RADIO FREQUENCY CIRCUIT DESIGN ORAL NOTES

By Giacomo Tombolan

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Guide to these notes:

- If you payed for these notes, **you've been scammed**. I always give my notes for free.
- I tried to explain and justify every step of each question, I hope you can follow my reasonings. If there's something that's "taken for granted", it means it was discussed in previous courses (i.e: fundamentals of electronics or analog circuit design)
- PDFs contain typos so beware of that! There are comments as errata corrige
- if my writing isn't clear, well, I'm sorry C: hope it helps anyway. Also, I speak maccheroni and I'm well aware of the English mistakes I made. My priority was to have a clear understanding of the topics.

If you're having any issue with this document just send an email to giacomo.tombolan@mail.polimi.it

Questions for the Oral Examination
RF Circuit Design

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Year 2020/21

RF Front-end Architectures

1. Effects of distortion.
2. Two-tone test and third-order intercept point (IIP3).
3. Theorem of maximum power transfer and its application to the impedance matching of amplifiers. Definition of power gains.
4. Matching networks: Resonant networks.
5. Matching networks: Transformers.
6. Noise figure of lossy circuits and cascaded systems.
7. *RF receivers*: Sensitivity and dynamic range.
8. *Heterodyne receivers*: Advantages. Image problem and filtering. Selectivity/Sensitivity trade-off. Block schematic from antenna to matched filter.
9. *Heterodyne receivers*: Problem of half-IF (IF/2).
10. Second-order nonlinearity. Intercept point IIP2 and link with 2nd-order harmonic distortion.
11. *Dual-IF receivers*: Architecture, advantages and drawbacks. Comparison with single-IF architecture.

12. *Zero-IF receivers*: Architecture, advantages and drawbacks. DC offsets and cancellation techniques.

13. *Zero-IF receivers*: Impact of I/Q mismatches on SNR. Impact of LO leakage.

14. *Image-reject receivers*: Shift-by-90 operation. Hartley architecture and effect of mismatches and Image-Rejection Ratio (IRR).

15. *Image-reject receivers*: Weaver architecture: advantages and drawbacks.

16. *Transmitters*: Effect of I/Q mismatches. Direct-conversion architecture.

17. *Transmitters*: Two-step transmitters. Single-Sideband (SSB) mixer.

Frequency Synthesizers

18. AM and FM disturbances of a carrier. Relationship between phase spectrum and voltage spectrum of the carrier.

19. Effects of phase noise in RF receivers and transmitters: EVM degradation. Reciprocal mixing in presence of blockers.

20. Phase detectors based on multiplier. Derivation of the phase model of the PLL. Nonlinear differential equation.

21. *Second-order PLLs*: Analysis of stability and transfer functions. Static phase error after n-th order input signal, frequency response.

22. *Second-order PLLs*: Frequency tracking and lock acquisition.

23. *Charge-pump PLLs*: Phase-frequency detector, phase-domain model, stabilizing zero, analysis of loop dynamics.

24. Limits of validity of the continuous-time model of PLLs.

25. Sources of ripple in a PLL. Reference spur problem in an integer-N loop. Methods to reduce the level of reference spur.

26. Design and simulation of a PLL.

RF Circuits

27. *LNAs*: Scattering parameters, insertion loss, reverse isolation, stability, linearity. Methods to increase reverse isolation.

28. *LNAs*: MOS noise model. Common-gate and shunt-feedback LNA topology.

29. *LNAs*: Inductor-degenerated topology.

30. *LNAs*: Noise canceling technique and application to shunt-feedback topology.

31. *Oscillators*: Feedback model and Barkahusen criterion. Negative-resistance model. Amplitude stabilization methods. Oscillation startup and effective gain.

32. *Oscillators*: Frequency stabilization. Effect of loop delay in oscillators. Meaning of quality factor in oscillators.

33. *Oscillators*: Phase Noise calculation in LC oscillators.

34. *Oscillators*: Noise/Power Trade-off.

35. *Oscillators*: Circuit topologies of voltage-controlled oscillators (VCOs). Noise on tuning voltage: calculation of FM noise.

36. *Oscillators*: Single-transistor and differential LC oscillator topologies: analysis with feedback and negative-resistor model.

37. *Oscillators*: Design and simulation of an RF oscillators in CMOS.

38. *Mixers*: Return-to-zero passive mixers in CMOS, conversion gain, noise.

39. *Mixers*: Single-balanced and double-balanced topologies, port-to-port isolation.

40. *Mixers*: Active mixers in CMOS, conversion gain, noise, port-to-port isolation.

RF - Recap through oral questions - Giacomo Tombolan 2020/2021

1) Effects of distortion

Consider a memoryless non-linear system: $x(t) \rightarrow \boxed{sys} \rightarrow y(t)$

$$y(t) = \underbrace{\alpha_1 x(t)}_{\text{small signal}} + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \dots \qquad x(t) = A \cos \omega t$$

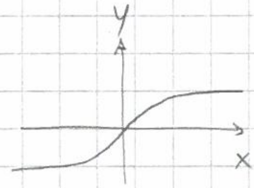
$$x^2(t) = A^2 \cos^2 \omega t = \underbrace{\frac{A^2}{2}}_{DC} + \frac{A^2}{2} \underbrace{\cos 2\omega t}_{\text{2nd harmonic}}$$

$$x^3(t) = A^3 \cos^3 \omega t = A^3 \cos \omega t \left(\frac{1 + \cos 2\omega t}{2} \right) = \frac{A^3}{2} \underbrace{\cos \omega t}_{\text{fundamental}} + \frac{A^3}{4} \underbrace{\cos 3\omega t}_{\text{3rd harm}}$$

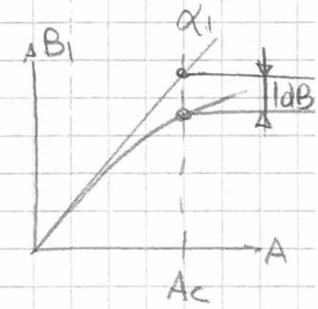
$$y(t) = B_0 + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t$$

$$B_0 = \alpha_2 \frac{A^2}{2} \quad B_1 = \alpha_1 A + \frac{3}{4} \alpha_3 A^2 \quad B_2 = \alpha_2 \frac{A^2}{2} \quad B_3 = \frac{1}{4} \alpha_3 A^2$$

Gain compression phenomenon:



$B_1 = \alpha_1 A + \frac{3}{4} \alpha_3 A^3 \rightarrow$ If $\alpha_1 \alpha_3 < 0$ we experience "gain compression" of the first harmonic $\cos \omega t$ with respect to just the small signal gain $\alpha_1 A$



Def: 1dB comp. point

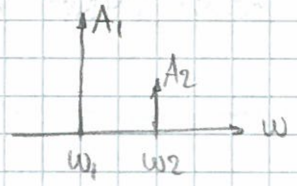
$$\frac{\alpha_1 A_c + \frac{3}{4} \alpha_3 A_c^3}{\alpha_1 A_c} = \underbrace{10^{-1/20}}_{1dB}$$

$$1 + \frac{3}{4} \frac{\alpha_3}{\alpha_1} A_c^2 = 0,89$$

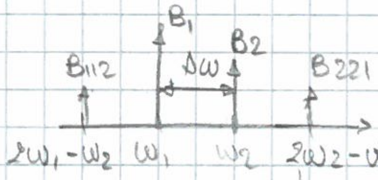
$$A_c = -9,6 dB + 10 \log_{10} \frac{4}{3} \left| \frac{\alpha_1}{\alpha_3} \right|$$

Distortion with two tones:

Consider now $x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$



We will experience 4 tones evenly spaced with an equal $\Delta\omega = \omega_2 - \omega_1$



$B_1 = \alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2} A_1 A_2^2$

$B_2 = \alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2} A_1^2 A_2$

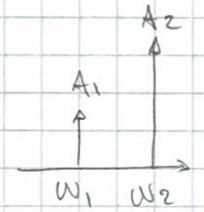
$B_{112} = \frac{3}{4}\alpha_3 A_1 A_2^2$

$B_{221} = \frac{3}{4}\alpha_3 A_1^2 A_2$

These are called IM3 (inter-modulation 3rd harmonics)

In RF we don't really care about harmonics (they get filtered out). We care about tones that fall near the signal of interest.

Blocking phenomenon: small wanted A_1, large unwanted A_2



$B_1 = \alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2 \approx \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1$

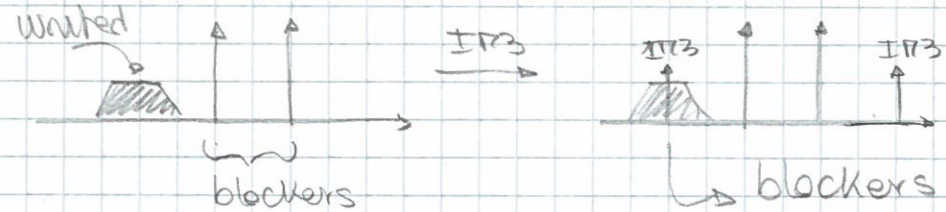
Negligible if $A_1^3 \ll A_1 A_2^2$

$\frac{B_1}{A_1} = \alpha_1 + \frac{3}{2}\alpha_3 A_2^2 \rightarrow 0$ for large A_2

harmonic gain of the sys

Intermodulation:

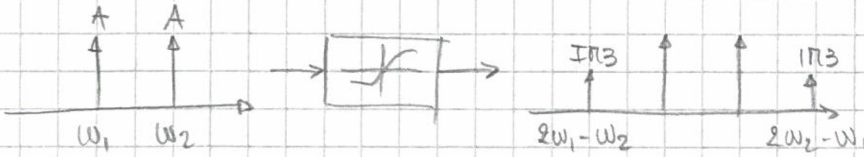
Consider two interferers near the wanted BW

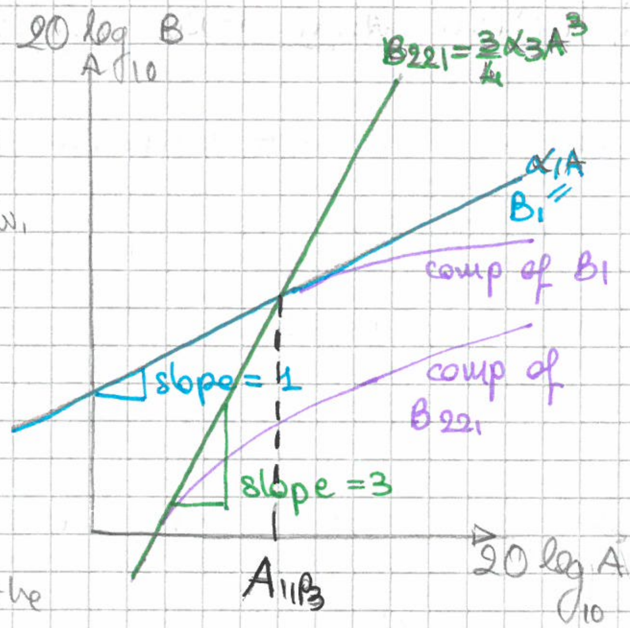


blockers degrade SNDR

SNDR = signal to noise/distortion ratio $= \dfrac{P_{sig}}{P_{noise} + P_{disturb}}$

2) Two tone test and IIP3

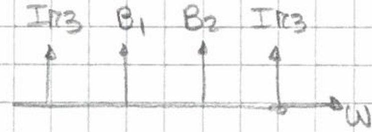




$$B_1\Big|_{A_1=A_2} = \alpha_1 A + o.t$$

$$B_{221}\Big|_{A_1=A_2} = \frac{3}{4}\alpha_3 A^3 + o.t$$

o.t = higher order terms that compress the two lines (see plot)



A_{IIP3} = point where linear and IM3 are equal (ideal)

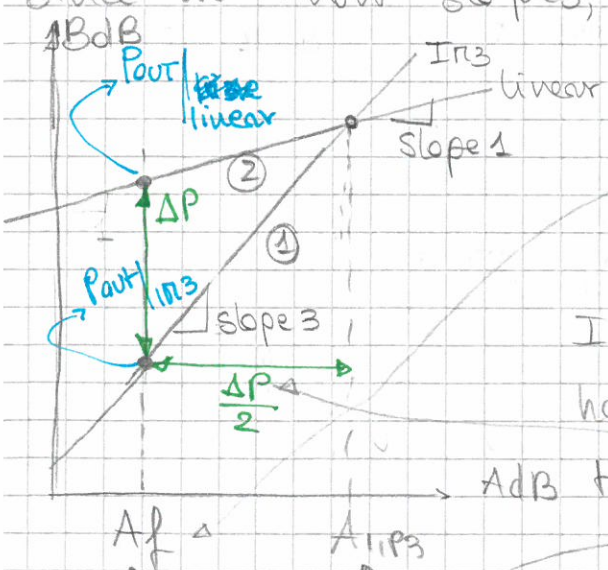
In reality $A_{IIP3}\Big|_{compressed} \geq A_{IIP3}\Big|_{IDEAL}$

$$\alpha_1 A_{IIP3} = \frac{3}{4}\alpha_3 A_{IIP3}^3 \longrightarrow A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

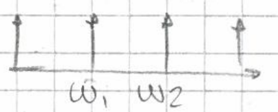
$$IIP3\,dB = 20\log_{10} A_{IIP3} = 10\log_{10}\left(\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|\right)$$

We can see that $IIP3dB = A\big|_{1dB\ comp\ point} + 9{,}6dB$

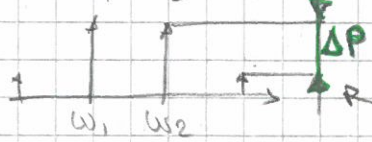
Practical measurements are taken for small amplitudes. Then, since we know slopes, we can determine the ideal A_{IIP3}:



@ A_{IIP3} as input:



@ Af as input:



I see that the difference between 1 and 2 has slope 2 therefore I'm $\frac{\Delta P}{2}$ distant from the IIP_3 point, leading to:

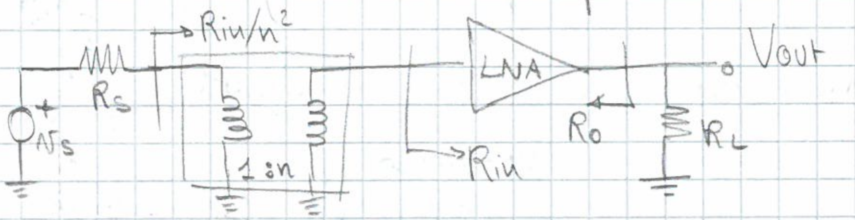
$$IIP_3\Big|_{dBm} = Pinput + \frac{\Delta P}{2}$$

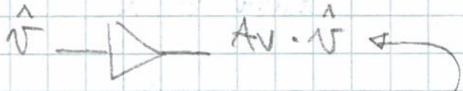
measured output difference

input power we test

3) Maximum power transfer and appl / power gains

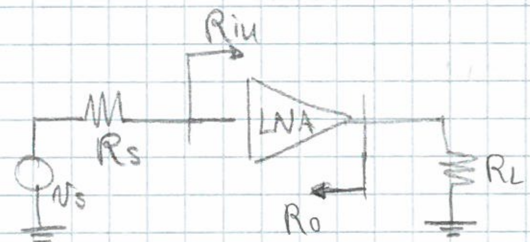
Consider a LNA with input matching





Unloaded voltage gain of LNA

$$\frac{V_{out}}{V_{in}} = \underbrace{\frac{R_{in}/n^2}{R_{in}/n^2 + R_s}}_{\text{out voltage division } \alpha} \cdot n \cdot A_v \cdot \frac{R_L}{R_o + R_L} = \alpha A_v \cdot \frac{R_L}{R_L + R_o}$$



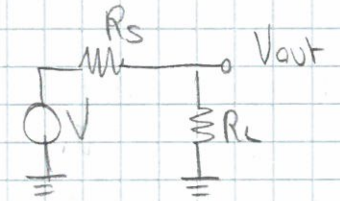
We're used to study

$$\frac{V_{out}}{V_{in}} = \alpha A_v \frac{R_L}{R_L + R_o} \quad \text{where} \quad \alpha = \frac{R_{in}}{R_{in} + R_s}$$

Max gain $\rightarrow$ (*1) $R_{in} \gg R_L$, $R_L \gg R_o$ $\rightarrow \left.\frac{V_{out}}{V_s}\right|_{max} = A_v$

We're led to think that (*1) is correct, but:

Max power transfer: consider the divider:



$$P_{out} = \underbrace{V^2\left(\frac{R_s}{R_s + R_L}\right)^2}_{V_{out}} \cdot \frac{1}{R_L} = V^2 \frac{R_L}{(R_s + R_L)^2}$$

We want to max power output with respect to load R_L:

$\frac{\partial P_{out}}{\partial R_L} = 0$ only if $R_L = R_s$ $\rightsquigarrow$ load is matched to source

This also works for impedances, where $Z_L = Z_s^*$

Therefore $P_{out} = P_{load} = \frac{V^2}{(2R_s)^2} \cdot R_s = \frac{V^2}{4 \cdot R_s} = P_L \Big|_{available}$

It is called output available power, since $V_{out} = V_{in}/2$

$$P_{L,\,available} = \frac{V_{out}^2}{8 R_s}$$

Therefore we can see that something is off because:

- to max voltage gain $R_L \gg R_S$
- to max power $R_L = R_S$

Consider α from LNA gain expression:

$$\alpha = \left(\frac{R_{in}/n^2}{R_{in}/n^2 + R_S}\right) n = \frac{n R_{in}}{R_{in} + n^2 R_S} \longrightarrow \text{maximize } \alpha$$

$$\frac{\partial \alpha}{\partial n} = \frac{R_{in}(R_{in} + n^2 R_S) - n R_{in} \cdot 2 R_S}{(R_{in} + n^2 R_S)^2} = 0 \longrightarrow R_{in} = n^2 R_S \quad (*1)$$

$$n\Big|_{opt} = \sqrt{\frac{R_{in}}{R_S}} \longrightarrow \alpha\Big|_{MAX} = \frac{n \cancel{R_{in}}}{2 \cancel{R_{in}}} = \frac{n_{opt}}{2} = \frac{1}{2}\sqrt{\frac{R_{in}}{R_S}}$$

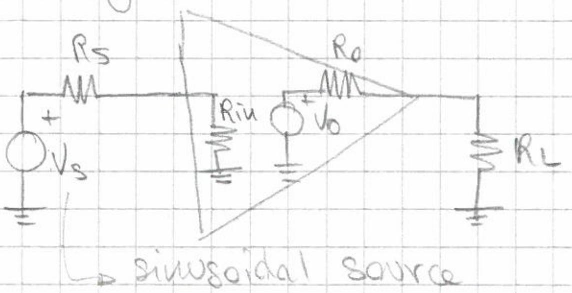
Important (*1) $\frac{R_{in}}{n^2} = R_S \rightsquigarrow$ impedances are matched

So, since we have $\alpha_{MAX} \rightsquigarrow$ Voltage gain is maximised through impedance matching:

$$\frac{V_{out}}{V_S}\Big|_{max} = \underbrace{\frac{1}{2} n_{opt}}_{\alpha_{MAX}} \; \underbrace{A_V \cdot \frac{R_L}{R_L + R_o}}_{\text{we can't really control these}}$$

e.g. $R_S = 50\,\Omega$, $R_{in} = 100\,\Omega$, $n_{opt} = \sqrt{\frac{100}{50}} = \sqrt{2}$ $\rightarrow \alpha = \frac{\sqrt{2}}{2}$

Power gain:



$$\frac{V_o}{V_S} = \alpha A_V = A_o$$

$$P_{S,\,available} = \frac{V_S^2}{8 R_S} \qquad P_{out,av} = \frac{V_o^2}{8 R_o}$$

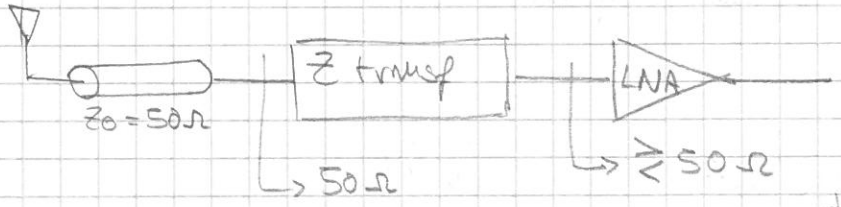
$$\frac{\left(\frac{V_S}{\sqrt{2}} \cdot \frac{1}{2}\right)^2}{R_S} = \frac{V_S^2}{8 R_S}$$

Available power gain $G_A = \frac{P_{out,av}}{P_{in,av}} = \frac{V_o^2 / 8 R_o}{V_S^2 / 8 R_S} = \left(\frac{V_o}{V_S}\right)^2 \frac{R_S}{R_o} =$

$G_A = (\alpha A_V)^2 \frac{R_S}{R_o} = A_o^2 \frac{R_S}{R_o}$ when $R_S = R_o$ $G_A = A_o^2$ but this is not true in general

1) Matching networks: resonant networks

If input impedance is not matched → reflections

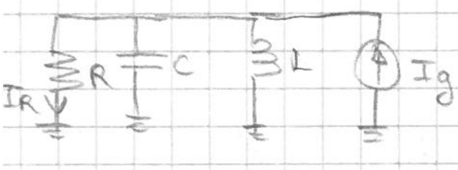


Impedance transformation network → downward (low to high Z) / upward (high to low Z)

Resonant circuits: RLC series/parallel

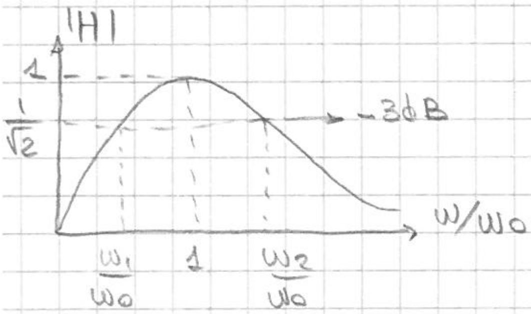
R models losses inside the RC reso.

$$\begin{cases} Q = \omega_0 RC \\ \omega_0 = 1/\sqrt{LC} \end{cases}$$



$$H(s) = \frac{I_R}{I_g} = \frac{s/RC}{s^2 + \frac{s}{RC} + \frac{1}{LC}} = \frac{s\frac{\omega_0}{Q}}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Input impedance will be $Z_{in} = \frac{V}{I_g} = R\frac{I_R}{I_g} = R \cdot H(s)$



-3dB bandwidth

$$|H(j\omega)^2| = \frac{1}{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2} = \frac{1}{2}$$

$$Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2 = 1 \qquad Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \pm 1$$

$$\omega^2 \mp Q\omega_0\omega - \omega_0^2 = 0 \rightarrow \omega_{1,2} = \omega_0\left(\mp\frac{1}{2Q} + \sqrt{1 + \frac{1}{4Q^2}}\right)$$

$$-3\text{dB BW} = \omega_2 - \omega_1 \rightarrow \frac{\omega_2 - \omega_1}{\omega_0} = \underline{\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}}$$

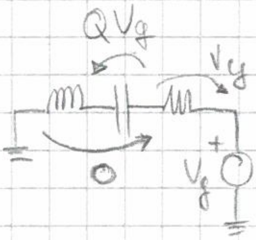
We can see that $Q = \frac{\omega_0}{\Delta\omega}$

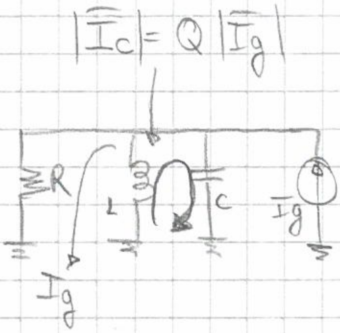
$$Q = \omega_0 RC = \omega_0 \frac{\frac{1}{2} C|V|^2}{\frac{1}{2}\frac{|V|^2}{R}} = \underbrace{\omega_0 \frac{E_{stored}}{P_{diss}}}_{\text{Important definition of } Q} = 2\pi \frac{E_{stored}}{E_{diss, \text{per cycle}}}$$

