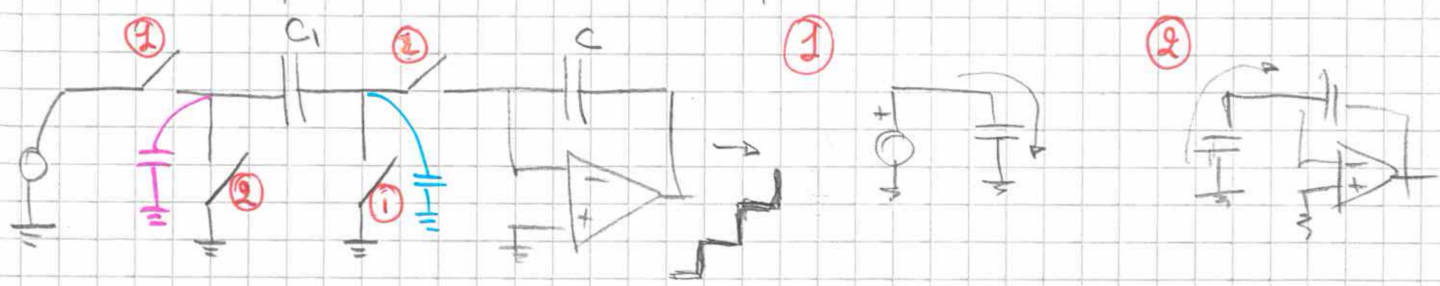




1 cap is tied to Vgnd during 1 and gnd during 2

2 cap is charged by Vin on 1, shorted by 2

Therefore $V_{OUT}$ will not suffer from Cp (ideally)

Moreover, if we now switch phases:

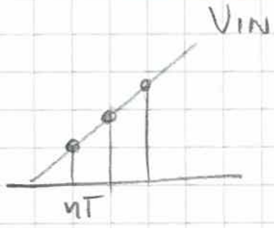


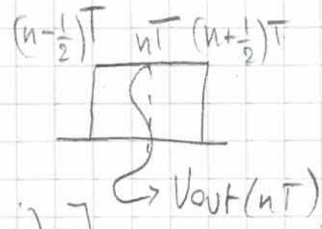

It's easy to see that Cp can't contribute either.

With these switched phases, we can also implement a non inverting integrator with ease, so Tow Thomas -R* can be implemented without additional opamps

# 33) SC : sampling, tf, out spectrum. Anti-aliasing filter and $f_{CK}$

**Vout spectrum** : series of rectangles :




$$V_{OUT}(t) = \sum_{n=0}^{+\infty} V_{OUT}(nT)\left\{ 1\left[t-\left(n-\frac{1}{2}\right)T\right] - 1\left[t-\left(n+\frac{1}{2}\right)T\right]\right.$$


Laplace

$$V_{OUT}(s) = \sum_{n=0}^{+\infty} V_{OUT}(nT)\,\frac{1}{s}\left\{ e^{-s(n-\frac{1}{2}T)}\; e^{-s(n+\frac{1}{2})T}\right\}$$

$$= \sum_{n=0}^{+\infty} V_{OUT}(nT)\,\frac{1}{s}\, e^{-snT}\left[ e^{s\frac{T}{2}} - e^{-s\frac{T}{2}}\right]$$

Fourier

$$V_{OUT}(j\omega) = \sum_{n=0}^{+\infty} V_{OUT}(nT)\,\frac{1}{2j\omega\frac{T}{2}}\cdot e^{-j\omega nT}\cdot\frac{2}{2}T\left[ e^{-j\omega\frac{T}{2}} - e^{+j\omega\frac{T}{2}}\right] \rightarrow \operatorname{sinc}\left(\frac{\omega T}{2}\right)$$

$s = j\omega$

$$V_{OUT}(j\omega) = \sum_{n=0}^{+\infty} V_{OUT}(nT)\, e^{-j\omega nT}\cdot T\operatorname{sinc}\left(\frac{\omega T}{2}\right)$$

$\hookrightarrow$ This is the nT delay of Vout (nT)

$$V_{OUT}(j\omega) = \sum_{n=0}^{+\infty} V_{OUT}(nT)\, e^{-j\omega nT} \Rightarrow V_{OUT}(z) = \sum_{n=0}^{+\infty} V_{OUT}(n)\, z^{-n}$$

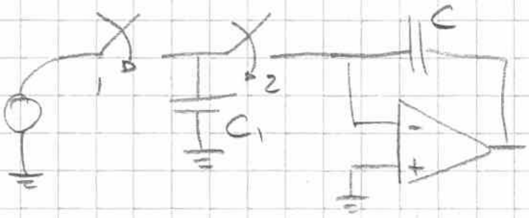
definition of z transform

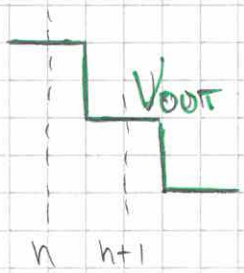
Therefore

$$V_{OUT}(j\omega) = \underbrace{V_{OUT}(z)\Big|_{z=e^{j\omega T}}}_{\text{output sequence z-transform}} \cdot T\operatorname{sinc}\left(\omega\frac{T}{2}\right)$$

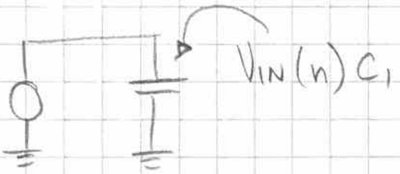
## Filter transfer function $H(z)$

We now need to find the relation between $V_{IN}$ and $V_{OUT}$:




@ time = n




@ time = n+1

$Q_C = C_1 \cdot V_{IN}$

$V_{OUT}(n+1) = -V_C(n+1)$

Therefore we can write

$$V_{OUT}(n+1) = V_{OUT}(n) - V_{IN}(n)\,\frac{C_1}{C}$$

$$V_{OUT}(z)\,z = V_{OUT}(z) - V_{IN}(z)\,\frac{C_1}{C} \longrightarrow \frac{V_{OUT}(z)}{V_{IN}(z)} = -\frac{C_1}{C}\,\frac{1}{z-1}$$

$$H(z) = -\frac{C_1}{C}\,\frac{1}{z-1}$$

$\rightarrow$ sampled transfer function of the circuit

# Total output spectrum

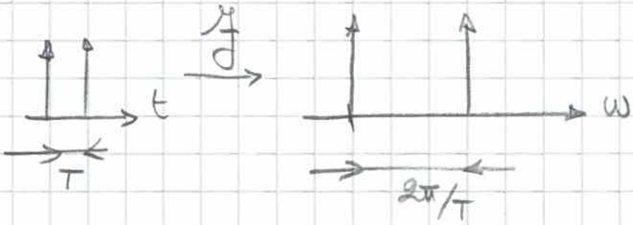
$$V_{OUT}(j\omega) = V_{IN}(z) \cdot H(z)\Big|_{z=e^{j\omega T}} \cdot T\,\mathrm{sinc}\left(\frac{\omega T}{2}\right)$$

Where:

$V_{IN}(z)\Big|_{z=e^{j\omega T}} = \sum_{0}^{+\infty} V_{IN}(n)\, e^{-j\omega T}$ which is the Fourier transform of $V_{IN}(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT)$ ⤳ sampled $V_{IN}$

$$V_{IN}(z)\Big|_{z=e^{j\omega T}} = \mathcal{F}\left[V_{IN}(t) \sum \delta(t-nT)\right] = V_{IN}(j\omega) * \mathcal{F}\left(\sum \delta(t-nT)\right) =$$

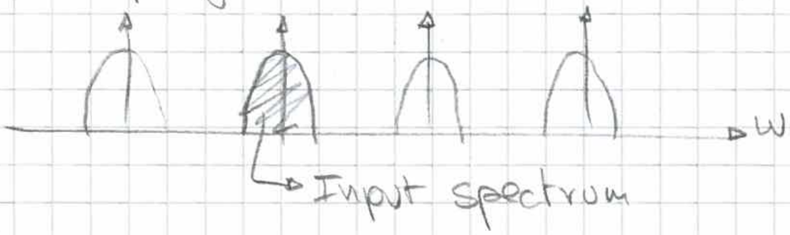
$$= V_{IN}(j\omega) * \sum_{k=-\infty}^{+\infty} \frac{2\pi}{T}\, \delta\left(\omega - \frac{2\pi}{T} k\right)$$




Therefore:

$$V_{out}(j\omega) = \underbrace{V_{IN}(j\omega) * \frac{2\pi}{T} \sum \delta\left(\omega - \frac{2\pi}{T} k\right)}_{*1} \cdot \underbrace{H(z)\Big|_{z=e^{j\omega T}}}_{*2} \cdot T\,\mathrm{sinc}\left(\frac{\omega T}{2}\right)$$

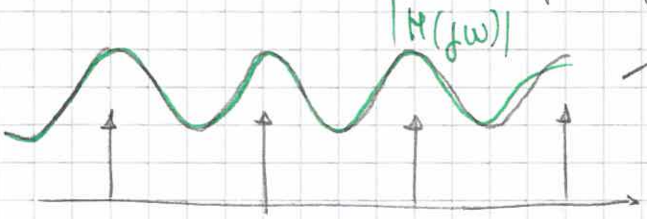
(*1) sampling in time domain → replicas in frequency domain




(*2) $H(z)\Big|_{z=e^{j\omega T}} = -\frac{C_1}{C} \frac{1}{e^{j\omega T}-1} = -\frac{C_1}{C} \frac{1}{\cos(\omega T) + j\sin(\omega T) - 1}$

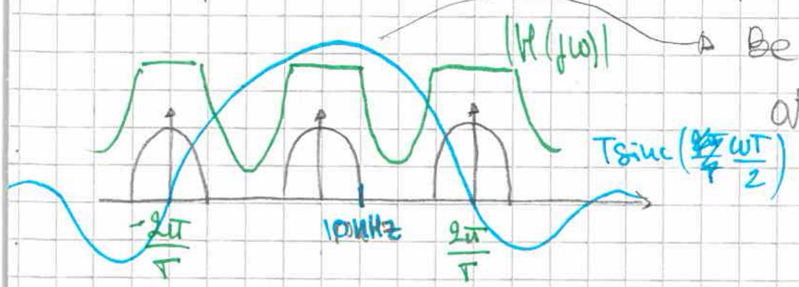
$$\left|H(z)\right|_{z=e^{j\omega T}} = \frac{C_1}{C} \frac{1}{\sqrt{\left[-1+\cos(\omega T)\right]^2 + \sin(\omega T)^2}}$$

→ This has a transfer function that is something like:




⤳ The simple integrator will have $H(j\omega) \to +\infty$ on each $\delta$ as it would do an analog one

Every filter will have a different $H(z)$, but the replicas + sinc function are typical of SC themselves, so

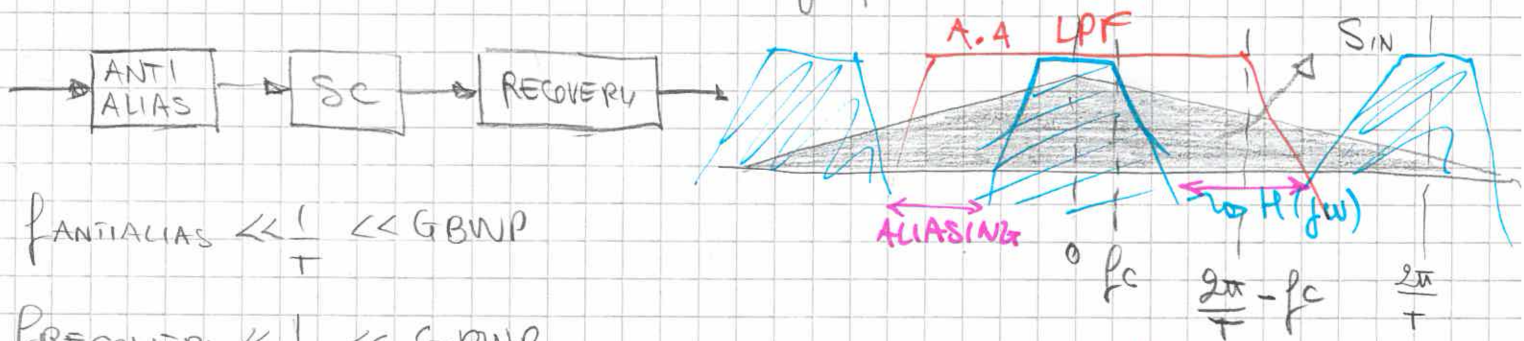



→ Be careful of the in-band attenuation given by the sinc

$H(j\omega)$ shows $2\pi f$ periodicity because after the sampling, the circuit is no more able to recognize $f_0$ harmonic from the others at $f_0 + \frac{2n\pi}{T}$.