For $\omega = \omega_0$, if:

① • RLC series: voltage amplification

② • RLC parallel: current amplification





② $|\bar{I}_C| = \omega_0 C \cdot |\bar{V}| = \omega_0 C R \cdot |\bar{I}_g| = Q\,|\bar{I}_g|$

① $|\bar{V}_C| = \frac{|\bar{I}|}{\omega_0 C} = \frac{|\bar{V}_g|}{\omega_0 RC} = Q\,|\bar{V}_g|$

L-match networks - upward L-match

lossless approximation: $R_S \simeq 0$

$|\bar{I}_L| = |\bar{I}_C| \simeq Q\,|\bar{I}_g|$ where $Q = \frac{1}{\omega_0 R_S C} = \frac{\omega_0 L}{R_S}$ ← series RLC

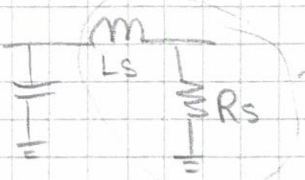
↳ Current amp for series RLC network

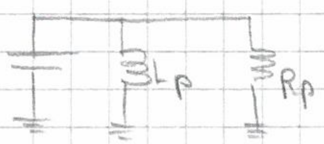
$Q\big|_{lossless} \gg 1$ because $R_S \to 0$

$|\bar{V}_{out}| \simeq |\bar{I}_L| R_S = |\bar{V}_{in}|\,\omega_0 C \cdot R_S = |\bar{V}_{in}| / Q$

So $|Z_{in}| = \frac{|\bar{V}_{in}|}{|\bar{I}_g|} \simeq \frac{Q\,|\bar{V}_{out}|}{|\bar{I}_L|/Q} = R_S \cdot Q_L^2$

If lossless approx is removed, then:





series to parallel

$sL + R_S = R_p // L_p = \frac{R_p\, sL_p}{R_p + sL_p}$

After calculations and separation for Re and Im we will get:

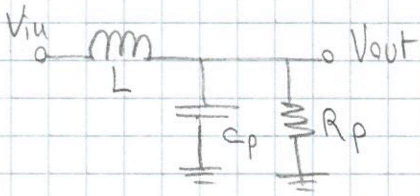
$R_p = R_S(1 + Q^2)$

$L_p = L_S \frac{1+Q^2}{Q_L^2}$

→ If $Q \gg 1$ → $R_p \simeq R_S Q^2$, $L_p \simeq L_S$

lossless network approx

L-match networks: downward L-match



We just flipped L with C

Using lossless we can say that $R_p \to \infty$

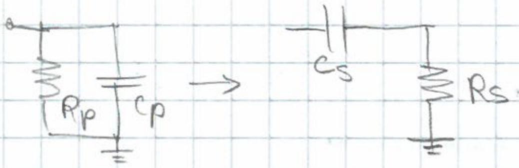
$|\overline{V_{out}}| = |\overline{V_c}| = Q\,|\overline{V_{in}}|$ so $|Z_{in}(j\omega_0)| \simeq R_p / Q^2$

↳ voltage amp

Where $Q = \dfrac{R}{\omega_0 L} = \omega_0 RC$

↳ parallel network

If lossless is removed, then



And we get that:

$$R_s = \frac{R_p}{1+Q^2} \qquad C_s = C_p \cdot \frac{1+Q^2}{Q^2}$$

How to remember this: $R_s \ll R_p$ always, then:

- upward $L_p > L_s$ by a small amount $\left(\dfrac{1+Q^2}{Q^2}\right)$
- downward $C_s > C_p$ " " " "

Always be careful to select the right Q factor (series or //)

π match network or colpitt's network

lossless $R_p \to \infty$

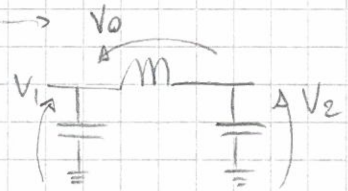
Series $\to$ $I = sC_2V_2 \simeq -sC_1V_1 \qquad \frac{V_1}{V_2} \simeq -\frac{C_2}{C_1}$

Consider now Rin, the dissipated power in the network is tied to Rp only $\to$ any equivalent resistor has to have the same dissipation of the circuit:

$$\frac{1}{2}\frac{V_{in}^2}{R_{in}} = \frac{1}{2}\frac{V_s^2}{R_p} \longrightarrow \underline{R_{in}} = R_p\left(\frac{V_1}{V_2}\right)^2 \simeq \underline{R_p\left(\frac{C_2}{C_1}\right)^2}$$

By choosing $C_2 \gtrless C_1$ (> up, < down) we can have upward / downward

To estimate Q we can say:



$$Q = \omega_0 \frac{E_{stored}}{P_{diss}} \qquad E_{stored} = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)|\bar{V}_0|^2$$

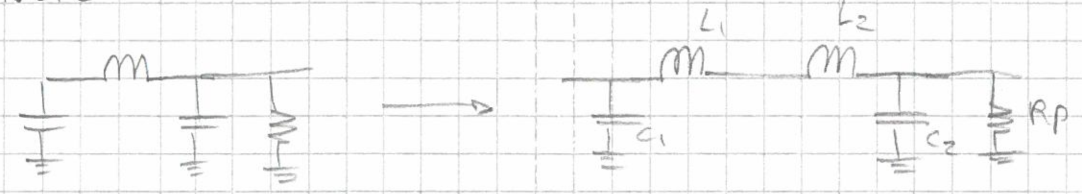
$$V_0 = V_1 - V_2 = -\frac{C_2}{C_1}V_2 - V_2 = -V_2\left(1+\frac{C_2}{C_1}\right) \qquad V_2 = V_0\left(\frac{C_1}{C_1+C_2}\right) = V_R$$

$$P_{DISS} = \frac{1}{2}\frac{|V_2|^2}{R_p} = \frac{1}{2}\left(\frac{C_1}{C_1+C_2}\right)^2|\bar{V}_0|^2$$

$\hookrightarrow$ AC voltage

$$Q = \omega_0 \frac{\cancel{\frac{1}{2}}\left(\frac{C_1C_2}{C_1+C_2}\right)\cancel{|\bar{V}_0|^2}}{\cancel{\frac{1}{2}}\left(\frac{C_1}{C_1+C_2}\right)^2\frac{\cancel{|V_0|^2}}{R_p}} = \underbrace{\omega_0 C_2 R_p}_{\text{L-match typical Q}}\overbrace{\left(1+\frac{C_2}{C_1}\right)}^{\text{Improvement factor}}$$

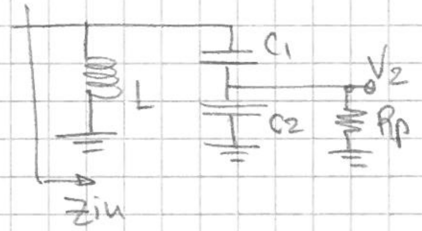
Note:



We can split L and therefore we see 2 L-match networks.

This can give more degrees of freedom

Tapped capacitor resonator



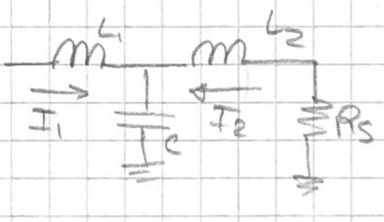
low loss

$$\frac{V_2}{V_0} \simeq \frac{C_1}{C_2 + C_1}$$

$$R_{in} = R_p \left(\frac{V_0}{V_2}\right)^2 = \left(1 + \frac{C_2}{C_1}\right)^2$$

from power dissipation equations

T-match network



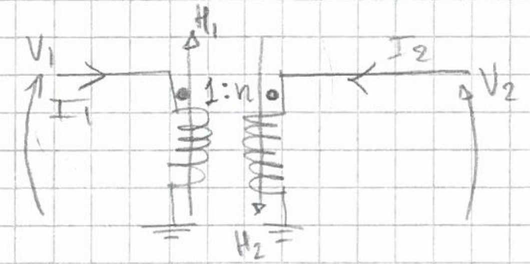
Low-loss $R_s \simeq 0$

$$-V = sL_1 I_1 \simeq sL_2 I_2 \qquad \frac{I_1}{I_2} \simeq \frac{L_2}{L_1}$$

From power dissipation $R_{in} = R_s \left(\frac{I_2}{I_1}\right)^2 \simeq R_s \left(\frac{L_2}{L_1}\right)^2$

5) Matching Networks : transformers

We use inductor coupling instead of resonance



In this case H_1, H_2 have the same orientation

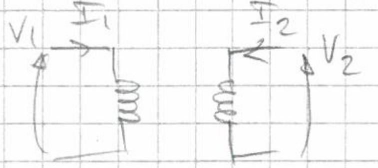
$$\mathcal{E}_m = \frac{\mu}{2} |\overline{H_1} + \overline{H_2}|^2 \, dv$$

$\mathcal{E}_m$ = magnetic energy

$|\overline{H}| = |\overline{H_1} + \overline{H_2}|$ total field

$$\mathcal{E}_m = \underbrace{\frac{\mu}{2} |\overline{H_1}|^2 dv}_{\text{coil 1 energy}} + \underbrace{\frac{\mu}{2} |\overline{H_2}|^2 dv}_{\text{coil 2 energy}} + \underbrace{\frac{\mu}{2} \cdot 2 |\overline{H_1}||\overline{H_2}| \, dv}_{\text{mutual energy}}$$

$$\vec{H} = \frac{\vec{B}}{\mu}$$



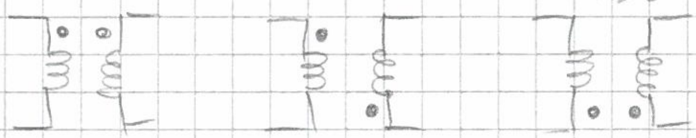
$$\begin{cases} \Phi_1 = L_1 I_1 + M I_2 \\ \Phi_2 = L_2 I_2 + M I_1 \end{cases}$$

M = coupled inductance

$V_1 = \dot{\Phi}_1 \quad V_2 = \dot{\Phi}_2$] → Lenz Law

$$\mathcal{E}_m = \int_0^t (V_1 I_1 + V_2 I_2) \, dt' = \frac{1}{2} L_1 I_1^2(t) + \frac{1}{2} L_2 I_2^2(t) + M I_1(t) I_2(t)$$

$M > 0 \qquad M < 0 \qquad M > 0$

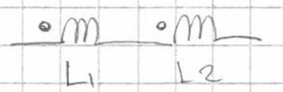


conservation of energy

$$K = \frac{|M|}{\sqrt{L_1 L_2}} \qquad 0 \leq K \leq 1$$

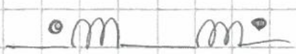
- $K = 1$ → max magnetic coupling
- $K = 0$ → zero coupling

Example: $I_1 = I_2 = I \qquad L_1 = L_2 = L$



$$\Phi = \Phi_1 + \Phi_2 = L_1 I_1 + M I_2 + L_2 I_2 + M I_1 = (L_1 + L_2 + 2M) I$$

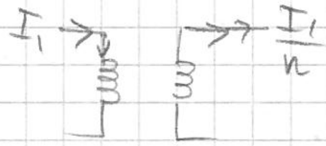
$$L_{TOT} = L_1 + L_2 + 2M \begin{cases} 4L & \text{if } K = 1 \\ 2L & \text{if } K = 0 \end{cases}$$



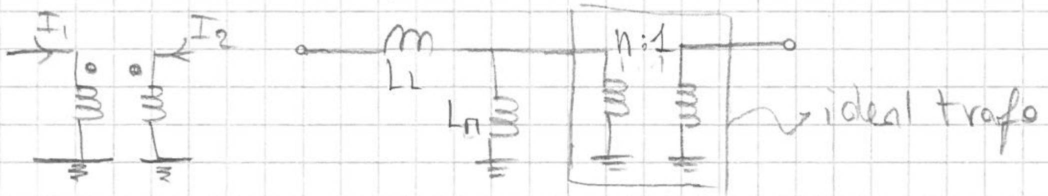
$$L_{TOT} = L_1 + L_2 - 2|M| \begin{cases} 2L & \text{if } K = 0 \\ 0 & \text{if } K = 1 \end{cases}$$

magneto motive force $mmf = n_1 I_1 + n_2 I_2 \rightarrow$ Ampere's law

$$\frac{I_1}{I_2} = -\frac{n_2}{n_1} \quad \text{for } 1:n \text{ trafo}$$



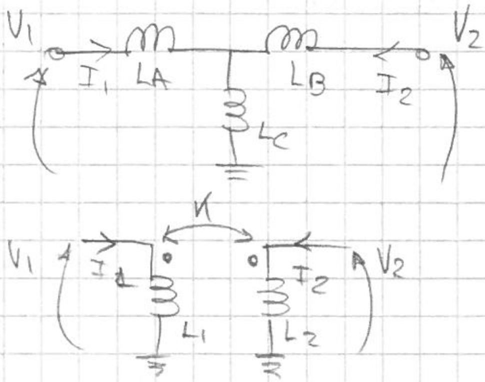
Equivalent model of coupled inductors



$L_L = (1-K^2) L_1 \qquad L_M = K^2 L_1 \qquad n = K\sqrt{\frac{L_1}{L_2}}$

L_L → leakage, L_M → magnetizing

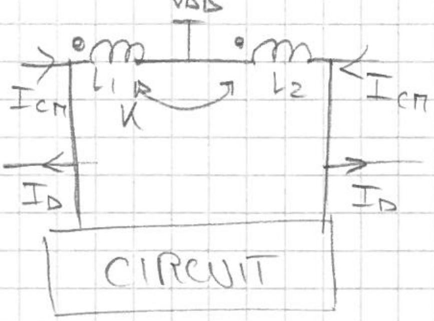
T-circuit for coupled inductors



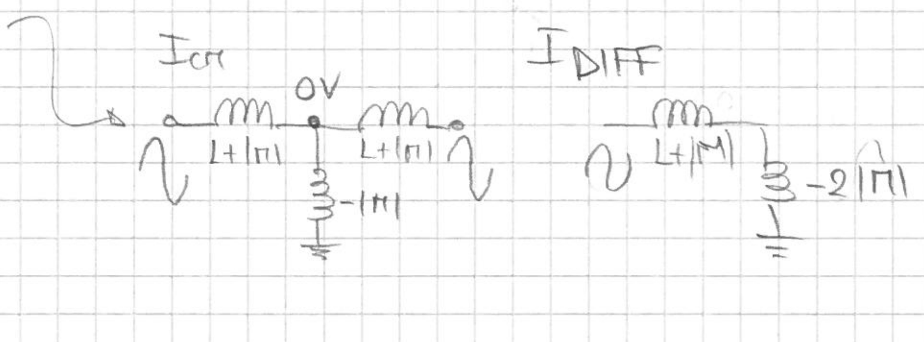
$$\begin{cases} L_A = L_1 - M \\ L_B = L_2 - M \\ L_C = M \end{cases}$$

Where $|M| = \frac{K}{\sqrt{L_1 L_2}}$

This is useful with common mode killer configuration



I_{CM} won't pass while I_D will



6) NF of lossy circuits and NF of cascaded systems

$$NF \triangleq \frac{SNR_{IN}}{SNR_{OUT}}$$

→ NF = 1 If stage is noiseless

→ NF = ∞ if input is noiseless → NF depends on input noise

$$NF = \frac{\overline{V_{SIG,IN}^2}}{\overline{V_{n,IN}^2}} \cdot \frac{\overline{V_{n\,out}^2}}{\overline{V_{SIG\,OUT}^2}} = \frac{1}{A_0^2} \cdot \frac{\overline{V_{n\,network}^2} + A_0^2\,\overline{V_{nRs}^2}}{\overline{V_{nRs}^2}}$$

$A_0^2 = \frac{\overline{V_{S\,IN}^2}}{\overline{V_{S\,OUT}^2}}$

$\overline{V_{n\,network}^2} + A_0^2\,\overline{V_{nRs}^2}$ → out total noise

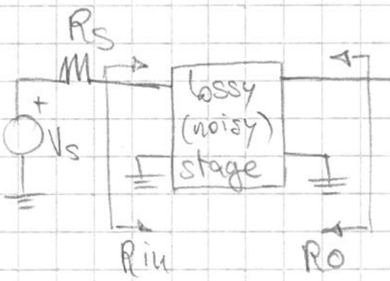
$\overline{V_{nRs}^2}$ → in noise

$$\frac{\overline{V_{out_n,TOT}^2}}{A_0^2} = NF \cdot \overline{V_{nRs}^2} \longrightarrow PSD\Big|_{R_o,\ \text{input referred}} = 4KTR_S \cdot NF_{R_o}$$

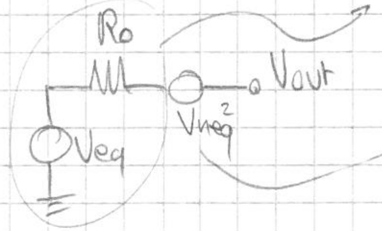
$\overline{V_{nRs}^2}$ → Resistor noise PSD

$\frac{\overline{V_{out_n,TOT}^2}}{A_0^2}$ → Input referred output noise

Where R_o is the output impedance of a noisy stage:



I can model the output port using Thevenin and Nyquist theorems



→ Thevenin eq

→ Nyquist $4KT\{Re(Z_o)\} = 4KTR_o$

$$NF = \frac{\overline{V_{nOUT,TOT}^2}}{A_0^2} \cdot \frac{1}{\overline{V_{nRs}^2}} = \frac{\overline{V_{neq}^2}}{A_0^2} \cdot \frac{1}{\overline{V_{nRs}^2}} = \frac{4KTR_o}{A_0^2} \cdot \frac{1}{4KTR_S}$$

$$NF = \frac{1}{A_0^2\,\frac{R_S}{R_o}} = \frac{1}{G_A} = L_A$$

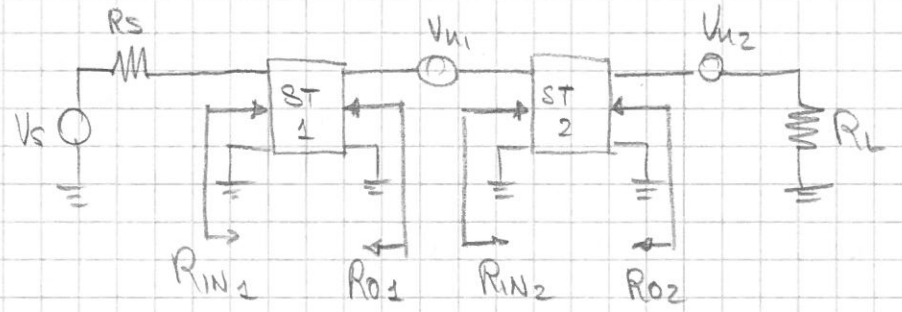
L_A → available power loss

G_A → available power gain

Knowing G_A, L_A → Noise of the stage is immediate

This is a very powerful result

NF in cascaded stages



$$NF = \underbrace{1 + \frac{\overline{V_{n1}^2}}{A_{o1}^2}\frac{1}{4KTR_S}}_{NF_1} + \frac{\overline{V_{n2}^2}}{A_{o1}^2 A_{o2}^2}\frac{1}{4KTR_S}$$

$$NF_2\Big|_{R_{o1}} = 1 + \frac{V_{n2}^2}{A_{o2}^2}\cdot\frac{1}{4KTR_{o1}}$$ → Now adjust NF equation using NF_2

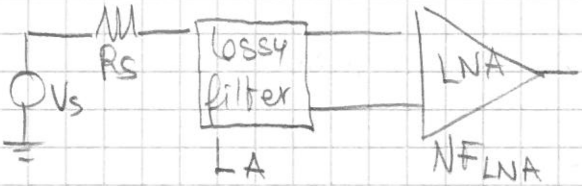
$$NF = NF_1 + \frac{\left(NF_2\big|_{R_{o1}} - 1\right)\cancel{4KT}R_{o1}}{A_{o1}^2\cdot\cancel{4KT}R_S} = NF_1 + \frac{NF_2\big|_{R_{o1}} - 1}{GA_1}$$

In general:

$$NF = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{GA_1} + \frac{NF_3 - 1}{GA_1\,GA_2} + \dots$$

1st stage is the most critical for NF because the following stages are attenuated by available power gains of the previous stages → more negligible

example:



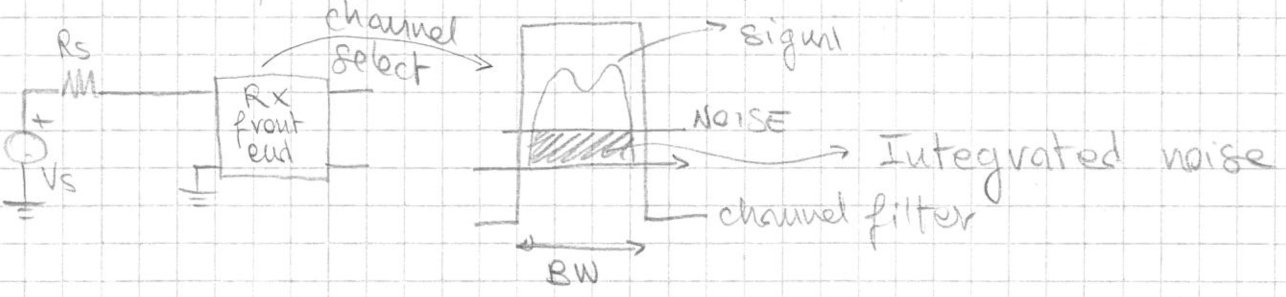
$$NF = NF_{filter} + \frac{NF_{LNA} - 1}{1/LA} = LA \cdot NF_{LNA}$$

where $NF_{filter} = LA$.