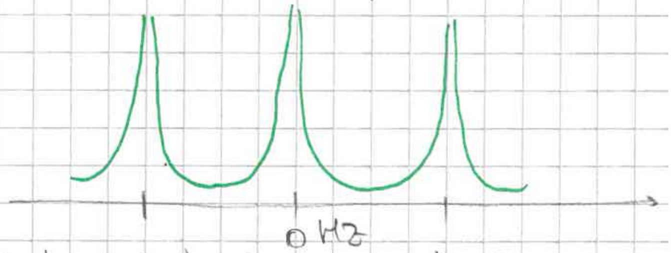
Sinc is related to the stepped output. However, if the SC out is read by an ADC, the sinc is lost since we need not to acquire the signal in continuous time domain.

Also, since output BW is limited to the filter, to get rid of replicas we can use an analog filter:




$f_{ANTIALIAS} \ll \frac{1}{T} \ll GBWP$

$f_{RECOVERY} \ll \frac{1}{T} \ll GBWP$ → 100M, ↑ 100k




Note: for a simple SC integrator for 0Hz it goes to ∞ like an analog integrator does

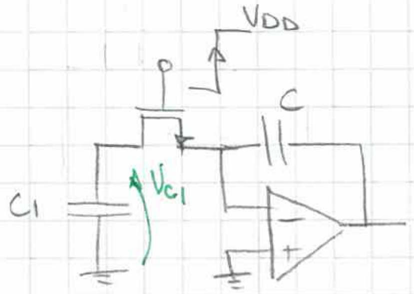
AA filter: $f_{AA} = f_{CK} - f_0$ ($f_0$ = cutoff of the integrator)

This means that for $f < f_0$ → no aliasing

For $f_0 < f < f_{CK} - f_0$ → ALIASING but it's filtered by H (and by sinc for a little part). This means that $f_{AA}$ can be relaxed with respect to $f_0$

## 34) Non idealities: trade-off between settling time and charge sharing in sizing the switches

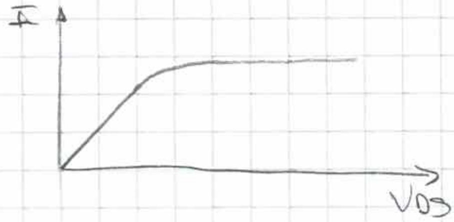
Switches are not ideal. Transient time needs to be low enough $\left( R_{ON} C_1 < \frac{T}{2} \right)$ so that the step isn't damaged.




Based on $V_{C1}$, MOS could be SAT or OHMIC.

Suppose that MOS is ohmic, then:

$$R_{ON} = \frac{1}{g_{DO}} = \frac{1}{2K\,V_{OV}}$$




$$V_{DS} = V_{C1} - V_{gnd} = V_{C1}$$

$$I_{DS}\Big|_{OHM} = 2K\left[ (V_{DD} - V_T) V_{C1} - \frac{V_{C1}^2}{2} \right]$$

The cap discharge current is the following:

$$I_{DS} = -C_1 \frac{dV_{C1}}{dt} \longrightarrow dt = \frac{-C_1}{I_{DS}}\, dV_{C1} \longrightarrow \text{solve this:}$$

$$\int_0^{T_{SW}} dt = -\frac{C_1}{K} \int_{V_I}^{V_F} \frac{dV_{C1}}{2 V_{OV} V_{C1} - V_{C1}^2}$$

$V_I > V_F$

See notes at the end of sheets

$$T_{SW} = \frac{C_1}{2K\,V_{OV}} \ln\left\{ \frac{(2V_{OV} - V_F)V_I}{(2V_{OV} - V_I)V_F} \right\}$$

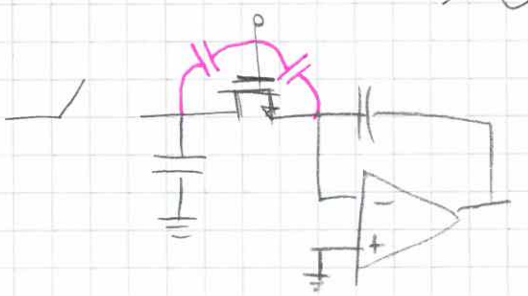
To first order, we can say $T_{SW} = \dfrac{C_1}{2K\,V_{OV}}$

Usually $C_1$ is set by noise requirements, therefore

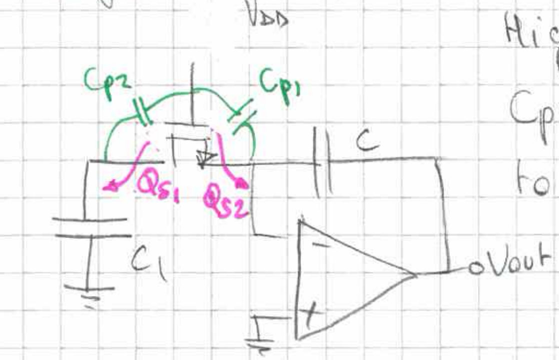
$$T_{SW} = \frac{C_1}{2\mu_n C'_{ox}(V_{DD} - V_T)} \frac{L}{W} \rightsquigarrow L = L_{min} \text{ to reduce } T_{SW}$$

The last choice would be $W \uparrow\uparrow$, but large area means larger $C_{ox} = WL\,C'_{ox}$

$\rightsquigarrow$ We now need to take into account the $C_{par}$

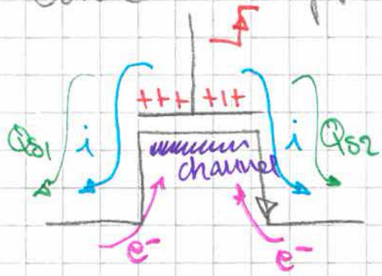


## Charge sharing



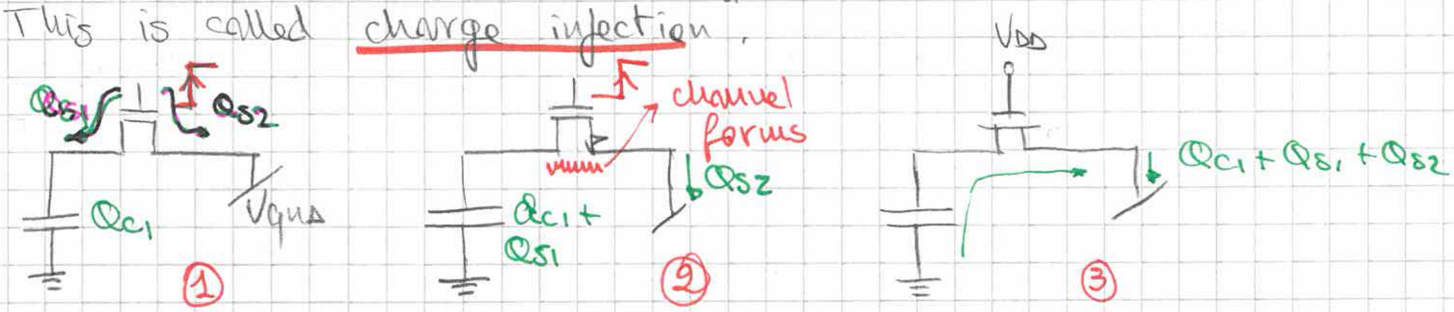

High W means that MOS has $C_{p1}$, $C_{p2}$ of a moderate value that needs to be controlled.

To form the channel, some electrons come from $C_1$ and some come from vgnd.




When we switch the transistor ON, some charge is needed to form the channel. This means that $e^-$ flow inside the MOS from $C_1$ and vgnd, therefore the current flows throug the other direction. This means that there's charge injected to $C_1$ and vgnd.

This is called "charge injection".

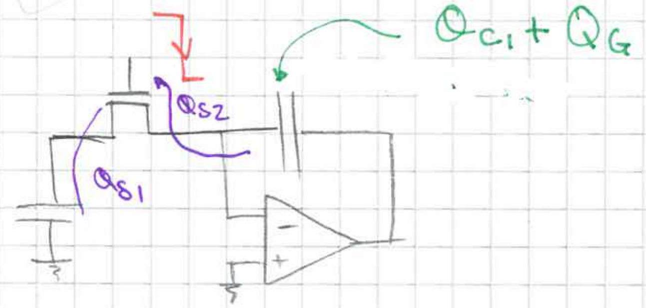



① ON transient, charge is moving to form the channel

② $Q_{S2}$ is injected, $Q_{S1}$ accumulates on $Q_{S1}$

③ MOS is on channel is formed and now $Q_{C1} + Q_{S1}$ can flow to vground too.

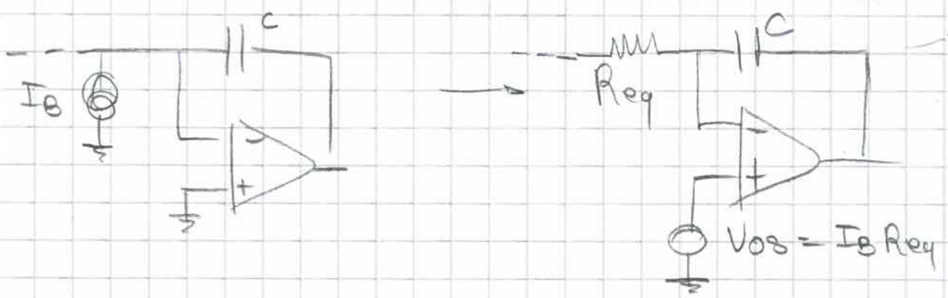
Total charge that will go on C will be $Q_{C1} + Q_{S2} + Q_{S1}$




On the OFF transient, $e^-$ flow towards $C_1$ and vgnd.
Some portion (because of different impedances and non-idealities of the opamp) will be lost on $C_1$, therefore after the two cycles there will be some net positive charge accumulated on $C \rightarrow Q_C = Q_{C_1} + \alpha Q_G$
where $Q_G = C_{ox} \cdot (V_{DD} - V_T)$ and $\alpha$ is a proportional factor

Some upgrade can be done by using NMOS + pMOS so that the differential switching on $V_{Gn}$ and $V_{Gp}$ will reduce the additional charge effect

We can see this accumulated charge for each clock cycle as an injected current $I_B = \alpha \frac{Q_{ch}}{T_{clk}}$, modeled like:




→ There's always a resistive path

$I_B$ would generate a ramp on C, but since SC filters always have resistive paths on vgnd, we can see $I_B$ as an input referred offset.
Therefore, SC filters will have higher DC offsets

## MOSFET sizing + clock

1) Ensure $T_{SW}$ is lower than $T_{CK}$:

$N$ = constant that divides the clock

$$R_{ON} C_1 = \frac{C_1}{2K V_{OV}} = \frac{T_{CK}}{N} = \frac{1}{N f_{CK}}$$

$V_{OV} = V_{DD} - V_T$

This leads to $C'_{ox} W (V_{DD} - V_T) > \frac{N f_{CK} C_{1L}}{\mu}$

2) Limit charge injection effects

$$\frac{\alpha Q_{ch}}{C} < \Delta V_{MAX} \rightarrow \frac{\alpha C'_{ox}(WL)(V_{DD}-V_T)}{C} < \Delta V_{MAX}$$