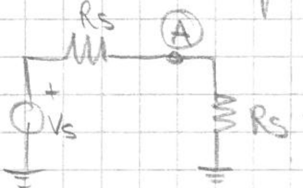
$$NF\big|_{dB} = LA\big|_{dB} + NF_{LNA}\big|_{dB}$$

7) RF receivers: sensitivity and dynamic range

RX sensitivity $\triangleq$ min detectable power



We need to find integrated noise. Consider matched inputs:



Suppose that $R_S|_{RX}$ is at 0K $\rightarrow \overline{V_n^2} = 4KTR_S = 0$ so that input R_S noise can deliver power to RX input

$$\frac{P_{n,av}}{\Delta f} = \frac{\frac{\overline{V_n^2}}{4R_S}}{\Delta f}$$

$\rightarrow$ matched RX frontend input (see max power transfer)

it's over a Δf because we want integrated noise over the channel BW

Considering the RX front-end:

$$\frac{P_{n,av}}{\Delta f} = \frac{\frac{\overline{V_{n}^2}_{TOT}}{4R_S}}{\Delta f} = \frac{\frac{\overline{V_n^2} \cdot NF_{RX}}{4R_S}}{\Delta f} = \frac{\frac{4KTR_S\ NF_{RX}}{4R_S}}{\Delta f} = \frac{KTNF_{RX}}{\Delta f}$$

Since $SNR|_{min} = \frac{P_{S,av\,(min)}}{P_{n,av}}$ $\rightarrow$ integrated power noise

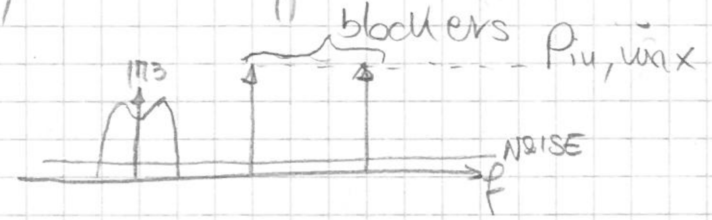
$$P_{S,av\,(min)} = SNR|_{min}\ P_{n,av} = SNR|_{min} \cdot KT \cdot NF_{RX} \cdot BW$$

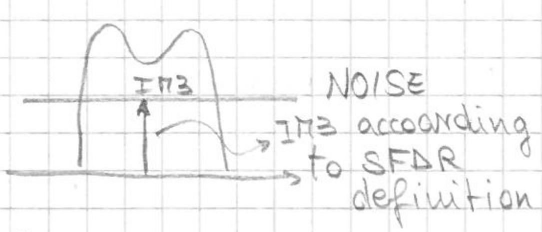
$$KT|_{dB} = -174\ dBW/Hz$$

6 ÷ 25 dB typically

$$P_{S,av}|_{min,\,dB} = -174\ \frac{dBW}{Hz} + SNR|_{min\,dB} + NF_{RX}|_{dB} + 10\log_{10}(BW)$$

Dynamic Range



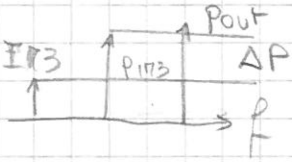


SFDR $\triangleq$ spurious free dynamic range $\triangleq P_{in,max}\big|_{dB} - P_{in,min}\big|_{dB}$

by the definition of "spurious free":

- $P_{in,max}$ = blockers power such that IM3 equals noise power
- $P_{in,min}$ = sensitivity

This way IM3 won't be detected because it's buried in noise



$$P_{IIP_3}\Big|_{dBm} = \underbrace{P_{in}\Big|_{max}}_{\text{max input blocker power}} + \frac{\Delta P}{2} = P_{in} + \frac{P_{OUT} - P_{IM3,OUT}}{2} =$$

$$P_{IIP_3}\Big|_{dBm} = P_{in}\Big|_{max} + \frac{\overbrace{P_{in} + \cancel{G_A}}^{P_{out}} - \overbrace{(P_{IM3,IN\ REFERRED} + \cancel{G_A})}^{P_{IM3,OUT}}}{2}$$

$$P_{IIP_3}\Big|_{dBm} = \frac{3}{2} P_{in}\Big|_{max} - \frac{1}{2} P_{IM3,IN\ REF} \quad \text{but we said} \quad P_{IM3}\Big|_{IN\ REF} = P_n$$

where P_n = input referred noise power, so

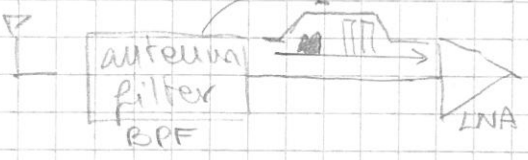
$$P_{IIP_3}\Big|_{dBm} = \frac{3}{2} P_{in}\Big|_{max} - \frac{1}{2} P_n \longrightarrow P_{in,max} = \frac{2 P_{IIP_3} + P_n}{3}$$

$$SFDR = P_{in,max} - P_{in,min} \qquad SNR_{min} = P_{in,min} - P_n \quad \text{so}$$

$$SFDR = \frac{2}{3} P_{IIP3} + \frac{1}{3} P_n - (P_n + SNR_{min})$$

$$SFDR\Big|_{dB} = \frac{2}{3} P_{IIP_3} - \frac{2}{3} \underbrace{P_n}_{KT \cdot NF_{Rx} \cdot BW} - SNR_{min}$$

8) Heterodyne receivers: advantages. Image problem and filtering. Selectivity / Sensitivity trade-off. Block schematic



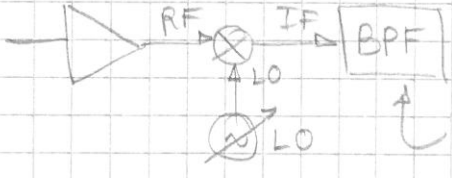
Antenna BPF filters out-of-band interferers

Channel selectivity ~ 60dB in wireless systems

Can't be achieved with RLC or butterworth BPFs. Solution:

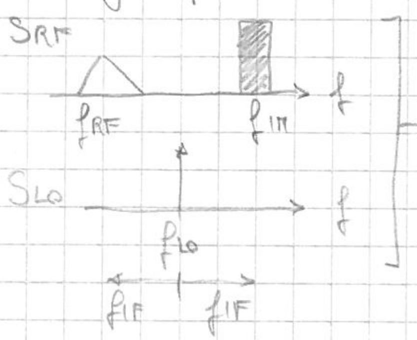
Heterodyne receiver: pros: - IF ≪ RF → higher selectivity at lower freq

- IF filter can be fixed → less complicate and expensive



off-chip SAW filter

Image problem:



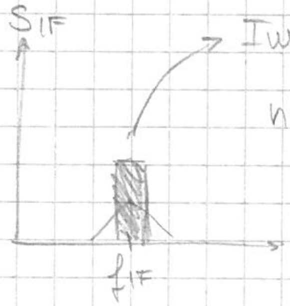
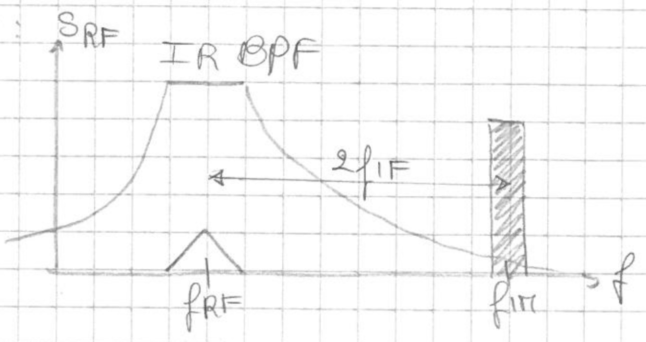


Image: interferer generated by numerous users nearby the RX receiver (police, WLAN, etc)

SNIR = signal to noise and image ratio is degraded.

Solution: Image Rejection filter:



$2f_{IF} = f_{IM} - f_{RF}$

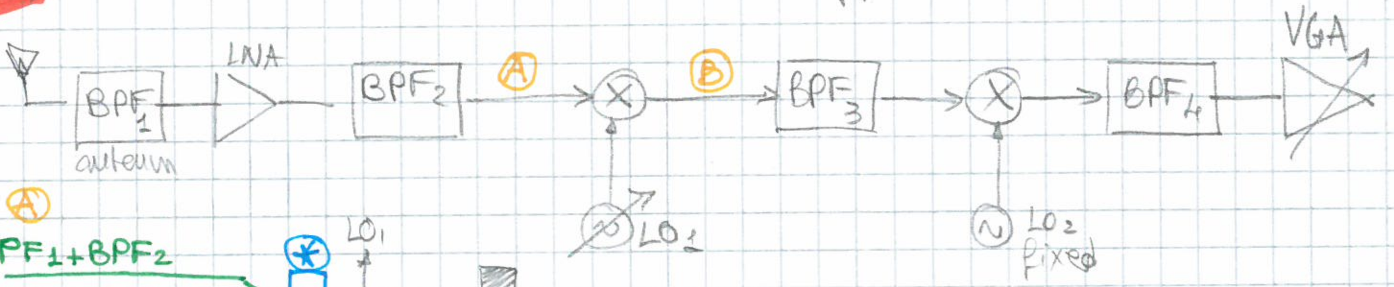
In order to have good rejection we would like high f_{IF}

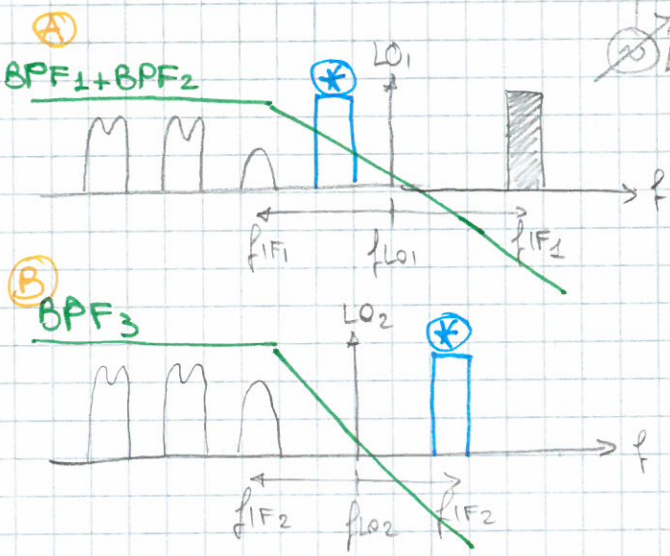
But:

- Selectivity → related to adjacent channels ⇒ low f_{IF}
- Sensitivity → related to in-band interferers → high f_{IF}

We can see that there is a tradeoff between sensitivity vs. selectivity.

11) Dual IF architecture: relax tradeoff (I moved the answer here)





Pros:

- large f_{IF_1}: relaxes IR (BPF_2)
- small f_{IF_2}: relaxes channel selection (BPF_4)

Cons:

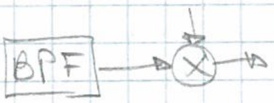
- more BPF + mixer + LO

Secondary image problem:

(*) another interferer can couple between f_{RF} and f_{LO_1} $\rightarrow$ We will get the same problem on f_{IF_2}

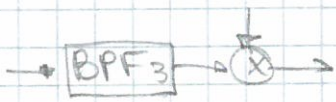
Solution: use BPF_3 to reject the secondary image

This still works because we're at f_{IF_1} $\rightarrow$ lower freq than f_{RF} $\Rightarrow$ BPF_3 requirement is still more relaxed:



Single IF

$$Q_1 = \frac{f_{RF}}{2 f_{IF}}$$

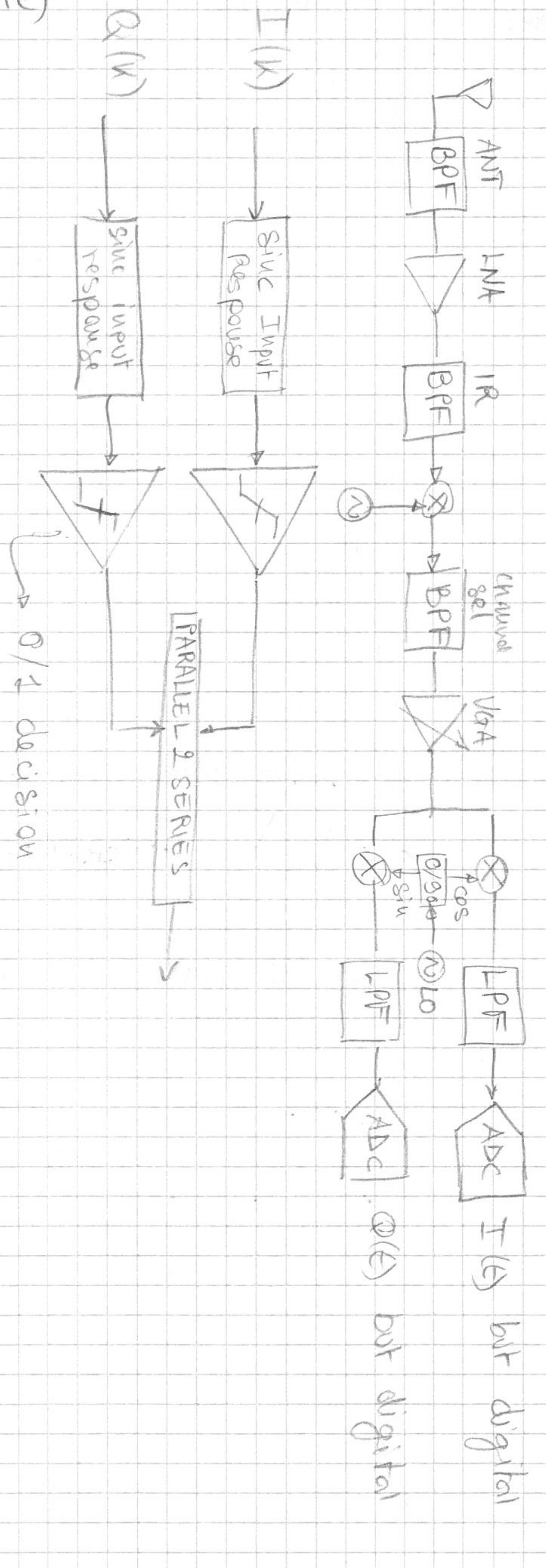


Dual IF

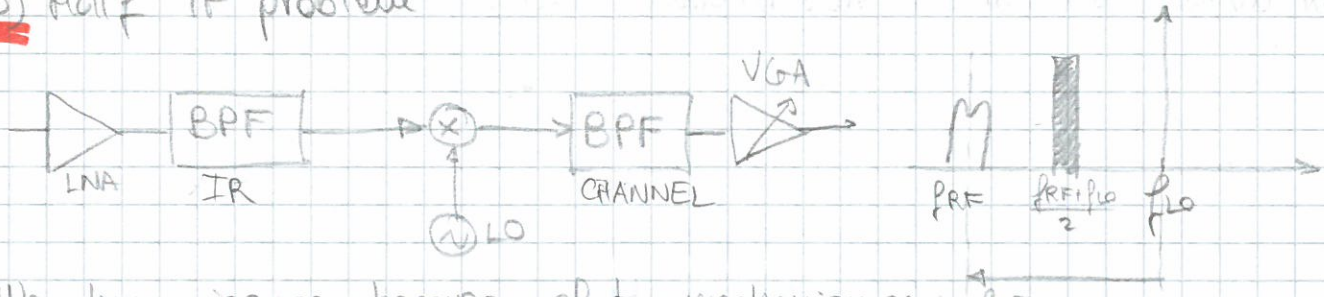
$$Q_2 = \frac{f_{IF_1}}{2 f_{IF_2}}$$

$\Rightarrow Q_2 \ll Q_1$

Full architecture (block schematic)



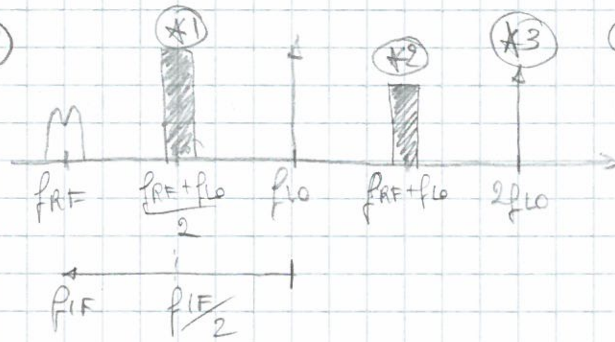
9) Half IF problem



We have issues because of two mechanisms:

- LO 2nd harmonic + LNA 2nd harmonic (A)
- VGA 2nd order non-linearity (B)

(A)

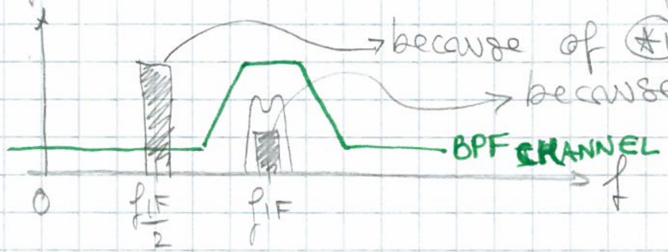


(*1): in-band interferer

(*2): 2nd harmonic of (*1) because of (A)

(*3): 2nd harmonic of LO because of duty cycle ≠ 50% or because of mixer non linearities

After the mixer we will get



→ because of (*1)

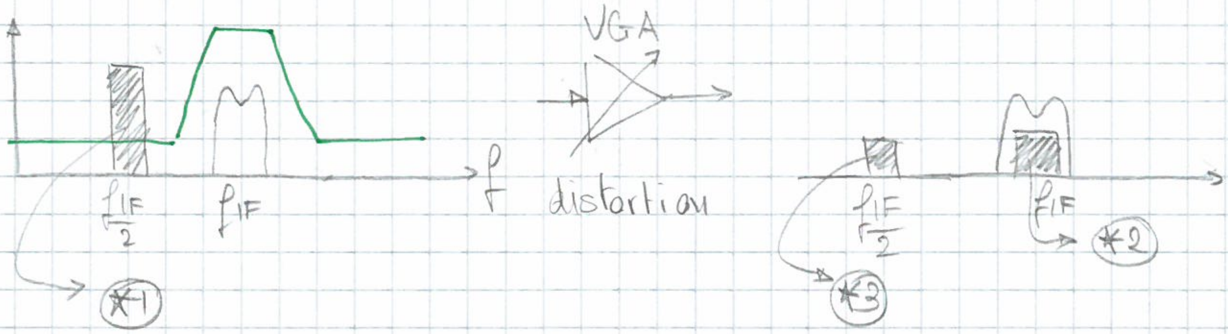
→ because of (*2) demodulated by (*3):

$$\underbrace{2f_{LO}}_{*3} - \underbrace{f_{RF} - f_{LO}}_{*2} = f_{IF}$$

BPF (channel select) can't filter demodulated (*2)

Result is that we have SNIR degradation.

(B)



(*1): half IF in-band interferer

(*3): (*1) but gets attenuated by BPF

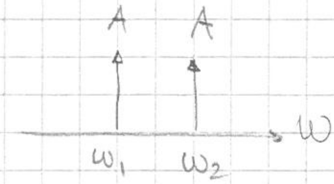
(*2): 2nd harmonic of (*3) generated by VGA distortion

So, after all, also 2nd harmonics can be <u>harmful</u>

10) Second-order non-linearity. IIP2 and link with HD2

Consider $y(t) = \underbrace{\alpha_1 x(t)}_{\text{linear coefficient}} + \alpha_2 x^2(t)$

Consider now the usual two tone test, where:

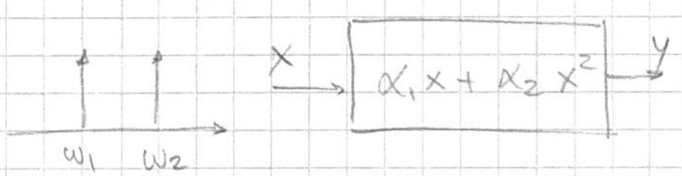


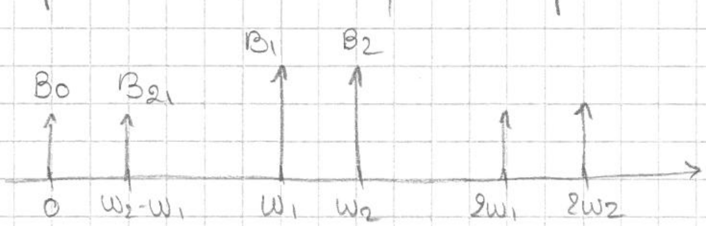
$x(t) = A\,(\cos\omega_1 t + \cos\omega_2 t)$

$y(t) = \alpha_1 A\,(\cos\omega_1 t + \cos\omega_2 t) + \alpha_2 A_2^2\,(\cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t + \dots)$

Quadratic term implies tone generation at low frequencies like $\omega_2 - \omega_1$.

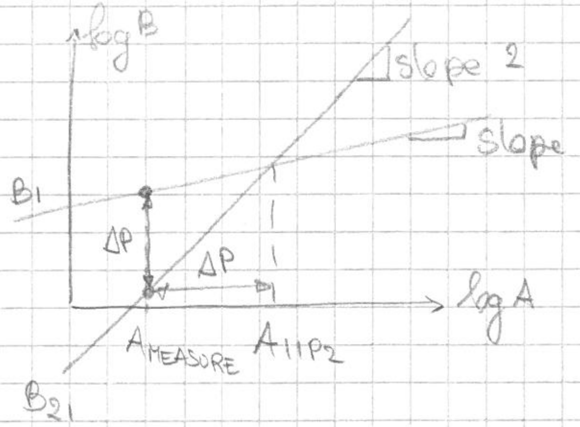
We see that generated tones depend on the square of A $\longrightarrow$ slope = 2 when A increases





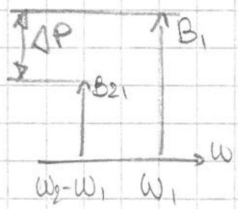
If we calculate B coefficients: $B_1 = B_2 = \alpha_1 A$

$B_0 = B_{21} = \alpha_2 A^2$

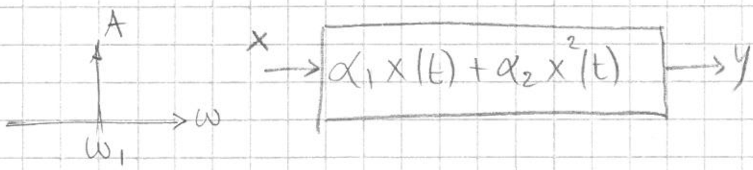


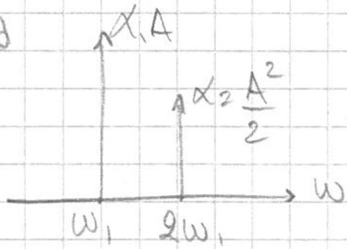
Using the same procedure for IIP3 (with slope difference 1 instead of 2), we found out that

$$IIP_2\big|_{dBm} = P_{in}\big|_{dBm} + \Delta P\big|_{dBm}$$



If we fed a single tone instead



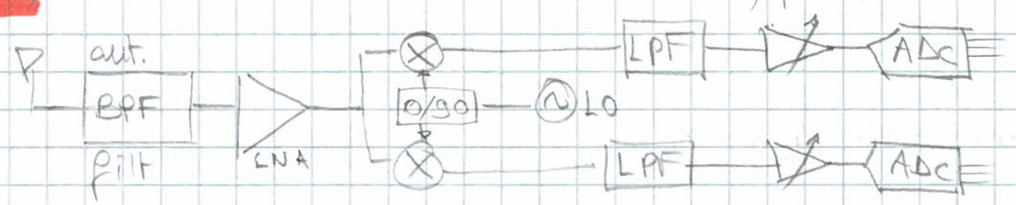


If we want to estimate HD2 throug IIP2:

$$HD2 = \frac{\alpha_2 A^2/2}{\alpha_1 A} = \frac{B_{21}}{B_1}\cdot\frac{1}{2} = \frac{A_{MEASURE}}{A_{IIP2}}\cdot\frac{1}{2}$$

$$HD2\big|_{dB} = -\Delta P\big|_{dB} - 6dB = P_{in}\big|_{dB} - IIP2\big|_{dB} - 6dB$$

12) Zero-IF receivers: architecture, pros/cons. DC os cancellation



In direct conv. $f_{LO} = f_{RF} \longrightarrow f_{IF} = f_{RF} - f_{LO} = 0 \longrightarrow$ Zero-IF

I need two mixers because it wouldn't be possible to recover I/Q

pros: → Direct conversion to baseband

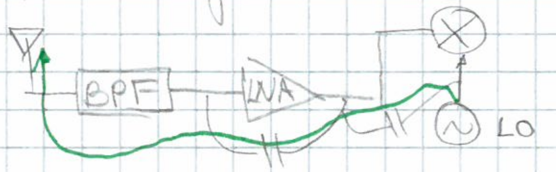
- Image problem "apparently" solved → No IR filter
- Channel select with LPF instead of BPF
- No need for off-chip SAW filters

→ Suitable for full-integration

- Mixing spurs are reduced in number → simpler to handle
- LNA (integrated) can be optimized for GA, NF, linearity without the 50 Ω requirement

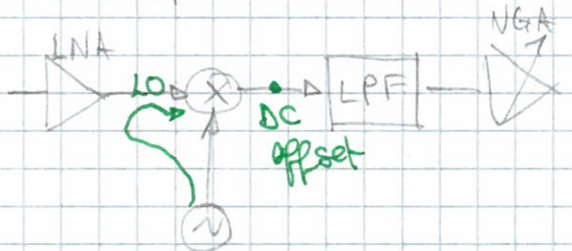
Cons:

- LO leakage



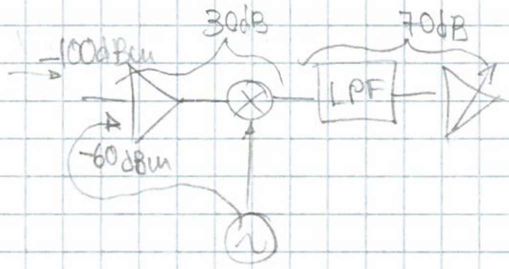
LO couples through capacitive tracks (mixer, LNA) or the chip substrate and propagates through antenna → it might violate radiation limits (< -50/80 dBm). This can be reduced using differential LOs.

- DC offset



LO couples through RF port of the mixer → self mixing introduces DC offsets

eg: signal = -100 dBm, LO leak = -60 dBm → LNA + mixer gain = +30 dB, LPF + VGA gain = +70 dB → 100 dB gain

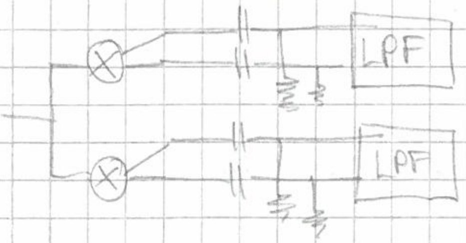


→ At LPF in → 100 μV signal, 10 mV DC os →

→ 30V at VGA out

DC offset cancellation techniques

- AC coupling: CR-LPF has to be low enough not to degrade BW of signal $\rightarrow \frac{f_{BW}}{1000} = f_{CR}$

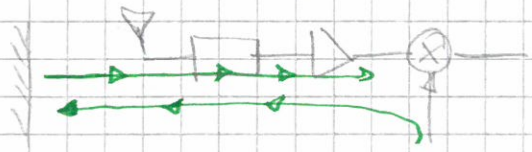


We have differential paths $\rightarrow$ implementation of 4 big capacitors to keep resistor noise low

- Switched cap

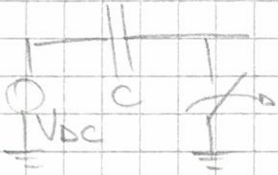
Using Time division of multiple samples (TDMA), we can do a zero setting to cancel DC offset.

Problem: we store interferers during sampling. DC os also varies over time because of reflections



We can average samples over time to isolate DC os.

Problem: switching cap noise

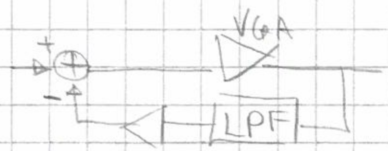


$V_C = V_{DC} + V_{noise}$ $\qquad \overline{V_{noise}^2} = KT/C$

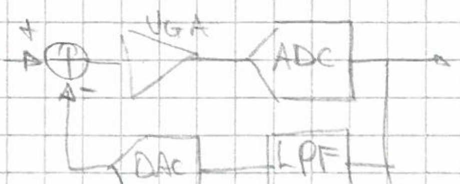
$$SNR = \frac{P_s'}{KT/C} = \frac{-93dBV @ 50\Omega}{-108dBV} > 15dB \rightarrow C > 250pF$$

So we have $4 \cdot 250pF = 1nF$ total capacitance $\rightarrow$ still too large

- Feedback

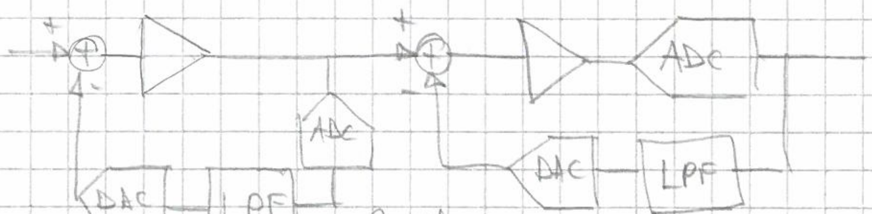


Using analog we can demonstrate that caps for LPF have to be even larger than CR-LPF



digital filter

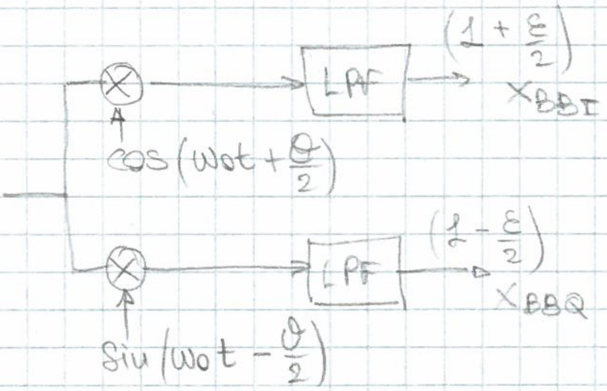
1-step



2-step

ADCs for OS estimation: they don't need to be high speed (we want ~~DC~~ only). They have to be precise. It's convinient to have ADC on chip. Two step approach is to avoid VGA saturation

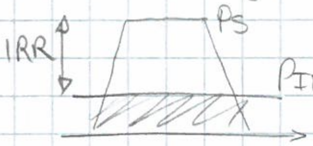
13) Zero-IF: I/Q mismatches on SNR. LO leakage impact



Consider a fixed delay $\tilde{\tau}$ on RF-mixer

$\sin\left(w_0 t-\frac{\theta}{2}\right)=\sin\left(w_0 t-\frac{w_0\tilde{\tau}}{2}\right)$

θ shift depends on w_0 → CRITICAL when using RF-to-0Hz conversion



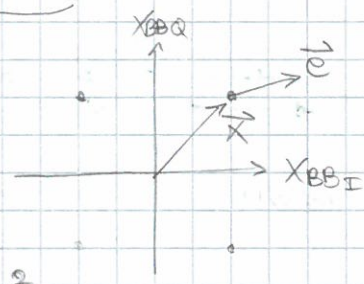
P_{IM} ~> generated by I/Q imbalances

$x_{RF}(t)=x_I(t)\cos w_0 t+x_Q(t)\sin w_0 t$

$x_{LO_1}=2\left(1+\frac{\varepsilon}{2}\right)\cos\left(w_0 t+\frac{\theta}{2}\right)$ $\quad x_{LO_2}=2\left(1-\frac{\varepsilon}{2}\right)\sin\left(w_0 t-\frac{\theta}{2}\right)$

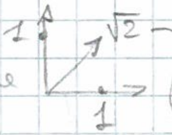
$$x_{BB_1}(t)=x_{RF}\cdot x_{LO_1}=\underbrace{x_I(t)\left(1+\frac{\varepsilon}{2}\right)\cos\frac{\theta}{2}}_{\text{wanted}}-\underbrace{x_Q(t)\left(1+\frac{\varepsilon}{2}\right)\sin\frac{\theta}{2}}_{\text{leaked}}$$

$$x_{BBQ}(t)=x_{RF}\cdot x_{LO_2}=\underbrace{x_Q(t)\left(1-\frac{\varepsilon}{2}\right)\cos\frac{\theta}{2}}_{\text{wanted}}-\underbrace{x_I(t)\left(1-\frac{\varepsilon}{2}\right)\sin\frac{\theta}{2}}_{\text{leaked}}$$



IRR = Image rejection ratio $=P_S/P_{IM}$

$$SNR=IRR=\frac{|\bar{x}|^2}{|\bar{e}|^2}=\frac{|\bar{x}|^2}{|x_{BB}-\bar{x}|^2}$$ where



$|\bar{x}|^2=2$

for small θ:

$$x_{BB_1}\simeq x_1\left(1+\frac{\varepsilon}{2}\right)\cancel{\cos\frac{\theta}{2}}^{\sim 1}-x_Q\left(1+\frac{\varepsilon}{2}\right)\cancel{\sin\frac{\theta}{2}}^{\sim\frac{\theta}{2}}$$

$$x_{BBQ}\simeq x_Q\left(1-\frac{\varepsilon}{2}\right)\cancel{\cos\frac{\theta}{2}}^{\sim 1}-x_1\left(1-\frac{\varepsilon}{2}\right)\cancel{\sin\frac{\theta}{2}}^{\sim\frac{\theta}{2}}$$

$$SNR=\frac{|\bar{x}|^2}{|\bar{e}|^2}=\frac{|\bar{x}|^2}{|\bar{x}_{BB}-\bar{x}|^2}=\frac{2}{(x_{BB_1}-x)^2+(x_{BBQ}-x)^2}=$$

$$=\frac{2}{\left[\left(1+\frac{\varepsilon}{2}\right)\left(1-\frac{\theta}{2}\right)-1\right]^2+\left[\left(1-\frac{\varepsilon}{2}\right)\left(1-\frac{\theta}{2}\right)-1\right]^2}=\frac{4}{\varepsilon^2+\theta^2}=\frac{1}{\left(\frac{\varepsilon}{2}\right)^2+\left(\frac{\theta}{2}\right)^2}$$

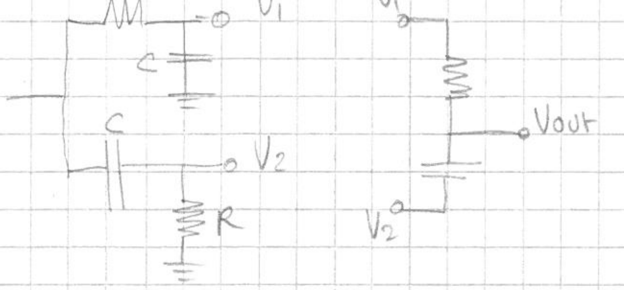
We can't get over ~30dB total.

e.g.: $\varepsilon\simeq 0{,}1$, $\theta=1^\circ=0{,}0174$ rad $\quad IRR=10\log\left(\frac{4}{\varepsilon^2+\theta^2}\right)=25{,}9\,dB$

Mixers used in direct conversion have to be accurate in order not to introduce large phase mismatch.

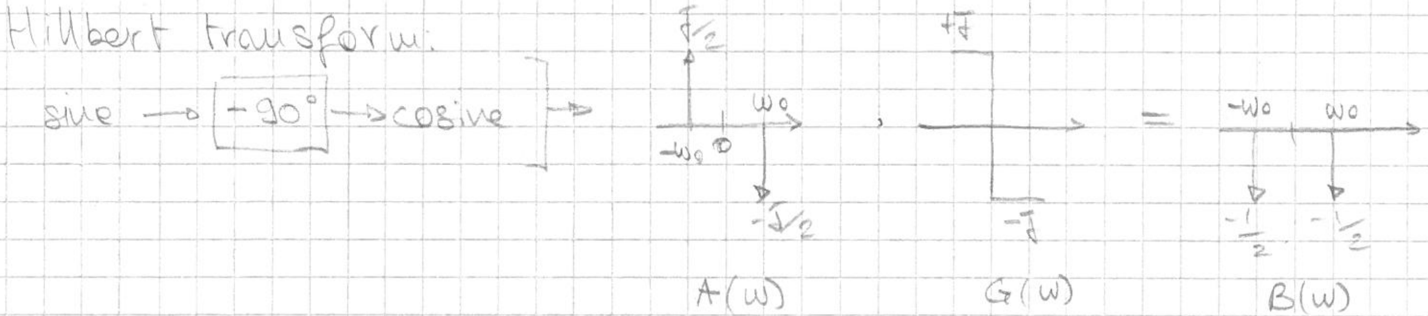
14) Image-reject receivers: 90°-shift. Hartley and IRR effects

Heterodyne receivers → OFF-chip SAW required (therefore 50 Ω matching required). Hartley receiver solves this



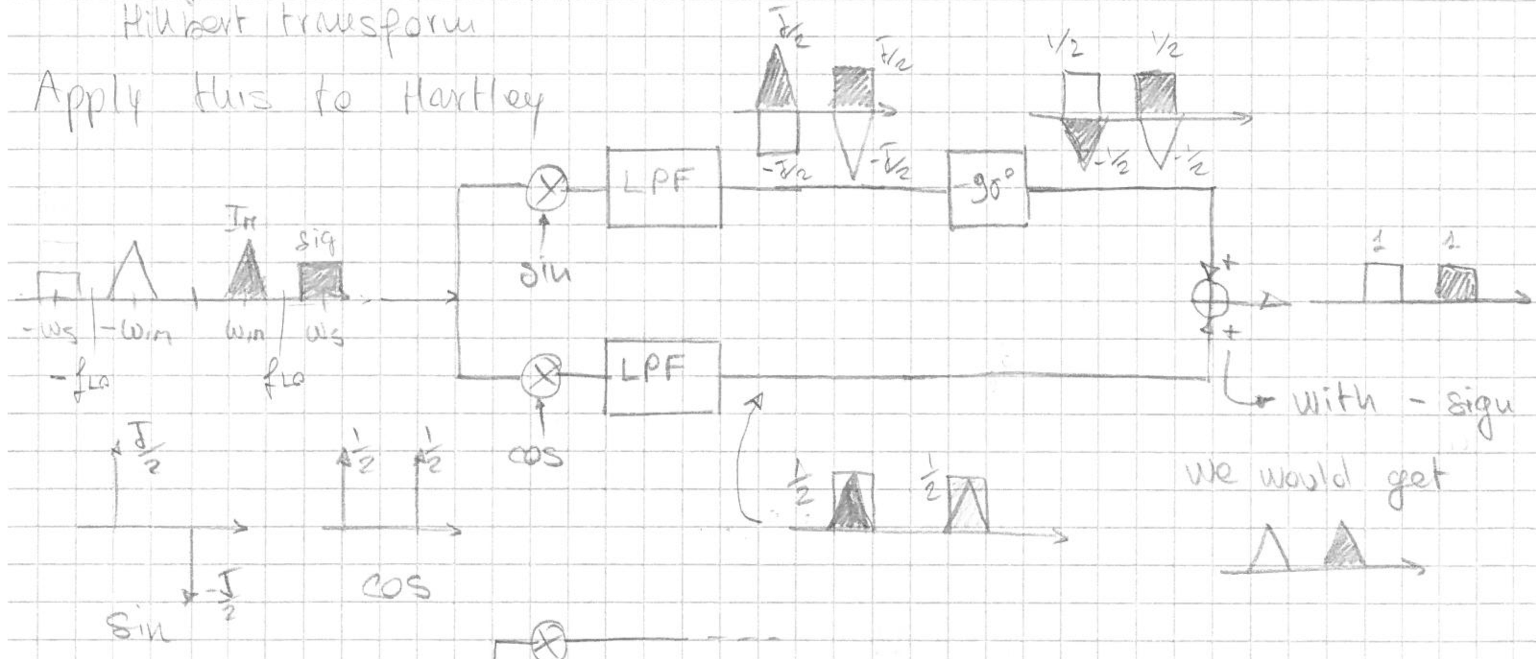
Two possible 90° shift implementations (Note: works for 1 frequency only)

Hilbert transform:



$$\underbrace{B(\omega) = G(\omega) \cdot A(\omega)}_{\text{Hilbert transform}} \quad \text{where} \quad G(\omega) = -J \, \mathrm{sign}(\omega)$$

Apply this to Hartley



If we consider

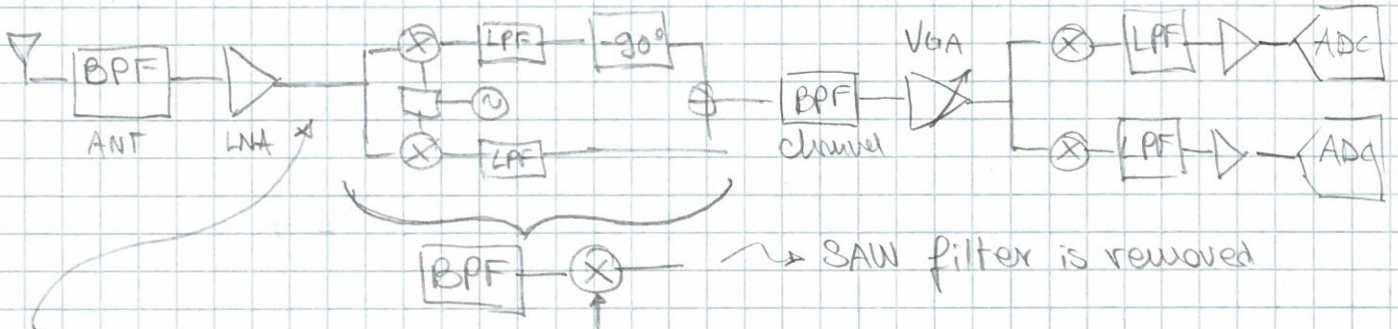
$(1-\frac{\varepsilon}{2})\sin(\omega_0 t - \frac{\theta}{2})$

$(1+\frac{\varepsilon}{2})\cos(\omega_0 t + \frac{\theta}{2})$

Then $IRR = \frac{P_s}{P_{IM}} = \frac{4}{\varepsilon^2 + \theta^2}$

Hartley architecture can't go over 30dB image suppression.
On the other hand, if Hartley + BPF is used, BPF requirements will be more relaxed thanks to the 30dB IRR

Hartley IR receiver



An additional IR BPF goes here in case 30dB IRR is not enough.

15) Image-reject receivers: Weaver architecture

Hartley cons:

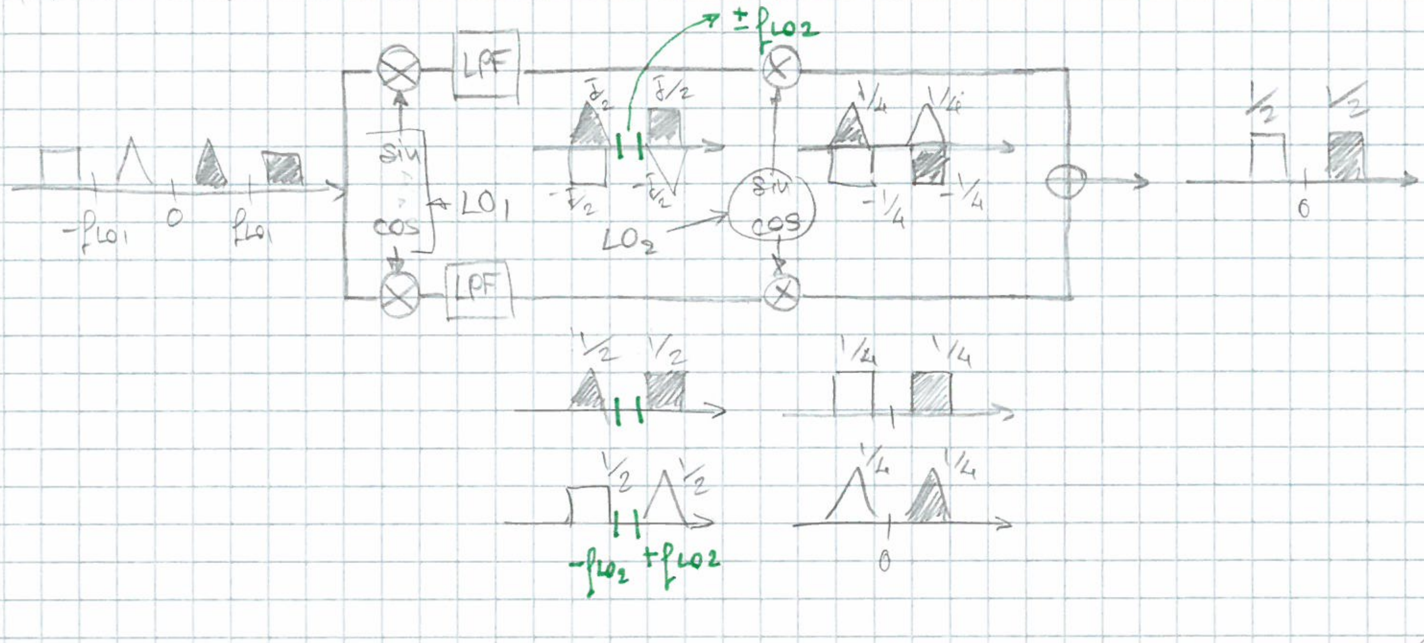
- phase shifting works only at f_{POLE} → narrow BW
- sensitive to RC absolute accuracy $\frac{\Delta\tau}{\tau} = \frac{\Delta R}{R} + \frac{\Delta C}{C}$ → low IRR
- phase shift introduces noise + power loss

Weaver architecture pros:

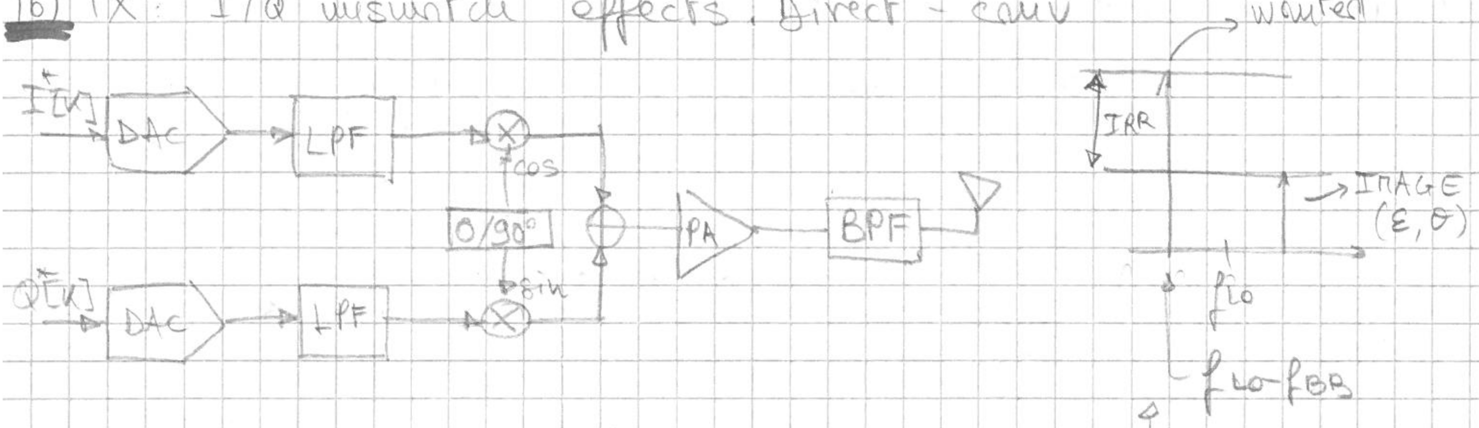
- Solves phase shifter issue

Weaver cons:

- requires more mixers
- suffers from secondary image problem → to mitigate this we would need BPFs instead of LPFs or a zero-IF IR (this would mean two more mixers



16) TX: I/Q mismatch effects. Direct - conv



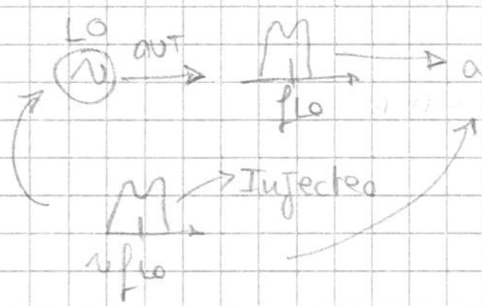
If $I(t) = \cos \omega_{BB}t$ $Q(t) = \sin \omega_{BB}t$ then

$\cos \omega_{bb}t \cos \omega_{LO}t - \sin \omega_{bb}t \sin \omega_{LO}t = \cos(\omega_{LO} - \omega_{BB})t$

If we introduce ε, θ imbalances we find $IRR = \frac{P_S}{P_{IM}} = \frac{4}{\varepsilon^2+\theta^2}$

In a direct TX, signal after PA self couples back into LO by em coupling or throug IC substrate → Injection locking