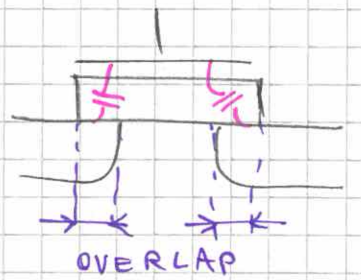
Combine 1 + 2:

$$f_{CK} < \frac{C}{C_1} \frac{\Delta V_{MAX}\, \mu}{L_{min}^2} \cdot \frac{1}{N \alpha}$$

$$W < \frac{C}{C'_{ox}(V_{DD}-V_T)} \frac{\Delta V_{MAX}}{\alpha L_{min}}$$

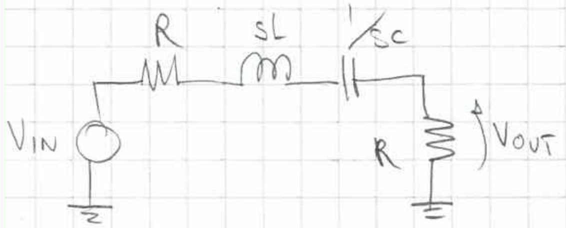
By selecting max transient time and the lowest offset allowable upper limits for $f_{CK}$ and $W$ (if we use $L_{min}$)

Note: overlap between gate and drain, source is there not to have issues with potential barriers



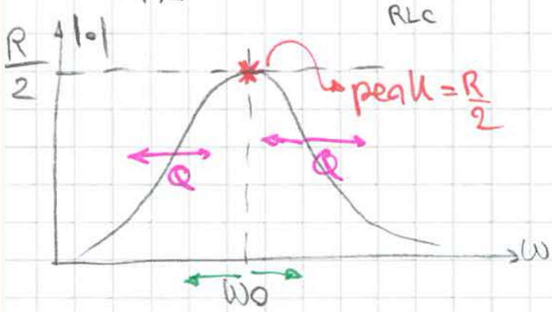

This generates additional parasitic capacitances

## Additional: Ordhard Theorem example

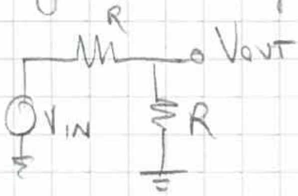



$$\frac{V_{OUT}}{V_{IN}} = \frac{R}{2R + \frac{1}{sC} + sL} = \frac{sRC}{s^2LC + s2RC + 1} =$$

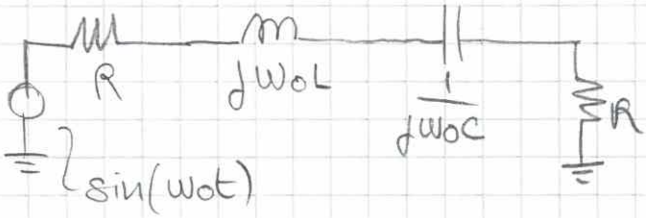
$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q\Big|_{\substack{\text{SERIES}\\ \text{RLC}}} = \frac{1}{\omega_0 2R \cdot C}$$




for a 10% value shift, $\omega_0$ changes by ~5% $\left(\frac{1}{\sqrt{C'L}} = \omega_0'\right)$, Q also changes but the peak value isn't influenced at all. In fact, at resonance, the reactance of the capacitor and inductor will have equal value but opposite signs, therefore they will be cancelled out, leading to:




To be more clear, at resonance $\omega_0$:




$L = 15\,\mu H$

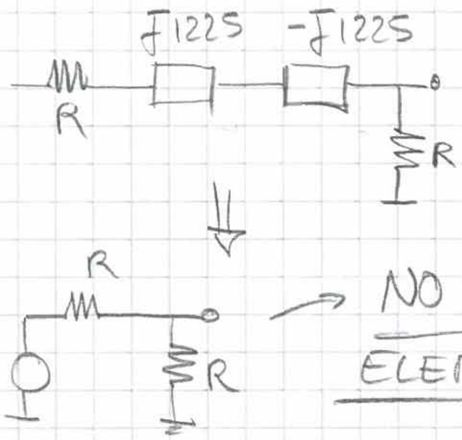
$C = 10\,pF$

$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi\,(13\text{ MHz})$

At 13 MHz:

$$j \cdot 2\pi\, 13 \cdot 10^6 \cdot 15\,\mu H \simeq j\,1225$$

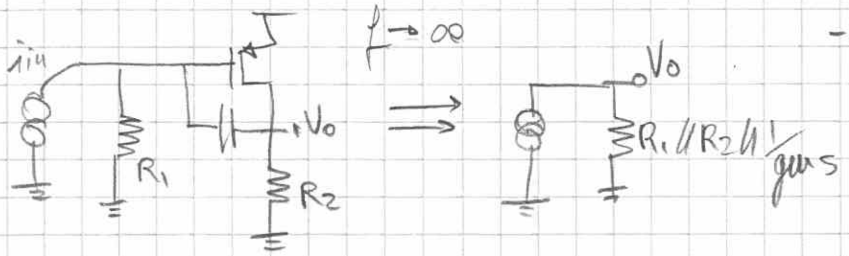
$$\frac{1}{j\, 2\pi\, 13 \cdot 10^6 \cdot 10 \cdot 10^{-12}} = -j\,1225$$

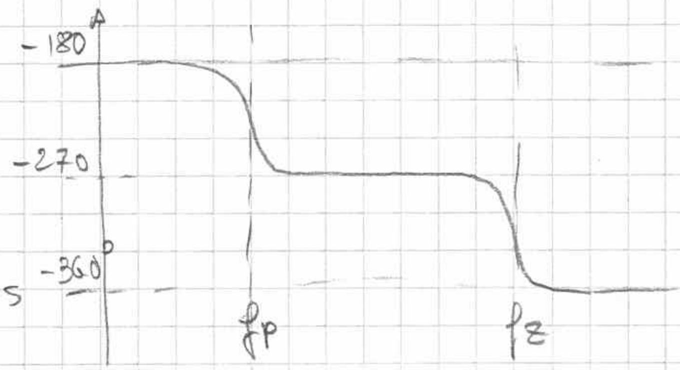



→ NO REACTIVE ELEMENTS!