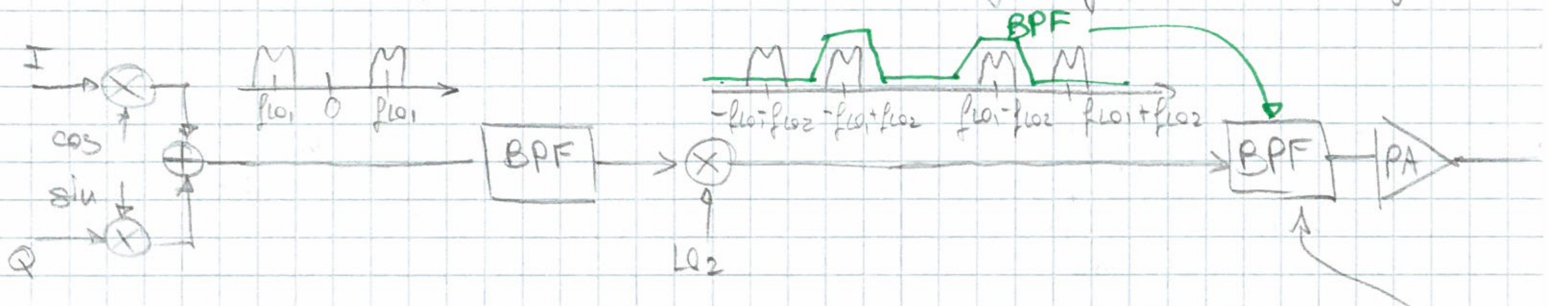
If coupled freq is $\sim f_{LO}$, LO will tend to follow that injection like in a PLL. This depends on Q factor of the osc. Same happens if injection isn't just a frequency but a band:



→ after locking, LO out and injected disturb will be the same

17) 2-step TX SSB mixer

We shift PA frequency band in order not to have it similar to LO frequency (thus reducing injection locking)



f_{LO_2} does not need to be really high. It's purpose is to change the $f_{out} = f_{LO_1} - f_{LO_2}$ frequency in order not to be similar to f_{LO_1}. BPF is used to reject the unwanted side-band

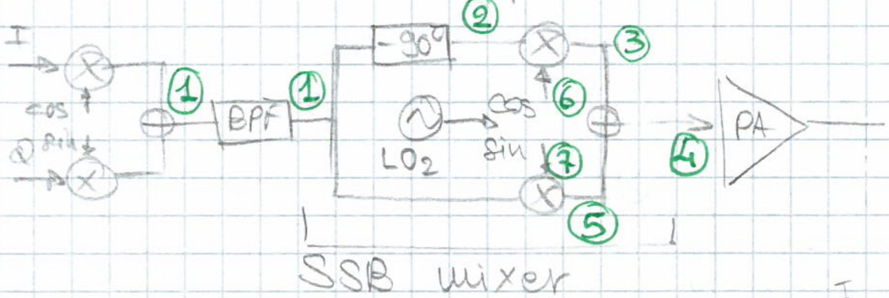
pros:

- Avoids LO_1 pulling from PA injection
- Improves I/Q matching

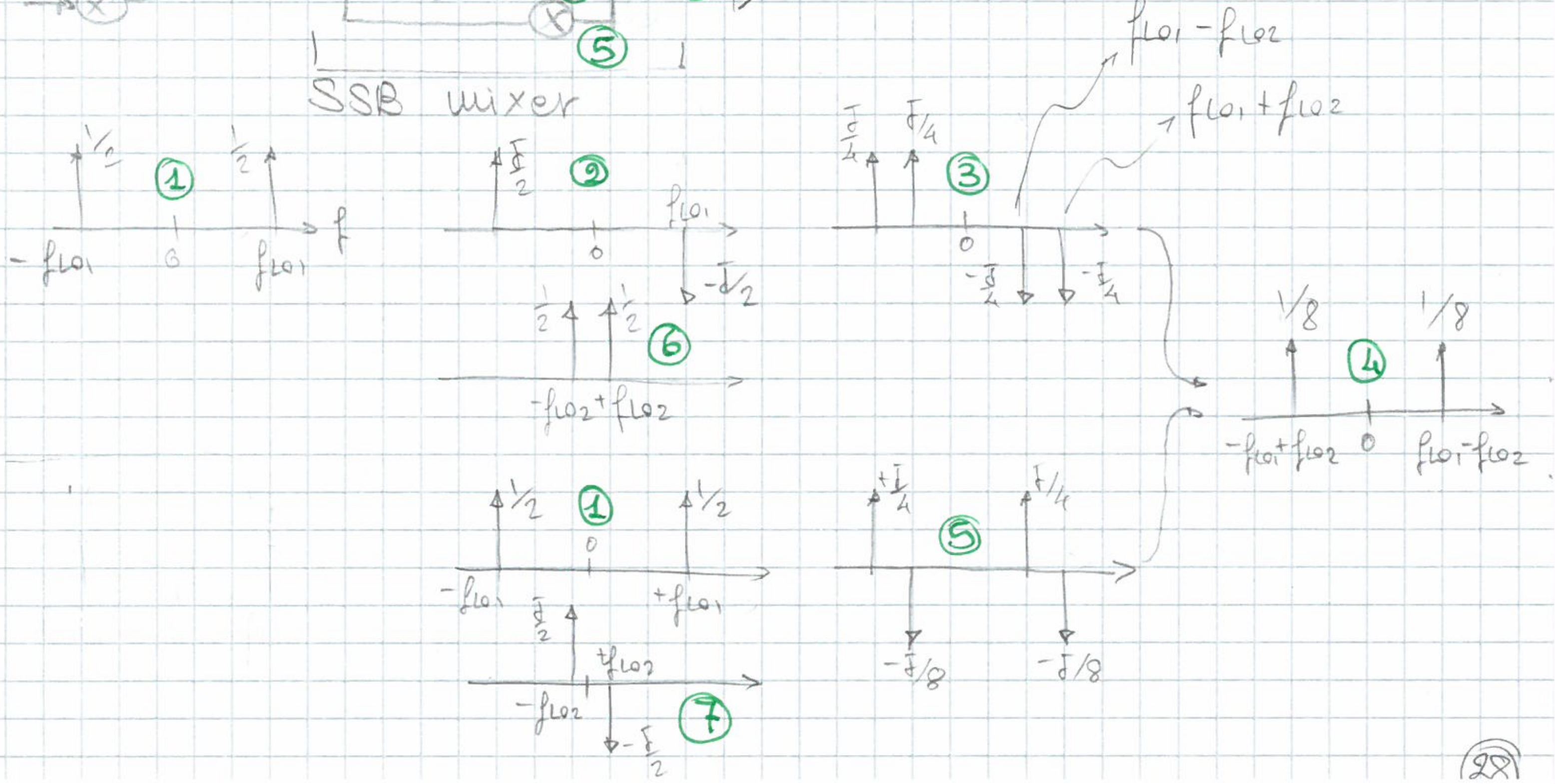
cons

- BPF used for reducing the side-band has to provide 50/60 dB

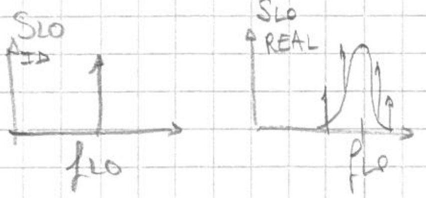
To solve the latter problem we use SSB (Single Sideband Mixer):



Suppose that ① is a cosine. (not a band)

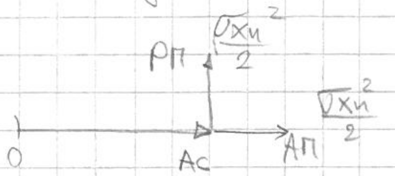


18) AM / FM disturbances. Phase/voltage spectrum relationship



Real LO spectrum will have a bandwidth + spurious tones that damage the ideal (wanted) δ for modulation.

According to Rice's theorem, noise will split into AM/PM:



AM can be avoided by hard limiting the carrier (the second order effect of PM by clipping AM signals is not considered here).

Why phase noise is a problem?

Recall $\varphi_n(t) = \int_{-\infty}^{t} \omega_n(\tau)\, d\tau$ → phase noise comes from the integration of frequency noise

If we consider spectrums, then:

$S_\varphi = |H(f)|^2 S_x$ for a LTI system. In our case we want integration of S_x, therefore $H(s) = \frac{1}{s} \to |H(f)|^2 = \frac{1}{4\pi^2 f^2}$

$S_\varphi = \frac{S_x}{4\pi^2 f^2}$ ⇝ If S_x is white ⇒ S_φ has $\frac{1}{f^2}$ component called random walk noise

Noise integration can build up a phase shift that can go to ∞, thus generating issues like sync etc..

Quantify PM noise in a spectrum

Consider:

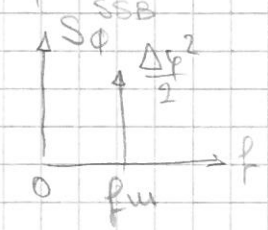
In general: disturbs + noise (unwanted)

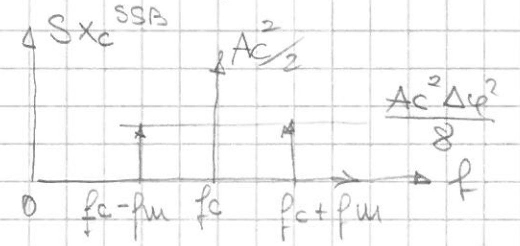
$X_c(t) = A_c \cos[\omega_c t + \varphi_m(t)]$ where $\varphi_m(t)$ is a sinusoidal spurious tone $\varphi(t) = \Delta\varphi \cos \omega_m t$.

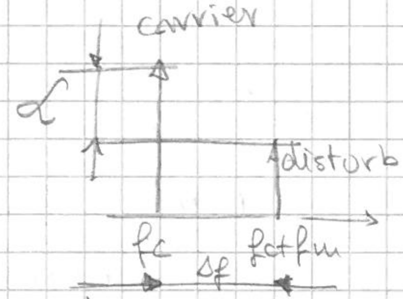
We then have:

$$X_c(t) \underset{\substack{\text{NBFM} \\ \Delta\varphi \ll 1\,\text{rad}}}{\simeq} A_c \cos\omega_c t - A_c \Delta\varphi \cos\omega_m t \cos\omega_c t =$$

$$A_c \cos\omega_c t - \frac{A_c \Delta\varphi}{2} \cos(\omega_m + \omega_c)t + \frac{A_c \Delta\varphi}{2} \cos(\omega_m - \omega_c)t$$







We can see that disturbance $\varphi_m(t)$ generates two spurs.

Let's define a sort of SNR called $\mathcal{L}$, in this case:

$$\mathcal{L}(\Delta f) = \frac{S(f_c + f_m)}{P_c}$$

$S(f_c+f_m)$ → Spectrum of a single band

P_c → power of the carrier

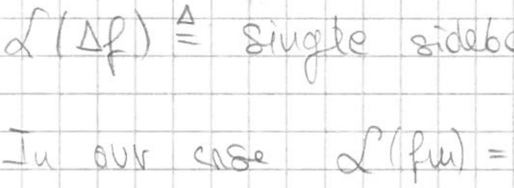
Δf → frequency offset Δf from the carrier frequency f_c

$\Delta f = (f_m + f_c) - f_c = f_m$

$\mathcal{L}(\Delta f) \triangleq$ single sideband to carrier ratio (SSCR)

In our case $\mathcal{L}(f_m) = \dfrac{\frac{\cancel{A_c^2}\Delta\varphi^2}{\cancel{16}\ 4}}{\frac{\cancel{A_c^2}}{\cancel{4}}} = \dfrac{\Delta\varphi^2}{4}$

Consider now white noise. We can apply the same reasoning



by selecting just a narrow band of the white noise at $\pm\Delta f$ offset from f_c

Typically defined in a 1Hz bandwidth

Consider now $1/f$ noise.

Same reasoning. Note that for low f noise diverges so NBFM cannot be applied anymore (we assumed $\Delta\varphi \ll 1$ rad)

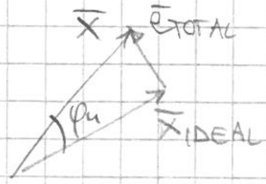
19) EVM degradation, reciprocal mixing

Amplitude error ε:

- can come from phase shifter (cos / sine have different amp)
- " " " mixer amplitude mismatches

Phase error θ:

- can come from tracks delay after mixers
- can come from bad phase shifting



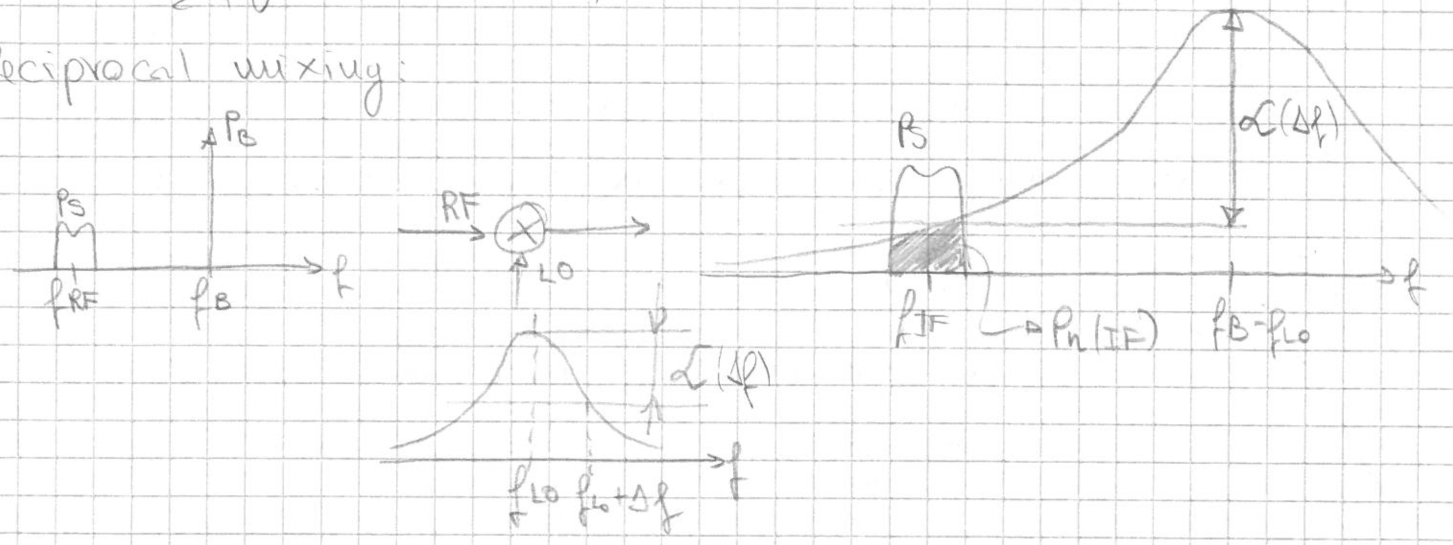
EVM = error vector magnitude $= \frac{1}{SNR}$

$$EVM = \frac{\frac{1}{N}\sum_{j=1}^{N} |\bar{e}_j|^2}{P_{average}} = \frac{|\bar{e}_{TOT}|^2}{|\bar{x}_{ID}|^2} = \frac{(|\bar{x}_{ID}|\, tg\, \varphi_n)^2}{|\bar{x}_{ID}|^2} \underset{\text{small } \varphi_n}{\simeq} \sigma_\varphi^2$$

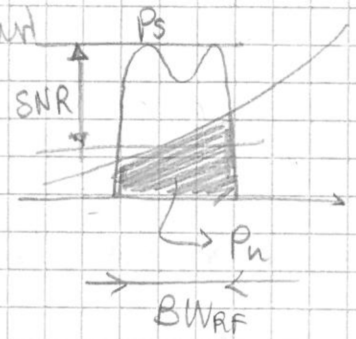
Regardless of power generated we still have the same EVM

$$EVM = \frac{4}{\varepsilon^2 + \theta^2}$$ → see question number 13 for demonstration

Reciprocal mixing:



A blocker couples in the mixer → power from blocker $*$ LO spectrum can be downconverted directly down to signal band



We defined: $\mathcal{L}(\Delta f) = \frac{P_{noise}(f_{IF}) \text{ in 1 Hz BW}}{P_{blocker}}$

So, we can say that:

$$SNR = \frac{P_S}{P_{noise}(f_{IF})\big|_{full\ BW}} = \frac{P_S}{\mathcal{L}(\Delta f)\cdot P_B \cdot BW_{RF}}$$

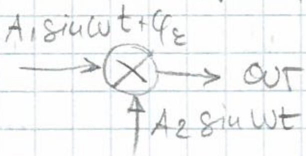
→ Not just 1 Hz

$$SNR\big|_{dB} = P_S\big|_{dB} - P_B\big|_{dB} - \mathcal{L}(\Delta f)\big|_{dBc} - 10\log_{10}(BW_{RF})$$

→ Be careful

20) PD based on multiplier. Phase model of PLL. Non-lin-diff eq.

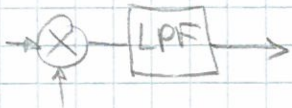
Multiplier: multiplies input and modulation sig



Consider

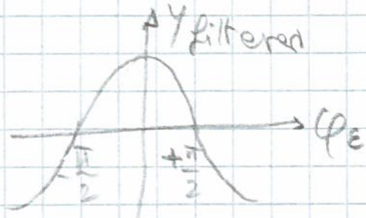
$x_1 = A_1 \sin(\omega t + \varphi_\varepsilon) \quad x_2 = A_2 \sin(\omega t)$ then

$$y = x_1 \cdot x_2 = \frac{A_1 A_2}{2}\Big(\underbrace{-\cos(2\omega t + \varphi_\varepsilon)}_{2\omega} + \underbrace{\cos \varphi_\varepsilon}_{\text{DC sig}}\Big)$$

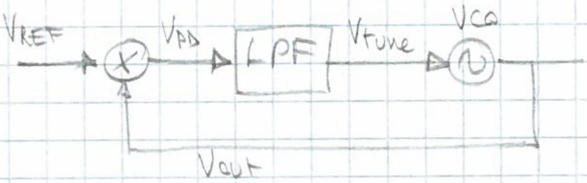


If we filter y with $f_{POLE}\big|_{LPF} \ll 2\omega$ then

$$y\big|_{filtered} \approx \frac{A_1 A_2}{2} \cos \varphi_\varepsilon \rightsquigarrow \text{no } \omega \text{ dependance}$$



Change of notation:



$V_{REF} = A_{REF} \sin(\omega_{ref} t + \phi_{ref})$

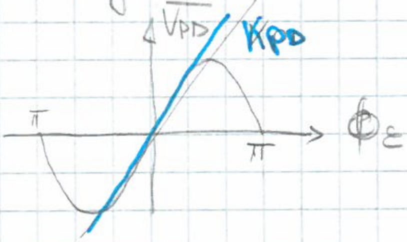
$V_{OUT} = A_{OUT} \cos(\omega_{out} t + \phi_{OUT})$

$V_{TUNE} = K_{PD} \sin(\phi_{ref} - \phi_{out}) = K_{PD} \sin \phi_\varepsilon$

$\phi_\varepsilon = \phi_{ref} - \phi_{out}$ = phase error

V_{out} is a cos instead of a sine because to have a "static" situation, $V_{PD} = 0$ therefore V_{REF} and V_{OUT} must be shifted by $\pm\frac{\pi}{2}$

Using this notation we get a new plot



for small ϕ_ε $\quad \overline{V_{PD}} = V_{tune} = K_{PD} \sin \phi_\varepsilon \sim K_{PD}\, \phi_\varepsilon$

K_{PD} = gain of the phase detector

Since $\omega_{OUT} = \omega_{FR} + K_{VCO} \cdot V_{tune}$ and since $\omega = \frac{d\phi}{dt}$ we can say:

$\frac{d\phi_{OUT}}{dt} = \omega_{FR} + K_{VCO} V_{tune}(t)$ and therefore get ϕ_ε derivative

$$\frac{d\phi_\varepsilon}{dt} = \frac{d\phi_{REF}}{dt} - \frac{d\phi_{OUT}}{dt} = (\omega_{REF} - \omega_{FR}) - K_{VCO} V_{tune}(t) = \Delta\omega - \underbrace{K_{VCO} K_{PD}}_{K} \sin \phi_\varepsilon =$$

operator is linear

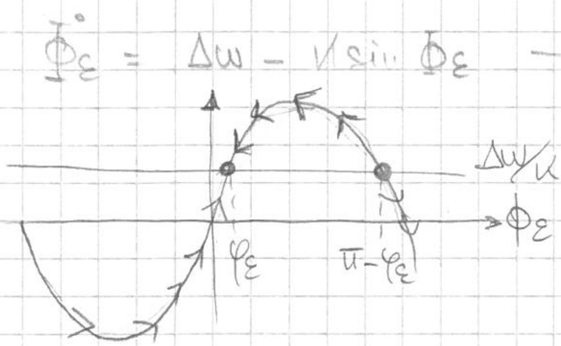
$\frac{d\phi_\varepsilon}{dt} = \Delta\omega - K \sin \phi_\varepsilon \rightarrow$ first order differential equation

(considering LPF contribution would complicate things)

Brief analysis of the PLL non lin differential eq.

- analyze stable equilibrium points

$$\dot{\Phi}_\varepsilon = \Delta\omega - K\sin\Phi_\varepsilon \rightarrow \dot{\Phi}_\varepsilon = 0 \rightarrow \sin\Phi_\varepsilon = \frac{\Delta\omega}{K}$$



Two equilibrium points at φ_ε and $\pi-\varphi_\varepsilon$

- $\sin\Phi_\varepsilon > \frac{\Delta\omega}{K} \rightarrow \Phi_\varepsilon$ is decreasing
- $\sin\Phi_\varepsilon < \frac{\Delta\omega}{K} \rightarrow \Phi_\varepsilon$ is increasing

$\rightarrow$ System will tend to φ_ε equilibrium point

If $\left|\frac{\Delta\omega}{K}\right| > 1 \rightsquigarrow$ no crossing with the plot $\rightarrow$ sys is out of lock

- lock state $\dot{\Phi}_\varepsilon = 0 \quad \omega_\varepsilon = \omega_{REF} - \omega_{OUT} = 0 \rightarrow \underline{\underline{\omega_{REF} = \omega_{OUT}}}$

For small phase perturbations; non linear diff eq becomes

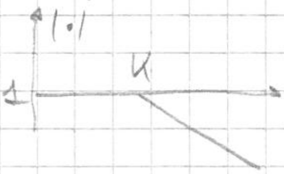
$$\dot{\Phi}_\varepsilon = \Delta\omega - K\sin\Phi_\varepsilon \underset{\text{linearize}}{\approx} -K\Phi_\varepsilon \rightarrow \text{laplace} \quad s\Phi_\varepsilon = -\Phi_\varepsilon \cdot K$$

Same thing happens for Φ_{out}.

$$s\Phi_{OUT} = \omega_{FR} + K\sin\Phi_\varepsilon \approx K\Phi_\varepsilon = \underline{\underline{K\left[\Phi_{REF} - \Phi_{OUT}\right]}} = s\Phi_{OUT} \quad (*1)$$

We want to find out response vs input:

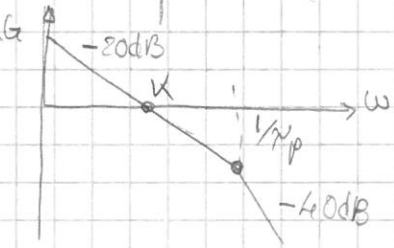
$$T(s) = \frac{\Phi_{OUT}}{\Phi_{REF}} \underset{(*1)}{=} \frac{K}{s+K}$$



Φ_{out} "follows" Φ_{ref} until pole frequency at K
If Φ_{ref} is noisy, we will see that Φ_{OUT} "filters" ref phase noise

21) 2nd order PLLs: stability and T(s). Static phase error after n-th order input, freq response

- Without filter $\longrightarrow LG(s) = K_{PD} \cdot \overset{1}{F(s)} \cdot \frac{K_{VCO}}{s} = \frac{K}{s}$
- With filter (consider single-pole LPF) $LG(s) = \frac{K}{s} \cdot \frac{1}{1+s\tau}$

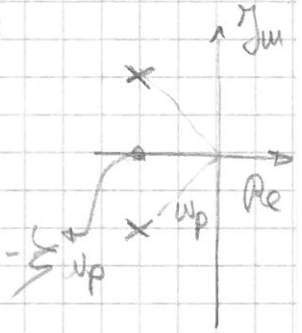


$\rightsquigarrow$ It has to cut at -20dB/dec, otherwise system will not be stable

Assume good phase margin

Let's design a system with 45° conjugate poles to have a good trade-off between overshoot/settling time:

$$T(s) = \frac{\Phi_{out}}{\Phi_{ref}} = \frac{LG(s)}{1+LG(s)} = \frac{1}{s^2\frac{\tau}{K} + s\frac{1}{K} + 1} = \frac{1}{\frac{s^2}{\omega_p^2} + \frac{2\xi}{\omega_p}s + 1}$$



$$\omega_p = \sqrt{\frac{K}{\tau}} \quad \xi = \frac{1}{2\sqrt{K\tau}} = \underbrace{\frac{\sqrt{2}}{2}}_{\text{trade-off}} \longrightarrow K = \frac{1}{2\tau}$$

We see that K has to be one octave before pole $\frac{1}{\tau}$

$$\omega_p = \sqrt{\frac{K}{\tau}} \underset{K = 1/2\tau}{=} \sqrt{2}\,K \longrightarrow \text{BW is now } \sqrt{2}\,K\text{, slightly higher}$$

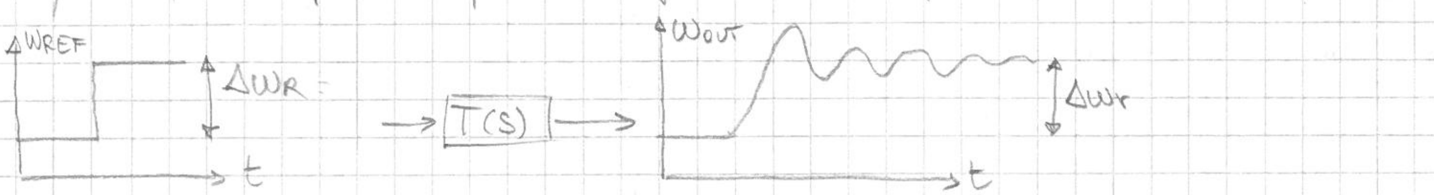
Note:
BW of closed loop is often approximated to just 0dB cut of LG(s).