## Additional: why the two stages zero is negative?

Consider just $C_\pi$:



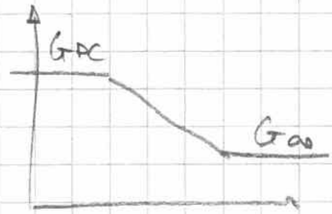

At 0Hz : gain is negative → $\phi = -180°$

At ∞Hz : gain is positive → $\phi = -360°$ or $0°$

From circuit inspection we find the usual pole at

$$f_L = \frac{1}{2\pi\, g_{ms} R_1 R_2 C_\pi}$$ and the gain $G_{DC} = -R_1 g_{ms} R_2$

$$G_\infty = +\frac{1}{g_{ms}}$$




Therefore $G_{DC} f_L = G_\infty f_Z$ $\qquad f_Z = \frac{g_{ms}}{2\pi C_\pi}$

To have $V_{OUT} < 0$ at 0Hz and $V_{OUT} > 0$ at ∞Hz, the only way to achieve this is to have a RHP zero

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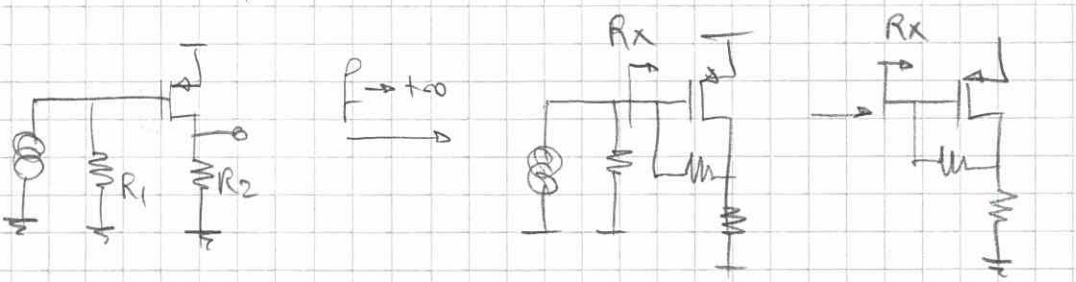
With nulling resistor:

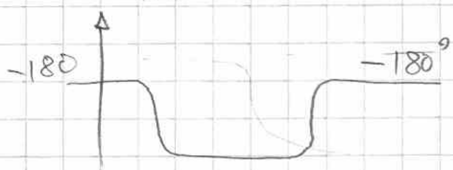



$$R_x = \frac{R_2 + R_N}{1 + g_{ms} R_2} \approx \frac{1}{g_{ms}}$$

$G|_{DC} = R_1 g_{ms} R_2$

$$G_\infty: \quad V_{OUT} = i_{in} \cdot (R_1 // \overbrace{R_x}^{1/g_{ms}}) - i_{in} \cdot \overbrace{\frac{R_1}{R_1 + R_x}}^{\approx 1} \cdot R_N \quad \to R_x$$

$$= i_{in} \left( \frac{1}{g_{ms}} - R_N \right)$$




If $R_N > \frac{1}{g_{ms}}$, $V_{OUT}$ at ∞ is $< 0$ → LHP zero recovers the 90° lost

If $R_N < \frac{1}{g_{ms}}$, $V_{OUT}$ at ∞ is $> 0$ → RHP zero again

## Additional : Solving $T_{switch}$

$$T_{sw} = \frac{C_1}{K}\int_{V_I}^{V_F} \frac{1}{2V_{ov}x - x^2}\,dx \Rightarrow \frac{1}{x(2V_{ov}-x)} = \frac{A}{x} + \frac{B}{2V_{ov}-x}$$

$$\frac{A(2V_{ov}-x)+Bx}{\cancel{(\quad)}} = \frac{1}{\cancel{x(\quad)}} \qquad A2V_{ov} + x(B-A) = 1$$

$$\begin{cases} 2V_{ov}A = 1 \\ B - A = 0 \end{cases} \longrightarrow \quad A = B = \frac{1}{2V_{ov}}$$

$$\int_{V_i}^{V_f} \frac{1}{2V_{ov}}\cdot\frac{1}{x} + \frac{-1}{2V_{ov}}\cdot\frac{-1}{2V_{ov}-x}\,dx =$$

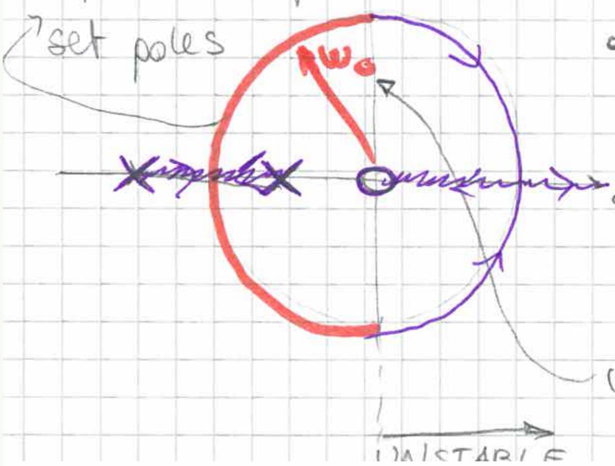
$$\frac{1}{2V_{ov}}\Big[\ln x - \ln(2V_{ov}-x)\Big]_{V_i}^{V_f} = \frac{1}{2V_{ov}}\ln\left\{\frac{(2V_{ov}-V_F)V_I}{(2V_{ov}-V_I)V_F}\right\}$$


If $V_F = 0 \longrightarrow T_{sw} \longrightarrow +\infty$ meaning that there will be some error in the transferred charge because $C_1$ will never be fully discharged for a finite clock period

## Additional: why we use the SK cell

We have multiple ways to implement complex conjugate poles, but the SK cell can:

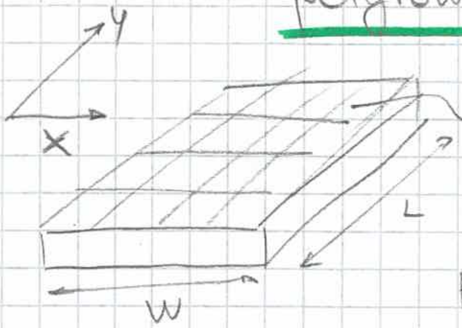
useful locus part to set poles




- Low component number
- Q can be independently set by just using K ($w_0$ is the same)
- Just one amp for two complex conjugate poles

$$w_0 = \frac{1}{RC} \qquad Q = \frac{1}{3-K}$$

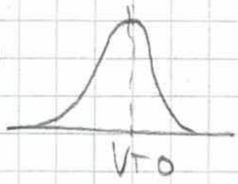
Additional : how do we choose the "elementary cells" in pelgrom formulas




→ This is a $\Lambda \times \Lambda = A_0$ square
$\Lambda$ → length
$A_0$ → area

How do we choose $A_0$? (This is valid for both $K_{V_T}$ and $K_R$ calculations).

Each square must be spatially uncorrelated with the adjecent squares.




If the square was too large, it wouldn't be a gaussian distribution anymore, therefore all the statistical reasoning we make would be affected by deterministic processes. In few words:

$$\sigma^2_{V_T} = \sigma^2_{V_{T1}} + \sigma^2_{V_{T2}} + \sigma^2_{V_{T3}} + \dots$$
$$\sigma^2_{G_T} = \sigma^2_{G_1} + \sigma^2_{G_2} + \sigma^2_{G_3} + \dots$$

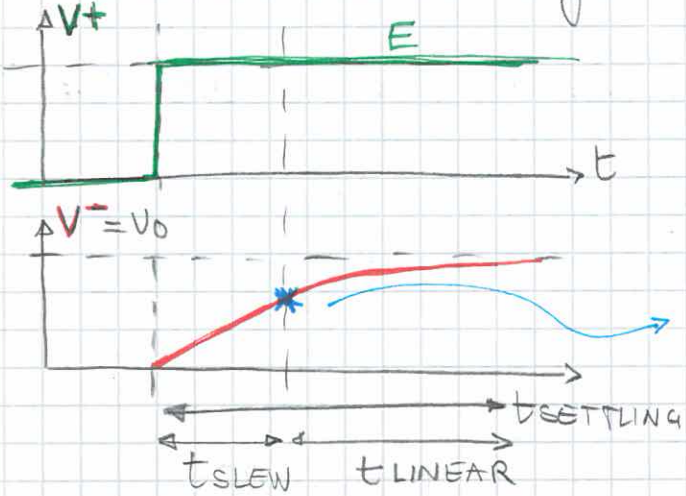
→ These wouldn't be valid anymore

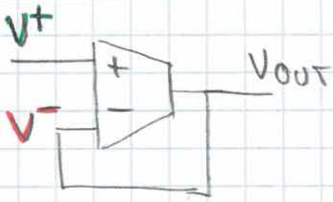
To select the right $A_0$, $\Lambda$ we would need to compute the double autocorrelation (across x and y axis) in order to have uncorrelated squares (or at least, autocorrelation length is small enough).

## Additional: notes on SR

We have an OTA buffer


At the start we have the SR limitation (see SR question). Right in this time, we can say that the big input (on $V^+$ node) step will shut off the MOSFETs and etc, etc...
But after the $t_{SLEW}$, when Vout has risen enough, $V^+$ won't be that big with respect to $V^-$:

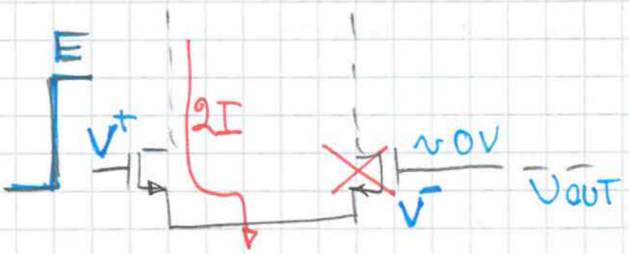


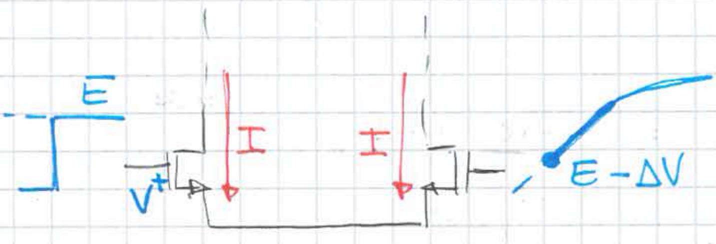




$V^- = V_{OUT}$

$V^+ = E$ while $V^- = E - \Delta V$

This means that $V^+$ and $V^-$ are similar enough so that the differential stage will recover the linear operation:

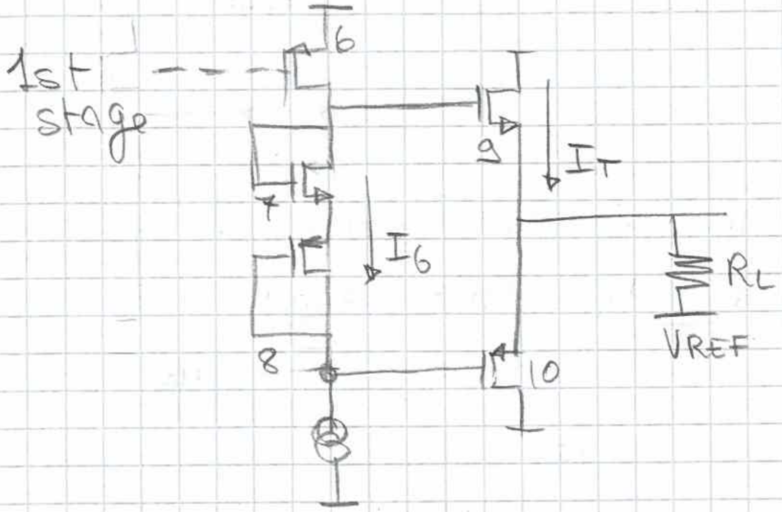


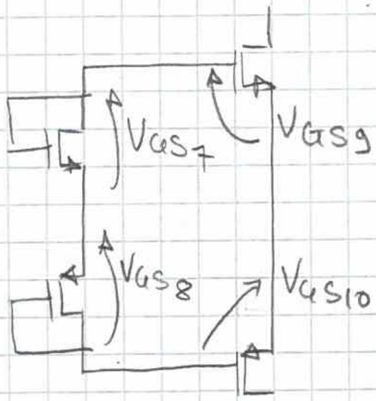




At the start of SR        When linearity is recovered

Condition will be met when $\frac{\Delta V}{\tau} = SR$

# Additional : class AB voltage shifter




We could think of using just nMOS as transdiodes, but if we mimic the output configuration we can compensate process variability on $V_T$

$$V_{GS7} + V_{GS8} = V_{GS9} + V_{GS10}$$

$$\cancel{V_{Tn}} + \sqrt{\frac{I_7}{K_7}} + \cancel{V_{Tp}} + \sqrt{\frac{I_8}{K_8}} = \cancel{V_{Tn}} + \sqrt{\frac{I_9}{K_9}} + \cancel{V_{Tp}} + \sqrt{\frac{I_{10}}{K_{10}}}$$

$$\sqrt{\frac{I_7}{K_7}} + \sqrt{\frac{I_8}{K_8}} = \sqrt{\frac{I_9}{K_9}} + \sqrt{\frac{I_{10}}{K_{10}}}$$

$$\left.\begin{array}{l} I_7 = I_8 = I_6 \\ I_9 = I_{10} = I_T \end{array}\right] \rightarrow I_T = n I_6$$

$$\sqrt{\frac{I_T}{I_6}} = \frac{\sqrt{\frac{1}{K_{10}}} + \sqrt{\frac{1}{K_9}}}{\sqrt{\frac{1}{K_7}} + \sqrt{\frac{1}{K_8}}}$$

$$K_7 = n K_9$$

$$K_8 = n K_{10}$$

$$\frac{I_T}{I_6} = n \left( \frac{1 + \sqrt{\frac{K_9}{K_{10}}}}{1 + \sqrt{\frac{K_7}{K_8}}} \right)^2$$

$\rightsquigarrow$ Result :

If $n \nearrow\nearrow$, $I_T \nearrow\nearrow$ → distortion ↘ because output transistors are well biased. This means that higher current leads to more power burned → $\eta$ of the stage ↘↘

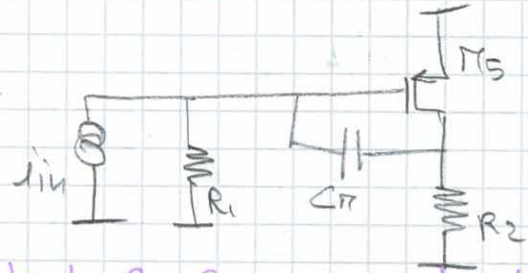
We clearly see a tradeoff between distortion and efficiency

## Additional : root locus for the RHP zero

Consider to use the OTA in a feedback configuration, then:

$$G_{REAL} = G_{IDEAL} \frac{-G_{LOOP}(s)}{1 - G_{LOOP}(s)}$$ → denominator has roots

Root locus : locus of the singularities that solve $1 - G_{loop}(s)$ for a varying $G_{loop}(0)$. Consider the circuit below




$$G_{loop}(s) = G_0 \frac{1 - s\tau_z}{1 + s\tau_p}$$

$$\tau_z = \frac{C_\pi}{g_{mS}} \qquad \tau_p \cong C_\pi R_1 g_{mS} R_2$$

Neglect $C_1$, $C_L$, we can do the same reasoning with those

$$1 - G_{loop}(s) = 0 \rightarrow G_0 \frac{(1 - s\tau_z)}{1 + s\tau_p} = 1 \rightarrow -G_0 \frac{\tau_z}{\tau_p} \cdot \frac{s - \frac{1}{\tau_z}}{s + \frac{1}{\tau_p}} = 1$$

The solution of this is split in two equations

$$|G_0| \frac{|\tau_z|}{|\tau_p|} \frac{\left|s - \frac{1}{\tau_z}\right|}{\left|s + \frac{1}{\tau_p}\right|} = |1|$$ → absolute value equation

$$\arg\left(-G_0 \frac{\tau_z}{\tau_p}\right) + \underbrace{\arg\left(s - \frac{1}{\tau_z}\right)}_{\text{zero}} - \underbrace{\arg\left(s + \frac{1}{\tau_p}\right)}_{\text{pole}} = 0°$$ phase equation

Note that $G_0 < 0$, $\tau_z > 0$, $\tau_p > 0 \rightarrow G_x = -G_0 \frac{\tau_z}{\tau_p} > 0$ so

$G_x$ is real and $> 0 \rightarrow \arg(G_x) = \tan^{-1}\left(\frac{0}{G_x}\right) = 0°$

$$\arg\left(j\omega - \frac{1}{\tau_z}\right) = \tan^{-1}\left(\frac{\omega}{-1/\tau_z}\right) = -\tan^{-1}(\tau_z \omega)$$

$$\arg\left(j\omega + \frac{1}{\tau_p}\right) = \tan^{-1}\left(\frac{\omega}{1/\tau_p}\right)$$

→ We can rewrite the requirement on phase

→ zero gives a $-90°$ contribution!!

$$\begin{cases} 0° = 0° - \tan^{-1}(\omega\tau_z) - \tan^{-1}(\omega\tau_p) \\ |G_0| \frac{|\tau_z|}{|\tau_p|} \frac{\left|s - \frac{1}{\tau_z}\right|}{\left|s + \frac{1}{\tau_p}\right|} = 1 \end{cases} \rightarrow$$