Phase margin $\varphi_m = 90° - \arctg\left(\frac{\omega_0}{\omega_p}\right) \underset{\omega_0 = \frac{1}{2}\omega_p}{\cong} 90° - 27° \cong 63$

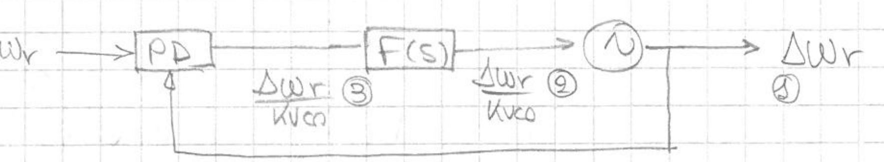
φ_m is limited by the overshoot/settling time condition

Static phase error for a 2nd order PLL

$T(s) = \frac{\phi_{out}}{\phi_{ref}} \Rightarrow \frac{s}{s} \cdot \frac{\phi_{out}}{\phi_{ref}} = \frac{\omega_{out}}{\omega_{ref}} \rightsquigarrow T(s)$ is valid as reference to output transfer function. We can then analyze the frequency response for a reference step.



Like it happens for the phase, we experience overshoot and settling time for output frequency ω_{out}.



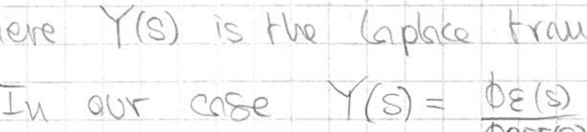
Qualitative approach: consider steady-state condition where

$\underset{①}{\omega_{out}} = \Delta\omega_r \rightarrow \underset{②}{V_{TUNE}} = \frac{\Delta\omega_r}{K_{VCO}}$ at steady-state $F(s) = 1$ so

$\underset{③}{V_{PD}} = \frac{\Delta\omega_r}{K_{VCO}}$ as well $\rightarrow \phi_\varepsilon = \frac{\frac{\Delta\omega_r}{K_{VCO}}}{K_{PD}} = \frac{\Delta\omega_r}{K} \rightarrow$ There is a small, finite error

This error is not null because of the final value theorem:

$\underbrace{\lim_{t\to\infty} y(t)}_{\text{steady-state error}} = \lim_{s\to 0} s \cdot Y(s)$ where $Y(s)$ is the laplace transform of $y(t)$. In our case $Y(s) = \underline{\frac{\phi_\varepsilon(s)}{\phi_{REF}(s)} \cdot \phi_{REF}(s)}$



Ingredients we need:

- Transfer function $\rightarrow \frac{\phi_\varepsilon(s)}{\phi_{REF}(s)} = \frac{1}{1 + LG(s)} = 1 - T(s) = \underline{\frac{s(1+s\tau)}{K + s(1+s\tau)}}$
- Input step $\rightarrow \frac{d\phi_{REF}}{dt} = \omega_{REF} \xrightarrow{\mathcal{L}} s\phi_{REF} = \omega_{REF} \rightsquigarrow$ if ω is an input step then (using laplace) $\omega_{REF}(s) = \frac{\Delta\omega_r}{s} \rightarrow \phi_{REF}(s) = \frac{\omega(s)}{s} = \underline{\frac{\Delta\omega_r}{s^2}}$

We can assemble the two, so $Y(s) = \phi_\varepsilon(s) = \frac{\phi_\varepsilon(s)}{\phi_{REF}(s)} \cdot \phi_{REF}(s)$

$\phi_\varepsilon(s) = \frac{\Delta\omega_r}{s^2} \cdot \frac{s(1+s\tau)}{K + s(1+s\tau)}$

Let's now apply the theorem:

$$\underbrace{\lim_{t\to\infty} \varphi_\varepsilon(t)}_{\text{static phase error}} = \lim_{s\to 0} s\cdot\Phi_\varepsilon(s) = \lim_{s\to 0} \cancel{s}\,\frac{\Delta\omega_r}{\cancel{s^2}}\cdot\frac{\cancel{s}(1+\overbrace{s\tau}^{\to 0})}{K+\underbrace{s}_{\to 0}(1+\underbrace{s\tau}_{\to 0})} = \frac{\Delta\omega_r}{K}$$

We found the same result we got intuitively

What would happen if we applied a phase step instead?

$$\Phi_{REF}(s) = \frac{\Delta\phi}{s} \rightarrow \lim_{s\to 0} \cancel{s}\,\frac{\Delta\phi}{\cancel{s}}\cdot\frac{s\,(\;)}{K+\cancel{s(\;)}} = 0$$

- frequency step $\rightarrow$ error is $\Delta\omega_r/K$
- phase step $\rightarrow$ null error

If we wanted to null the frequency step static phase error?

In general: $$\lim_{s\to 0} s\cdot\frac{\Delta}{s^m}\,\frac{s^n H(s)}{s^n H(s)+K} = \lim_{s\to 0}\frac{\Delta}{K}s^{n-m+1}$$

n: number of integrators

m: order of input perturbation

$$\lim_{s\to 0}\frac{\Delta}{K}s^{n-m+1} \begin{cases} \rightarrow \Delta/K & \text{for } n=m-1 \\ \rightarrow 0 & \text{for } n\geq m \end{cases}$$

Static phase error is null if # integrators is at least equal to the input perturbation order

22) lock acquisition and frequency tracking

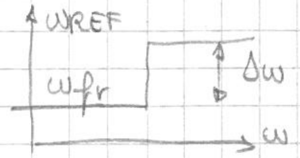
$X_{REF}(t) = \cos(\omega_{fr})t$ before a step

$X_{OUT}(t) = \sin(\omega_{fr})t$ out is locked to input (in quadrature)

Let's now apply a step $\Delta\omega$

$X_{REF}(t) = \cos(\omega_{fr} + \Delta\omega)t$ where $(\omega_{fr}+\Delta\omega)t = \Phi_{ref}$

$X_{out}(t) = \sin\underbrace{\left(\omega_{fr} + \int_{-\infty}^{t} K_{vco} V_{tune}(\tau)d\tau\right)}_{\Phi_{out}}$



→ We expect output to rise, overshoot and settle, but suppose that step is fast and filter is very slow in its action, we can express $X_{out}(t)$ like:

$X_{OUT}(t) = \sin(\omega_{fr} + \underbrace{\int \ldots}_{\text{neglect}}) \simeq \sin(\omega_{fr} + \underbrace{0.t}_{\text{omitted terms}})$

So output is still at ω_{fr}, it does not change. Therefore, initially:

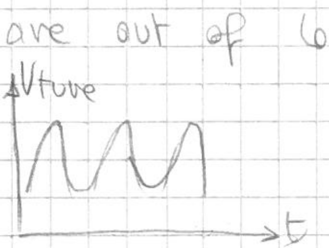
$V_{PD} = X_{out} \cdot X_{REF} \simeq K_{PD} \sin(\Delta\omega t - 0.t)$

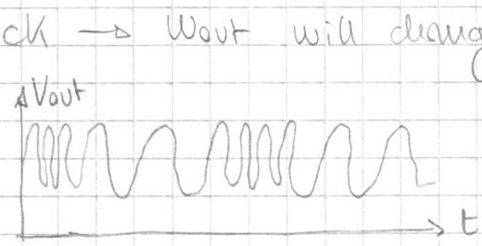
Note: there's a sinusoidal behaviour between input phase step and V_{PD}.

Filtering introduces amplitude/delay change in the signal, so:

$V_{TUNE} \simeq K_{PD} \underbrace{|F(\Delta\omega)|}_{\text{amp. change}} \sin[\Delta\omega t + \underbrace{\measuredangle F(\Delta\omega)}_{\text{phase change}} + 0.t]$

The sinusoidal behaviour is the same that we found when we are out of lock → W_{out} will change in a sinusoidal matter:





But let's look for a locking range:

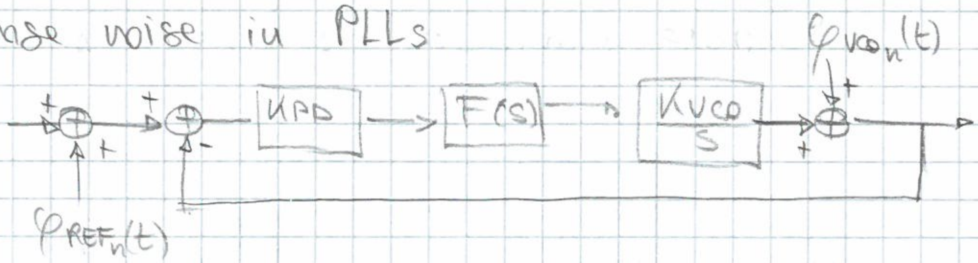
$V_{tune} = K_{PD} |F(\Delta\omega)| \sin[\ldots]$ Since $|\sin| \leq 1$ then

$|V_{tune}| \leq K_{PD} |F(\Delta\omega)| \rightarrow \left|\frac{\Delta\omega}{K_{vco}}\right| \leq K_{PD} |F(\Delta\omega)|$

There is a limit $\Delta\omega$ that can generate a lock:

$\underbrace{\Delta\omega_c}_{\text{capture range}} = K_{vco} K_{PD} |F(\Delta\omega)| = K |F(\Delta\omega_c)|$ → LPF narrows the original $\Delta\omega_c = K$

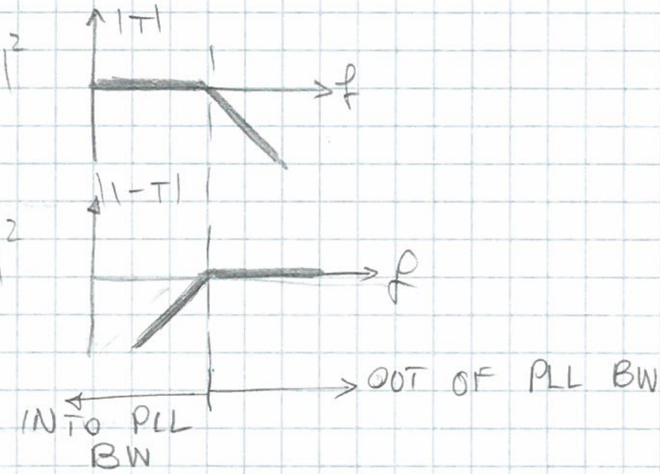
Phase noise in PLLs



If we consider VCO and REF phase noise sources:

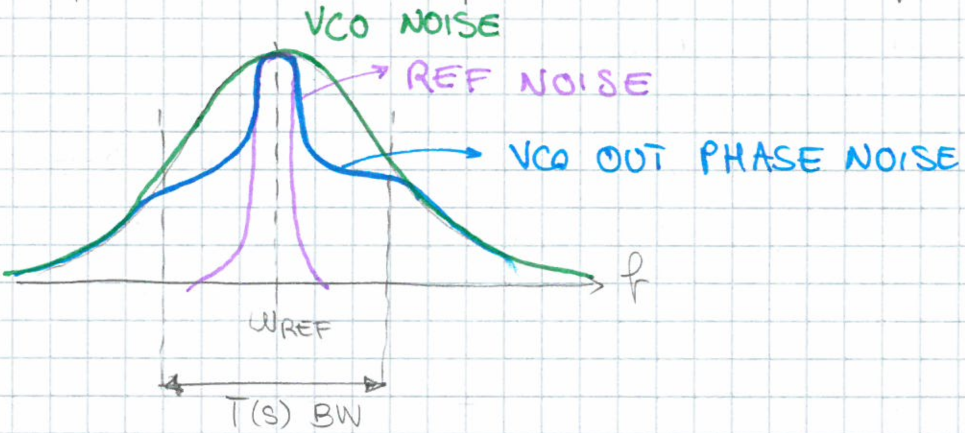
$$S_{\varphi out}(f)\Big|_{\substack{REF\\NOISE}} = S_{\varphi REF_n} \, |T(f)|^2$$

$$S_{\varphi out}(f)\Big|_{\substack{VCO\\NOISE}} = S_{\varphi VCO_N} \, |1-T(f)|^2$$



Meaning:

- within PLL BW: VCO follows REF phase noise
- out of PLL BW: VCO follows its own phase noise

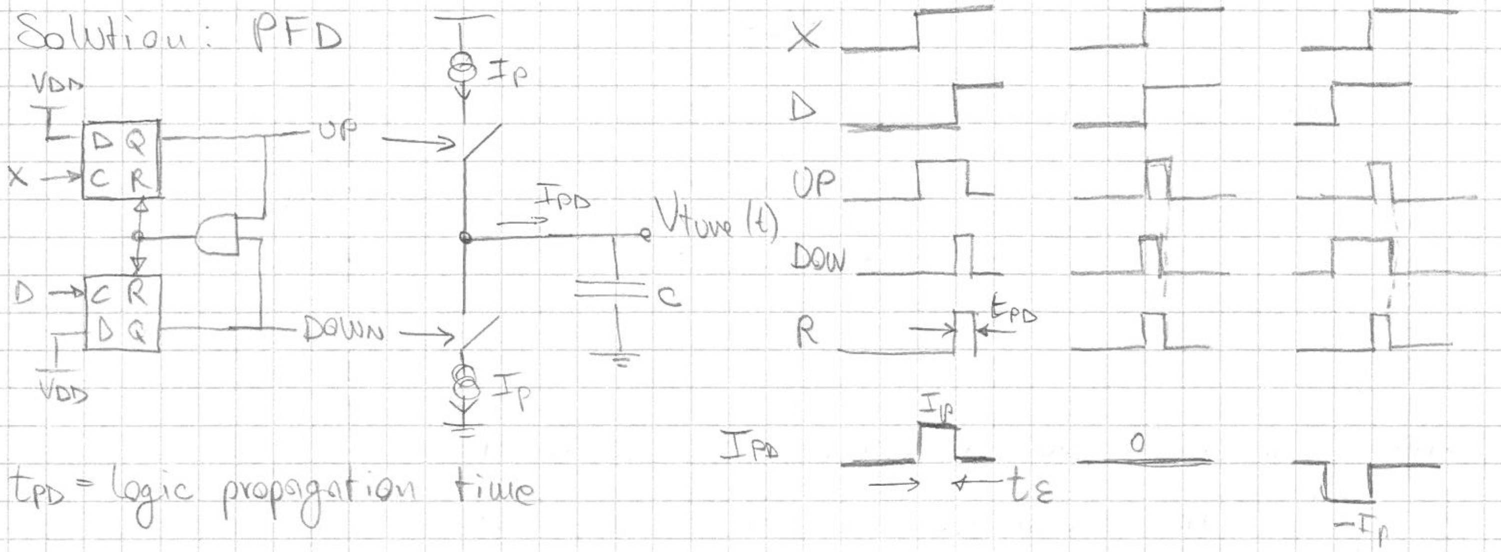


23) Charge pump PLLs: PFD, phase-domain model, stabilizing zero, loop dynamics

Problems:

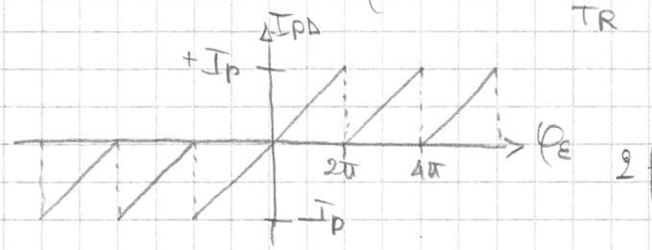
- conventional PDs do not track frequency (phase only)
- " " generate spurs at $2\omega_{REF}$ (analog mixer / XOR) or at ω_{REF} (SR-FF). Type II PLL keeps only $\overline{V_{PD}} = 0$, but does not solve the $2\omega_{REF}$ / ω_{REF} issue

Solution: PFD

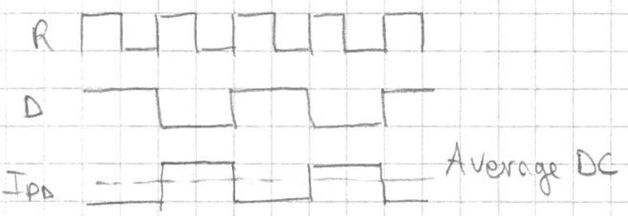


t_{PD} = logic propagation time

We know that $\varphi_\varepsilon = 2\pi \frac{t_\varepsilon}{T_R}$ therefore:



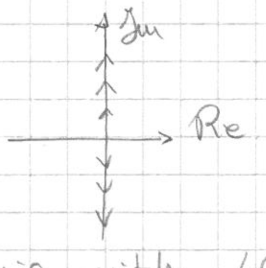
How can it detect frequency? Suppose $2 f_{REF} = f_{DIV}$

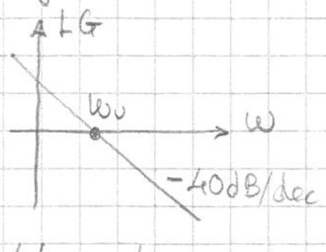


It can detect frequency because when $f_{REF} \gg f_{DIV}$ output current is always > 0 (DC) $\rightsquigarrow$ VCO will increase out frequency therefore letting f_{DIV} approach f_{REF}. This means that it always (ideally) locks.

$K_{PD} = \frac{I_P}{2\pi}$ If we just have the charge pump + 1 capacitor:

$$LG(s) = -K_{PD} \cdot \frac{K_{VCO}}{s} \cdot \frac{1}{sC}$$

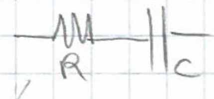




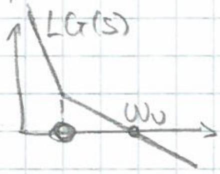
This $LG(s)$ cuts 0dB axis with 40dB/dec slope → unstable

We can think of recovering stability by modifying the filter:

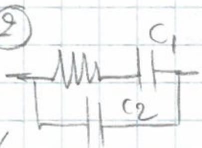
(1)



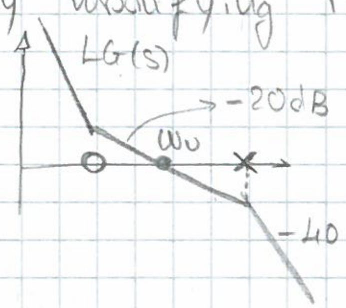
↳ introduces a zero



(2)



↳ zero + pole



$$LG(s)\Big|_{(1)} = K_{PD}\frac{K_{VCO}}{s}\cdot\frac{1+sRC}{sC} \overset{\text{High freq}}{=} K_{PD}\frac{K_{VCO}}{s}\cdot R$$

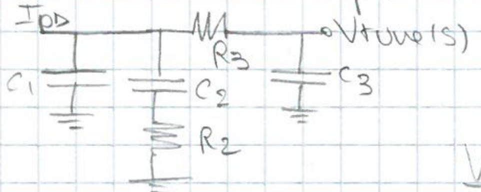
Issue: R could be high value → $I_p R = V$ could go above PLL's dynamic range

Issue: shallow filtering (−20dB/dec) of high frequency spurious tones → VCO will show unwanted modulation tones at its out

$$LG(s)\Big|_{(2)} = K_{PD}\frac{K_{VCO}}{s}\cdot\frac{1}{s(C_1+C_2)}\cdot\frac{1+sC_1R}{1+sR\,C_1/\!/C_2}$$

Second solution lowers R value and with its −40dB/dec filtering on HF, attenuates HF spurs more than solution (1)

Note: another pole can be added with the following:



$$Z_1(s) = C_1/\!/(C_2+R_2) = \frac{1}{s(C_1+C_2)}\;\frac{1+s\tau_z}{1+s\tau_p}$$

$$\frac{V_{tune}(s)}{I_{PD}(s)} = \frac{Z_1(s)}{Z_1(s)+R_3+\frac{1}{sC_3}}\cdot\frac{1}{sC_3} = \text{heavy calculations}$$

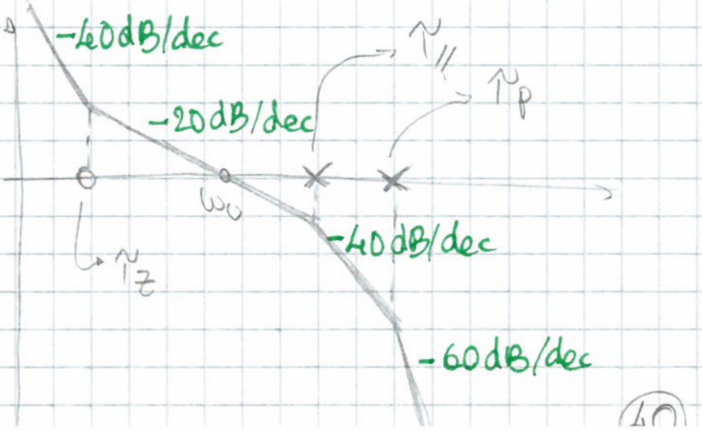
and rearrangements $= \dfrac{1}{s(C_1+C_2)}\;\dfrac{1+s\tau_z}{(1+s\tau_{/\!/})(1+s\tau_p)+\frac{C_3}{C_1+C_2}(1+s\tau_z)}$

Where $\tau_p = R_3C_3$ $\tau_z = R_2C_2$ $\tau_{/\!/} = R_2\,\dfrac{C_1C_2}{C_1+C_2}$

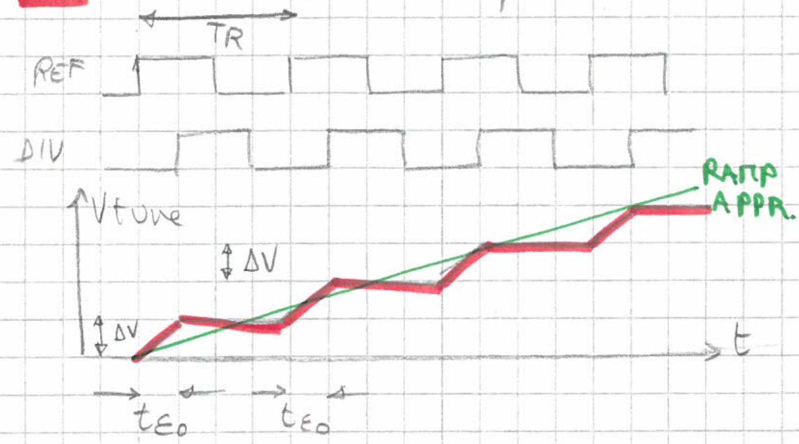
If $C_3 \ll C_1+C_2$ and $C_3 \ll C_1 + \dfrac{R_3}{R_1}\left(1+\dfrac{C_1}{C_2}\right)C_3$ the third pole does not interact, leading to

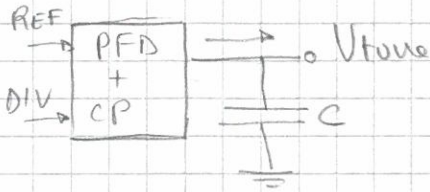
$$\frac{V_{tune}(s)}{I_{PD}(s)} = \frac{1}{s(C_1+C_2)}\;\frac{1+s\tau_z}{(1+s\tau_{/\!/})(1+s\tau_p)}$$

−60dB/dec slope further attenuates high frequency spurs (see answer 25)



24) Limits of validity of continuous-time PLL model



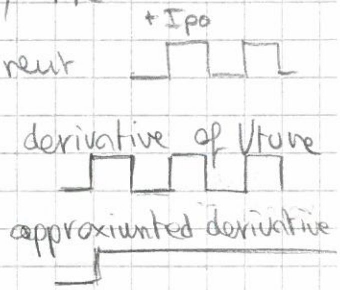


CP is simple because:

- Performs sum with currents → No need for opamps

Vtune is a discrete-time process that updates every TR

$C = \frac{dV}{Q}$ $\quad i = \frac{dQ}{dt}$ $\quad \rightarrow \Delta V = \frac{t_\varepsilon \cdot I_{PO}}{C}$ → I_{PO} peak current



We can approx multiple steps (discrete time integrator) to an average single step (CT integrator → ramp) if the time scale of interest is much longer than input period. Therefore the "train" of pulses in Vtune's discrete derivative can be averaged to just one

Gardner estimated that $BW|_{PLL,\ \text{Gardner's limit}} = f_{REF}/10$! $= f_{REF}/20$ (safer)

Exploiting this: $\quad t_{\varepsilon 0} = \frac{\varphi_{\varepsilon 0} \cdot TR}{2\pi}$

$$V_{tune} \simeq \frac{\Delta V}{TR} \cdot t = \frac{t_{\varepsilon 0} \cdot I_{PO}}{C \cdot TR} \cdot t = \frac{I_{PO}}{C} \cdot \frac{\varphi_{\varepsilon 0}}{2\pi} t$$

by using Laplace transform

$$V_{tune}(s) = \frac{I_{PO}}{C} \frac{\varphi_{\varepsilon 0}}{2\pi} \cdot \frac{1}{s^2} = \frac{I_{PO}}{2\pi} \cdot \frac{1}{sC} \cdot \frac{\varphi_{\varepsilon 0}}{s}$$

$\frac{I_{PO}}{2\pi} = K_{PD}$

$\frac{1}{sC} = Z(s)$ of the CP

$\frac{\varphi_{\varepsilon 0}}{s}$ = phase error input step

$$\rightarrow \frac{V_{tune}(s)}{\varphi_\varepsilon(s)} = \frac{I_{PO}}{2\pi} \cdot \frac{1}{sC} = K_{PD} Z(s)$$

It can be shown that, using Z-transform for PLLs with BW above Gardner's limit can still give a good estimate (to be verified in simulations)

25) Sources of ripples in PLLs: ref spur problem and methods to reduce the level

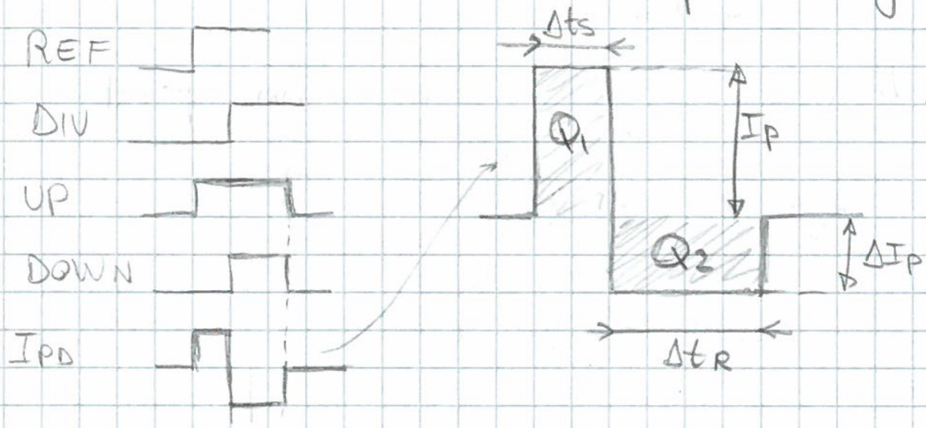
Sources of ripples:

1. varactor current leakage in VCO discharge the capacitor
2. disturbances at ω_{REF} coupled throug GND or PSU lines
3. CP imbalances

Solutions:

1. Use MOS varactors, they greatly reduce leakage current
2. Shield Vtune line from disturbances
3. Better fabbrication process and calibration of PFD + CP

CP current mismatch and spurs amplitude



A Ron (switch) or current generator mismatch can cause the creation of a current difference ΔI_p

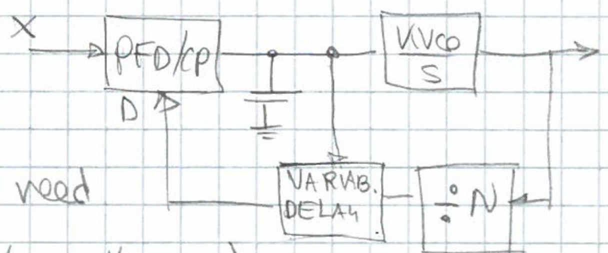
At steady state no net current shuld enter the filter → charges Q_1 and Q_2 must be the same.

Therefore the PLL automatically adjust REF / DIV delay Δt_S so that $Q_1 = I_p \cdot \Delta t_S = Q_2 = \Delta I_p \cdot \Delta t_R$

That way, on average $\langle I_{PD} \rangle = 0$ → steady state

Spur reduction methods:

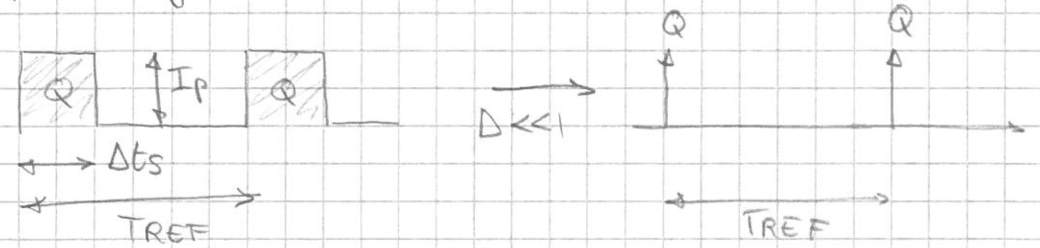
- add poles (see answer 23)
- "sampling loop filter" (→ see Razavi section 10.4)
- Add a variable delay in the path (see Razavi section 10.4).



Loop gain will see a zero without the need for an additional resistor. $LG(s) = \frac{I_p}{2\pi} \frac{1}{sC} \left(\frac{K_{vco}}{s} + K_D \cdot \omega_{REF} \right)$

Where $\tau_{delay} = \tau_0 + K_D V_{tune} \rightarrow K_D [ns/V]$

Spur magnitude estimation:

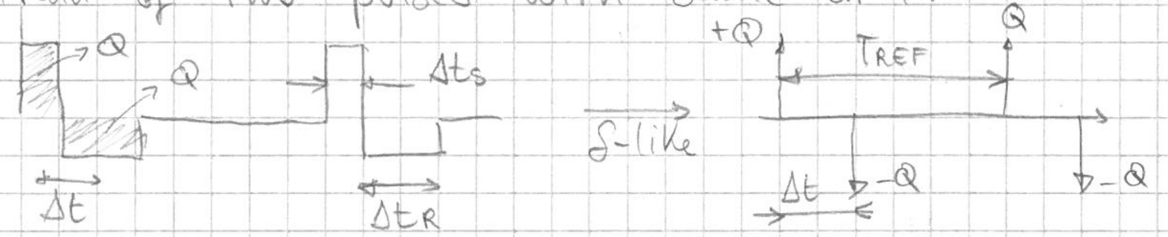


Where $D = \frac{\Delta t_S}{T_{REF}}$ → Duty cycle

Fundamental harmonic of the train of pulses will be:

$$\frac{2}{\pi} I_p \sin(\pi D) \underset{D \ll 1}{\approx} 2 I_p \frac{\Delta t_S}{T_{REF}} = \frac{2Q}{T_{REF}}$$

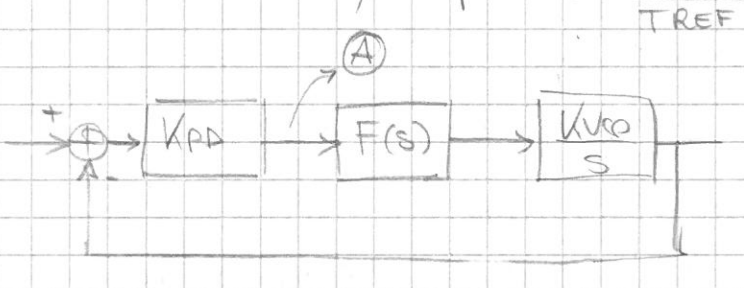
Train of two pulses with same area:



Assuming ideal deltas: $\Delta t = \frac{\Delta t_S + \Delta t_R}{2}$ then the first harmonic will be $I_p^{(\omega_{REF})} = \frac{2Q}{T_{REF}} \left(1 - e^{-j 2\pi \Delta t / T_{REF}}\right)$

For low Δt we say $I_p^{(\omega_{REF})} \approx \frac{2Q}{T_{REF}} \left(j 2\pi \frac{\Delta t}{T_{REF}}\right)$ $\quad 1 - e^{-x} \approx x$, $x \to 0$

→ $j\, \omega_{REF} \cdot \Delta t$



$$\mathcal{L}(\omega_x) = \frac{S_\varphi}{2} = \frac{1}{2} \frac{K_{VCO}^2}{\omega_x} |F(j\omega_x)|^2 \cdot \left|\frac{I(\omega_x)}{2}\right|^2 \cdot \left|\frac{1}{1 - G_{loop}(\omega_x)}\right|^2$$

Loop is usually negligible because spurs at $\sim \omega_{REF}$ are well above the $BW|_{PLL}$ → Loop is practically open

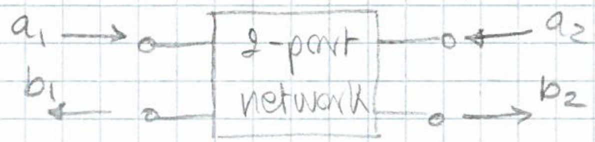
For a leakage current we have

Δt_S will be adjusted to match the two areas (charges): procedure that follows will be the same as the single pulse train

27) LNA: Scattering parameters, insertion loss, reverse isolation, stability, linearity. Methods to increase reverse isolation

Microwave design focusses on power transfer instead of voltage or current transfer

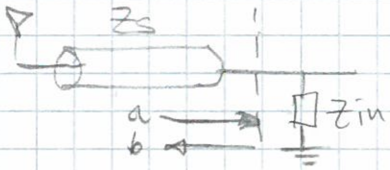
HF measurement of I/V is very difficult while power is more straight forward



Consider a 2-port network

a = incident power wave at port 1/2

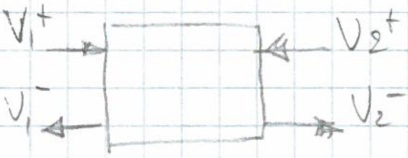
b = reflected " " " " "



We can say, for a single port $b = \Gamma a$ where $\Gamma = \frac{Z_{in} - Z_S}{Z_{in} + Z_S}$

If we consider power transfer between ports:

$$\begin{cases} b_1 = S_{11} a_1 + S_{12} a_2 \\ b_2 = S_{21} a_1 + S_{22} a_2 \end{cases} \qquad \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \rightarrow \text{Scattering parameters matrix}$$



We can express S-params in voltages

$$\begin{cases} V_1^- = S_{11} V_1^+ + S_{12} V_2^+ \\ V_2^- = S_{21} V_1^+ + S_{22} V_2^+ \end{cases} \rightarrow S_{11} = \frac{b_1}{a_1} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+ = 0}$$

When defining S_{11}, $V_2^+ = 0$ means that load is assumed to be matched at port 2.

What do S-parameters represent?

- $\underline{S_{11}}$: accuracy of input matching

  Input return loss $RL_{IN} = 10 \log_{10} \frac{1}{|S_{11}^2|} = -20 \log_{10} \{S_{11}\}$

- $\underline{S_{12}}$: how much output couples through input network

  Reverse isolation $= -20 \log_{10} \{S_{12}\}$

- $\underline{S_{22}}$: accuracy of output matching

  Output return loss $RL_{OUT} = -20 \log_{10} \{S_{22}\}$

- $\underline{S_{21}}$: gain of input-to-output transfer

  Forward gain $= -20 \log_{10} \{S_{21}\}$

Why don't we use V/I to measure in/out port impedances?
For an impedance measurement we need to short or open in/out ports. At HF this is detrimental because:

- Short ports → magnetic coupling (no matching)
- open ports → capacitive coupling (no matching)

Stability: based on conditions of the environment (user hand on phone, etc...) antenna impedance slightly changes.
LNA has to take into account this → LNA must be stable for all frequencies.
If LNA is stable only in a small frequency range (the working one), non-stability on other frequencies can cause oscillations → it becomes non-linear and it heavily compresses the gain even in the working narrow band.
For this reason, we care a lot about reducing reverse gain. (S_{12}).
Common gate and shunt feedback are two robust

topologies that are naturally stable → no stability issues
Reverse isolation of an LNA is important to reduce
LO leakage into LNA input (causing LO propagation to
the antenna)

28) MOS noise model. Common-gate and shunt-feedback
MOS noise:
Transconductance definition $g_{do} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{DS}=0}$

• Triode region $I_D = \frac{1}{2}\mu C'_{ox}\frac{W}{L}\left[2V_{ov}V_{DS} - V_{DS}^2\right]$

$$g_{do} = \mu C'_{ox}\frac{W}{L}V_{ov} = \frac{1}{R_{on}}$$

• Saturation region $I_D = \frac{1}{2}\mu C'_{ox}\frac{W}{L}V_{ov}^2$

$$g_m = g_{do}$$

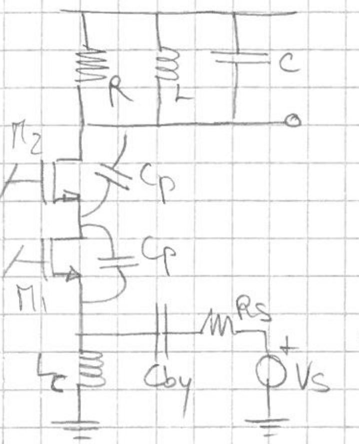
Consider carrier saturation (HV field, short channel MOS)

$g_m = \alpha \cdot g_{do} < g_{do}$ → $\alpha = 1$ no saturation
→ $\alpha < 1$ with carrier saturation

• No saturation → $PSD|_{MOS} = 4KT\gamma g_{do}$

• In saturation → $PSD|_{MOS} = 4KT\frac{\gamma}{\alpha}g_m$

Common-gate Topology



L_c = choke inductor

RLC = tuned load to maximize gain

M_2 = cascode to improve reverse isolation because of parasitic capacitances C_p

- Matching input: $\frac{1}{g_{m_1}} = R_S$

- Voltage output $= \frac{V_{out}}{V_S} = \frac{R_L}{2R_S}$

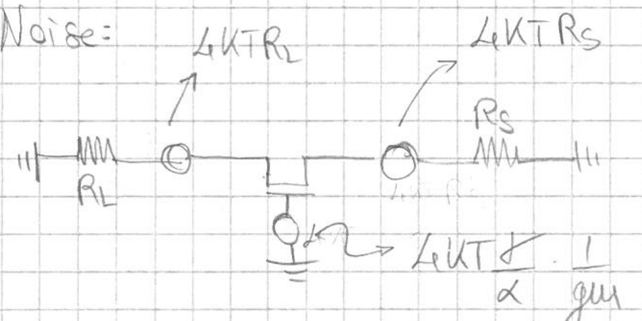
R_L comes from the LC resonator and it's limited by the Q of the reso $R_L = Q\omega_0 L$ $\quad R_L \sim 100\Omega \div 1K\Omega$

Since $R_S = 50\Omega$ then $A_0 \sim 1 \div 10 \Rightarrow A_0 \sim 0 \div 20dB$ max

Maximum gain of the transistor can't be achieved at RF

eg: $f_0 = 1GHz$ $\quad L = 1nH$ $\quad Q = 10 \rightarrow R_L = 62{,}8\Omega$

- Noise:



$R_S = 1/g_m$

$$NF = 1 + \frac{\cancel{4KT}\frac{\gamma}{\alpha}\cdot\cancel{\frac{1}{g_m}}}{\cancel{4KT}\,\cancel{R_S}} + \frac{\cancel{4KT}R_L}{\cancel{4KT}R_S\left(\frac{R_L}{2R_S}\right)^2}$$

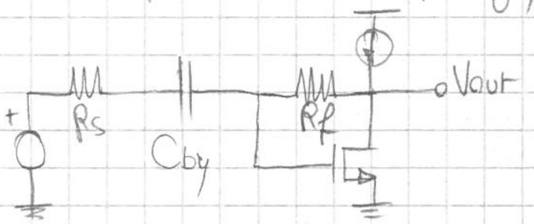
$$NF = 1 + \frac{\gamma}{\alpha} + 4\frac{R_S}{R_L}$$

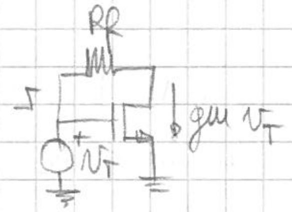
Since $A_0 = \frac{R_L}{2R_S} \rightarrow 4\frac{R_S}{R_L} = \frac{2}{A_0} \rightarrow NF = 1 + \frac{\gamma}{\alpha} + \frac{2}{A_0}$

$$\text{Power gain} \triangleq \frac{P_{out,av}}{P_{in,av}} = \frac{\frac{1}{2}\frac{V_{out}^2}{R_L}}{\frac{1}{8}\frac{V_{in}^2}{R_S}} = \overbrace{A_0}^{\frac{R_L}{2R_S}}{}^2 \cdot 4\frac{R_S}{R_L} = \frac{R_L}{R_S}$$

because of input matching

Shunt-feedback topology

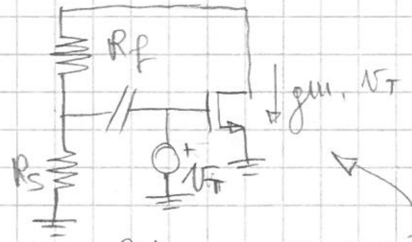




- Input impedance $Z_{in} = \frac{1}{g_{m1}}$

- $A_0 = \frac{T_{ID}}{1-G_{loop}} + \frac{T_{DIRECT}}{1-G_{loop}}$ → forward gain is not high, T_{DIRECT} will have an effect

$$= \frac{-\frac{R_f}{R_S}}{1+g_m R_S} + \frac{1}{1+g_m R_S} = \frac{1-g_{m1}R_f}{1+g_{m1}R_s}$$



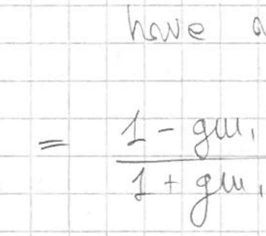
Gloop calculation

Consider matching input $\frac{1}{g_m} = R_S$, therefore:

- $A_0\Big|_{MATCHED} = \left(1 - \frac{R_f}{R_S}\right)\frac{1}{2} = -\frac{R_f}{2R_S}$

$R_f \gg R_S$

- Noise:



$$\frac{V_{out}}{i_n} = Z_{OUT} = \frac{R_f+R_s}{1+G_{loop}} = \frac{R_f+R_s}{2}$$

$$i_n^2\Big|_{MOS} = 4KT g_m \frac{\gamma}{\alpha}$$

$$V_n^2\Big|_{R_f} = 4KTR_f$$

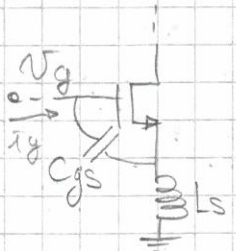
→ $\frac{V_{out}}{V_{nR_f}} = 1$

$$NF = 1 + \frac{\cancel{4KT}\frac{\gamma}{\alpha} \cdot g_{m1}\left(\frac{R_f+R_s}{\cancel{2}}\right)}{\cancel{4KT}R_S\frac{1}{\cancel{4}}\left(1-\frac{R_f}{R_S}\right)^2} + \frac{\cancel{4KT}R_f}{\cancel{4KT}R_S\frac{1}{4}\left(1-\frac{R_f}{R_S}\right)^2} =$$

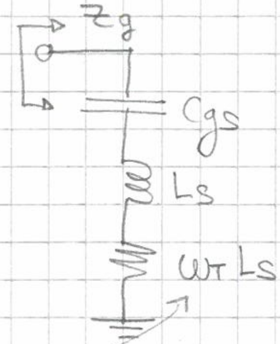
$$NF = 1 + \frac{\gamma}{\alpha} + \frac{4R_S}{R_f}$$ → the same as common-gate

→ Consider $\frac{1}{4}\left(1-\frac{R_f}{R_S}\right)^2 \overset{R_f \gg R_S}{\approx} + \frac{R_f^2}{4R_S^2}$

29) LNA: inductor degenerated CS topology



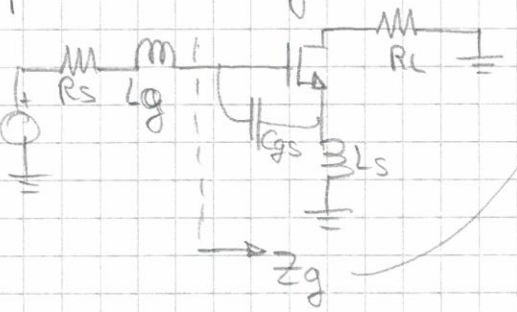
$$\begin{cases} V_g = V_{gs} + sL_s\,(g_m V_{gs} + i_g) \\ i_g = sC_{gs} \cdot V_{gs} \end{cases}$$



$$Z_g = \frac{V_g}{i_g} = \underbrace{\frac{1}{sC_{gs}}}_{\text{cap}} + \underbrace{sL_s}_{\text{inductor}} + \overbrace{g_m \frac{L_s}{C_{gs}}}^{\text{Real impedance}} \qquad \omega_T \simeq \frac{g_m}{C_{gs}}$$

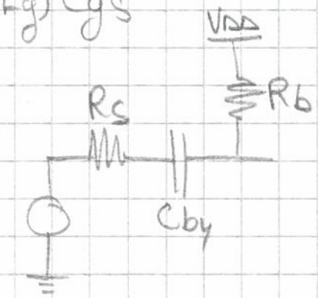
→ Neglects C_{gd}

Typical matching condition



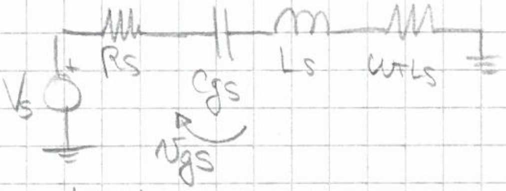
take into account L_g

$$\begin{cases} \omega_0 = \dfrac{1}{\sqrt{L_s C_{gs}}} \simeq \dfrac{1}{\sqrt{(L_s + L_g)\,C_{gs}}} \\ \omega_T L_s = R_s \end{cases}$$



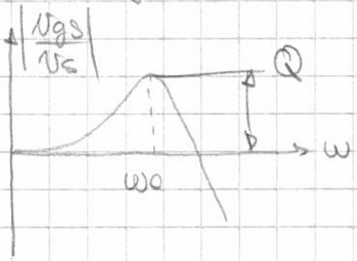
L_g is used to limit biasing network noise, that is:

We will neglect L_g for the following calculations:



$$V_{out} = -g_m V_{gs} R_L = -g_m R \cdot Q\, V_s$$

where $Q = \dfrac{1}{\omega_0 C_{gs}\, 2R}$ → input matching



$$A_0 = \frac{V_{out}}{V_s} = -g_m R_L\, Q = \frac{-g_m R_L}{\omega_0 C_{gs}\, 2R_L} = -\frac{\omega_T}{\omega_0}\frac{R_L}{R_s}$$

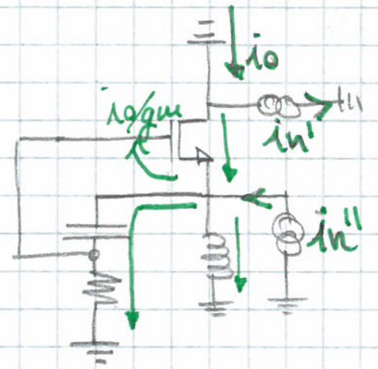
$\frac{g_m}{C_{gs}}$ → ω_T

where $-\dfrac{R_L}{2R_s}$ = common gate A_0, $\dfrac{\omega_T}{\omega_0}$ = boost factor

The boost factor is given by the resonant network that adds another gain without impacting on the MOS!

Since L, C are noiseless → NF will be better

NF of degenerated CS network



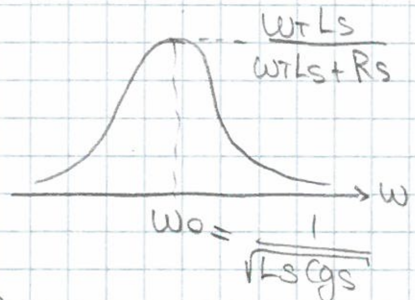
To compute MOSFET noise, we can use Norton
This way we don't need to take into account R_L
NF will be output noise current i_o

$$i_n'' + i_o = \underbrace{-\frac{i_o}{g_m} sC}_{\text{current in } C_{gs}} + \underbrace{\frac{-\frac{i_o}{g_m} sC \cdot R_s - \frac{i_o}{g_m}}{sL_s}}_{\text{current in } L_s}$$

$i_n', i_n'' \rightarrow$ superposition of effects

$$i_o\left(1 + \frac{sC_{gs}}{g_m} + \frac{R_s C_{gs}}{g_m L_s} + \frac{1}{s g_m L_s}\right) = -i_n''$$

$$\frac{i_o}{i_n''} = \frac{-s g_m / C_{gs}}{s^2 + s\left(\frac{g_m}{C_{gs}} + \frac{R_s}{L_s}\right) + \frac{1}{L_s C_{gs}}} \overset{\text{at resonance}}{=} -\frac{\omega_T L_s}{\omega_T L_s + R_s}$$



Since $\omega_T L_s = R_s \rightarrow \frac{i_o}{i_n''} = -\frac{1}{2}$ while $\frac{i_o}{i_n'} = 1$

Therefore $\frac{i_o}{i_n} = \frac{i_o}{i_n'} + \frac{i_o}{i_n''} = 1 - \frac{1}{2} = \frac{1}{2}$ $\rightsquigarrow$ half of i_n recirculates

$$\frac{\overline{i_{n\,out}}^2\Big|_{MOS}}{\overline{i_{n\,out}}^2\Big|_{R_s}} = \frac{4kT\frac{\gamma}{\alpha} g_m \cdot \left(\frac{1}{2}\right)^2}{4kT R_s \cdot \left(\frac{g_m}{\omega_0 C_{gs} 2R_s}\right)^2} = \frac{\gamma}{\alpha}\frac{R_s}{g_m}\omega_0^2 C_{gs}^2 =$$

$$= \frac{\gamma}{\alpha}\frac{\omega_0}{\omega_T} \cdot \frac{1}{Q_L}$$ $\rightsquigarrow$ Q_L of Z_g in matching conditions

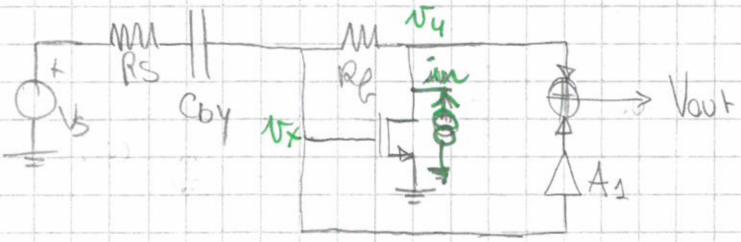
NF will be:

Reduction factor

$$NF\Big|_{DEGEN.\ CS} = 1 + \frac{\gamma}{\alpha}\frac{\omega_0}{\omega_T} \cdot \frac{1}{Q_L} + \underbrace{4\frac{R_L}{R_s}\left(\frac{\omega_0}{\omega_T}\right)^2}_{NF|_{R_L}}$$

Compare to common gate: $NF\Big|_{CG} = 1 + \frac{\gamma}{\alpha} + 4\frac{R_s}{R_L}$ We clearly see that the task to amplify given to RLC network instead of the FET reduces the noise that would be otherwise introduced by a CG or shunt-feedback

30) noise cancelling applied to shunt-feedback



We would like to cancel FET i_n by exploiting the the shunt.

v_x has same gain, different than v_y] → $V_{out}=0$ by A_1 compensation:

$$\frac{v_y}{i_n} = \frac{R_f + R_s}{2} = Z_{OUT} \qquad \frac{v_x}{i_n} = \frac{v_y}{i_n} \cdot \frac{R_s}{R_s+R_f} = \frac{\cancel{R_s+R_f}}{2} \cdot \frac{R_s}{\cancel{R_s+R_f}} > 0$$

$$\frac{V_{out}}{i_n} = A_1 v_x + v_y = A_1 \frac{R_s}{2} + \frac{R_s+R_f}{2} \rightarrow V_{OUT}=0 \text{ If } A_1 = -\left(1+\frac{R_f}{R_s}\right)$$

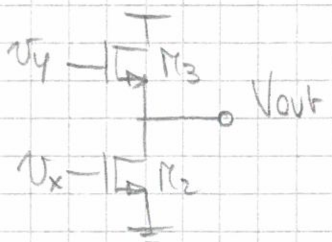
What about signal transfer V_{OUT}/V_s ?

$$\frac{V_{out}}{V_s} = \frac{v_y}{v_s} + A_1 \frac{v_x}{v_s} = \underbrace{\frac{1}{2}\left(1-\frac{R_f}{R_s}\right)}_{A_0 \text{ (matched)}} - \underbrace{\left(1+\frac{R_f}{R_s}\right)}_{A_1} \cdot \underbrace{\frac{1}{2}}_{v_x = \frac{1}{2} v_s \text{ (matched)}} = -\frac{R_f}{R_s}$$

$$\frac{V_{out}}{i_n} = 0 \quad \text{while} \quad \frac{V_{out}}{V_s} = -\frac{R_f}{R_s}$$] → Double signal gain, zero noise transfer (for FET's)

This is why we used v_x and v_y, we want to kill noise without damaging the signal transfer

A_1 + summer can be obtained by using two mosfets:



By using superposition principle:

$$\frac{V_{out}}{v_y} \simeq 1 \text{ (buffer)} \qquad \frac{V_{out}}{v_x} = -\frac{g_{m2}}{g_{m3}} \text{ (CS with active load)}$$

NF for noise cancelling topology:

We need to verify that M_2, M_3 do not introduce more noise

$$NF = 1 + \frac{\cancel{4KT} R_f}{\cancel{4KT} R_s \left(\frac{R_f}{R_s}\right)^2} + \frac{\cancel{4KT} \frac{\gamma}{\alpha} (g_{m2}+g_{m3}) \left(\frac{1}{g_{m3}}\right)^2}{\cancel{4KT} R_s \left(\frac{R_f}{R_s}\right)^2} + 0 \Big|_{M_1 \text{ current noise}}$$

$$\frac{\frac{\gamma}{\alpha}\left(\frac{g_{m2}}{g_{m3}} + 1\right)\frac{1}{g_{m3}}}{\frac{R_f^2}{R_s}} \rightarrow \frac{g_{m2}}{g_{m3}} = 1 + \frac{R_f}{R_s}$$

$R_f \gg R_s$

$$\frac{\frac{\gamma}{\alpha}\left(\frac{R_f}{R_s} + 1\right)\frac{1}{g_{m3}}}{R_f^2/R_s} \simeq \frac{\gamma}{\alpha} \cdot \frac{1}{R_f\, g_{m3}} \qquad \frac{R_s}{R_f} \rightarrow 0$$

$$NF\Big|_{R_f \gg R_s} \simeq 1 + \frac{\gamma}{\alpha}\frac{1}{R_f\, g_{m3}}$$

To have $NF\big|_{\text{NOISE CANC.}} < NF\big|_{\text{SHUNT-FEEDBACK}}$ we need that

$g_{m3} > \frac{1}{R_f} \rightarrow \frac{1}{g_{m3}} < R_f \Big] \rightarrow$ more current is drawn, more power is consumed, large area occupied

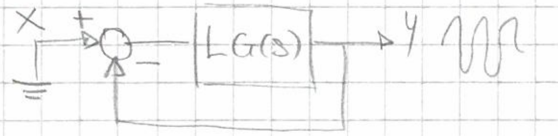
Moreover, gates of M_2 and M_3 have parasitic capacitances and we need to compensate those using inductors.

So, in order to have noise cancelling:

- $M_1 \rightarrow g_{m1} = 1/R_s$ input matching
- $R_f \gg R_s$ $\rightarrow$ to have higher gain
- $M_3 \rightarrow g_{m3} > 1/R_f$ to have low NF

31) Oscillators: feedback model + Barkhausen. R<0 model.

Amp stabilization methods, osc startup and effective gain

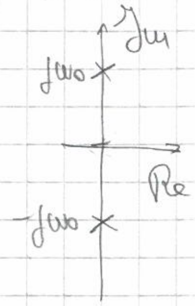


$$\frac{Y}{X} = \frac{LG(s)}{1 + LG(s)} \quad (1)$$

In order to oscillate $Y(j\omega_0) \neq 0$ with no input applied $X(j\omega_0) = 0$

So, from (1) we get the following

$$1 + LG(j\omega_0) = 0 \rightarrow LG(j\omega_0) = -1 \begin{cases} |LG(j\omega_0)| = 1 \\ \measuredangle LG(j\omega_0) = \pi = 180^\circ \end{cases}$$

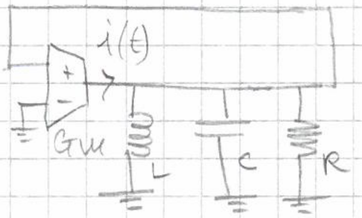


This is called the Barkhausen's criterion

It means that, in order to get proper oscillation, a negative feedback system should show $\pm j\omega_0$ solutions only

for a positive feedback sys $|LG(j\omega_0)| = 1 \quad \measuredangle LG(j\omega_0) = 0^\circ$

Consider a LC osc, where R models the energy dissipated in the tank:

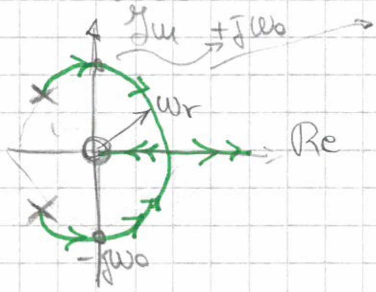


$$\mathcal{L}(s) = R \frac{s\frac{\omega_r}{Q}}{s^2 + s\frac{\omega_r}{Q} + \omega_r^2} \quad \text{Band pass } T(s)$$

where $\omega_r = \frac{1}{\sqrt{LC}}$ and $Q = \omega_r RC$

$$LG(s) = G_m R \frac{s\,\omega_r/Q}{s^2 + s\frac{\omega_r}{Q} + \omega_r^2}$$ ↳ we cut the loop at Gm input

Root locus:



→ Cut on the y axis is $\omega_0 = \omega_r$ $(s = \pm j\omega_0$

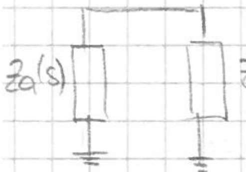
1) $\measuredangle LG(j\omega_0) = 0 \quad \measuredangle \left.\frac{s\,\omega_r/Q}{s^2 + s\frac{\omega_r}{Q} + \omega_r^2}\right|_{s = j\omega_0} = \frac{\pi}{2} - \arctan\frac{\omega_0\omega_r/Q}{\omega_r^2 - \omega_0^2} = 0$

Solution will be $\omega_r = \omega_0$

2) $|LG(j\omega_0)| = 1$ If $\omega_r = \omega_0$ $\quad G_m R \frac{\cancel{\omega_0\omega_r/Q}}{\sqrt{\underbrace{(\omega_r^2 - \omega_0^2)^2}_{0} + \cancel{\left(\frac{\omega_0\omega_r}{Q}\right)^2}}} = 1$

It becomes $G_m R$

Negative resistance mode

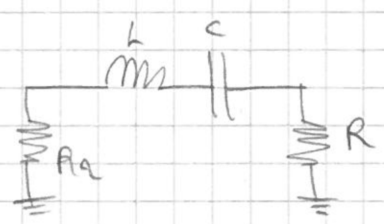


It's based on an energy approach:
balance between dissipated and active power:

$$\frac{1}{2}\frac{A_0^2}{R} = \frac{1}{2} G_m A_0^2 \longrightarrow G_m = \frac{1}{R}$$

Osc condition: $Z_a(j\omega_0) + Z(j\omega_0) = 0 \rightarrow Z(j\omega_0) = -Z_a(j\omega_0)$

$$\Re(Z_a(j\omega_0)) = -\Re(Z(j\omega_0))$$
$$\Im(Z_a(j\omega_0)) = -\Im(Z(j\omega_0))$$



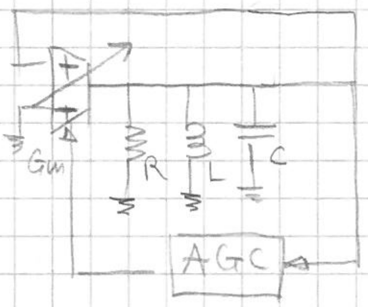
- L, C resonates $\omega_0 L + \frac{1}{\omega_0 C} = 0 \longrightarrow \omega_0 = \frac{1}{\sqrt{LC}}$
- Real part $R_a = -R$

Amplitude compensation

If $G_m \frac{A_0}{2} < \frac{A_0^2}{2R}$ oscillation fades out (active pwr < dissipated)

If $G_m \frac{A_0}{2} > \frac{A_0^2}{2R}$ oscillation diverges

We to have a sinusoid that is constant we must adjust the gain:



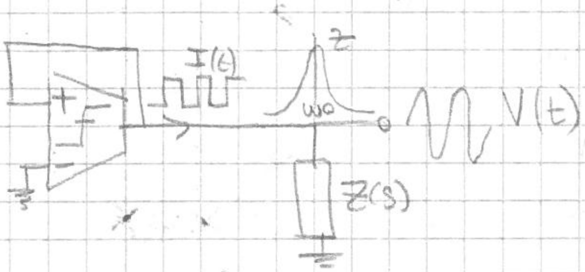
AGC = automatic gain control that reads output amplitude and automatically adjust the Gm to have
$G_m R = 1$

Hard limiting for amplitude stabilization:

Instead of a linear Gm we can think of a non linear:

We feed a sine input and get a square output. This becomes particularly useful considering that:

RLC tank → high Q → narrow band → only certain harmonics survive (ideally just the oscilation one):



Even though behaviour is non linear, if we just take into account only ω_0 harmonic, we can still use linear analysis tools.

$\overline{I_1}, \overline{V_1}$ = current / voltage of the 1st harmonic.

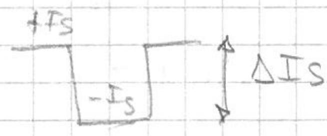
Apply the osc condition:

$\overline{I_1} Z(\omega_0) = \overline{V_1} \quad Z(\omega_0) = \frac{\overline{V_1}}{\overline{I_1}} = G_{mh}$ where G_{mh} takes the name of "harmonic transconductance".

Therefore $Z(j\omega_0) = \frac{1}{G_{mh}}$

Consider:

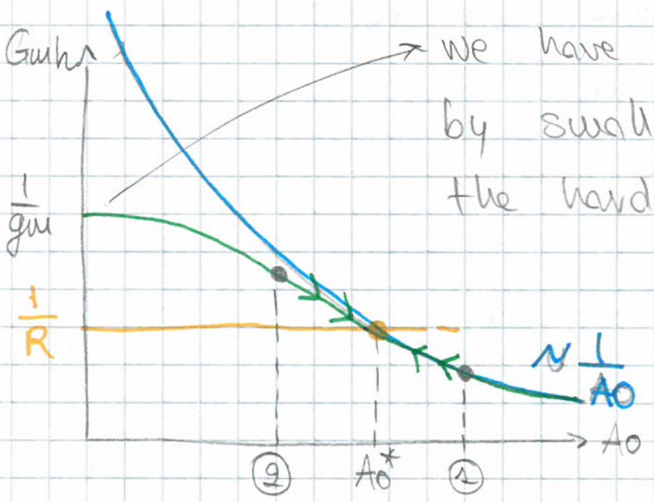
- hard limiting $I(V(t)) = I_S \cdot \text{sign}\{V(t)\}$
- $V(t) = A_0 \cos \omega_0 t \longrightarrow \overline{V_1} = A_0$
- Square current $\pm I_S$ with 50% Duty cycle therefore $\overline{I_1} = \frac{2}{\pi} \Delta I_S = \frac{4}{\pi} I_S$



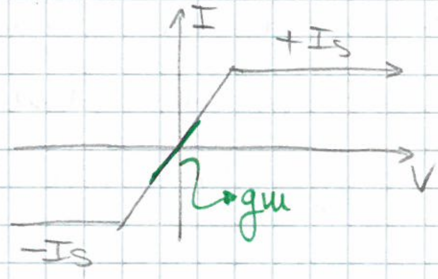
Now apply the Barkhausen condition for the 1st harmonic

$$LG_h(j\omega_0) = 1 \quad \begin{cases} G_{mh} R = 1 \longrightarrow G_{mh} = \frac{\overline{I_1}}{\overline{V_1}} = \frac{4}{\pi} I_S \cdot \frac{1}{A_0} \\ \omega_0 = \frac{1}{\sqrt{LC}} \end{cases}$$

So $\frac{4}{\pi} \frac{I_S}{A_0} \cdot R = 1 \longrightarrow$ OSC Amplitude $A_0 = \frac{4}{\pi} I_S R$

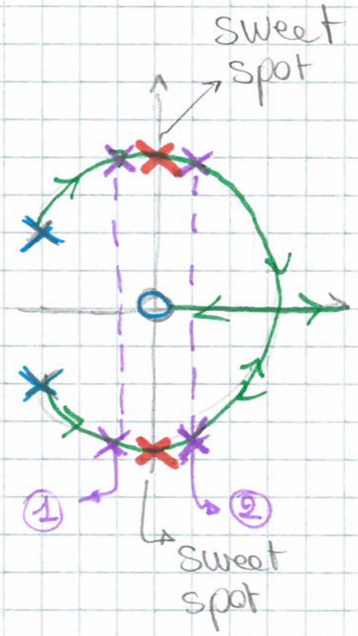


we have a limit imposed by small signal gm of the hard limiter



① $A_0 > A_0^*$: $G_{mh} R < 1$ poles in LHP $\Rightarrow A_0 \downarrow$ → fading

② $A_0 < A_0^*$: $G_{mh} R > 1$ poles in RHP $\Rightarrow A_0 \uparrow$ → diverging



We can see that hard limiting compensates oscillation amplitude when it changes.

But, to compensate A_0 we first need to build up one.
For this reason:

At startup → divergence → active power > dissipated power

$$\frac{1}{2} G_m A_0^2 > \frac{1}{2} \frac{A_0^2}{R} \rightarrow G_m R > 1$$

Typically: $\underline{G_m R > EG}$ where EG = excess gain ~ 2

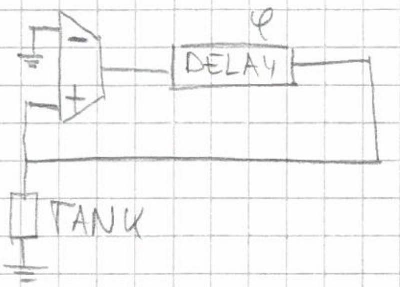
After startup, we then need to keep the oscillation up, so:

$\underline{LG_h(j\omega_0) = 1}$

These are the two important ingredients to be used when designing a real oscillator.

32) Frequency stability. Loop delay effect on OSC. Meaning of Q factor

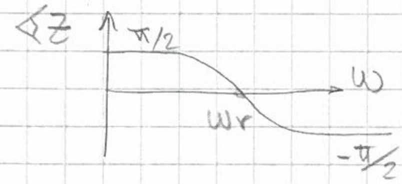
Goal: achieve a stable frequency that is robust with respect to phase delays induced in the loop:



OSC condition:

- $|LG(j\omega_0)| = 1$
- $\sphericalangle LG(j\omega_0) = 0$ but $LG(j\omega) = G_m Z(j\omega) e^{-j\varphi}$

loop gain has now a phase shift



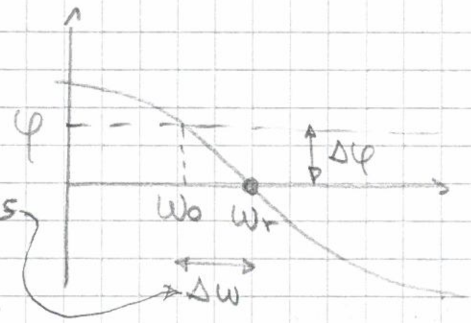
$$\sphericalangle LG(j\omega_0) = 0 \longrightarrow -\varphi + \sphericalangle Z(j\omega_0) = 0$$

$$+\frac{\pi}{2} - \operatorname{atan}\left\{\frac{\omega_0\omega_r/Q}{\omega_r^2 - \omega_0^2}\right\} = \varphi$$

Since $\frac{\pi}{2} - \operatorname{atan} x = \operatorname{atan}\frac{1}{x}$ we get

$$\operatorname{atan}\left\{\frac{\omega_r^2 - \omega_0^2}{\omega_0\omega_r/Q}\right\} = \varphi$$

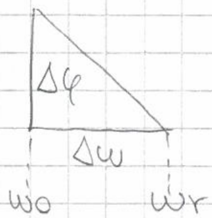
frequency shifts back from tank resonance ω_s



We can observe that $\omega_0 < \omega_r$ with a positive $\varphi > 0$

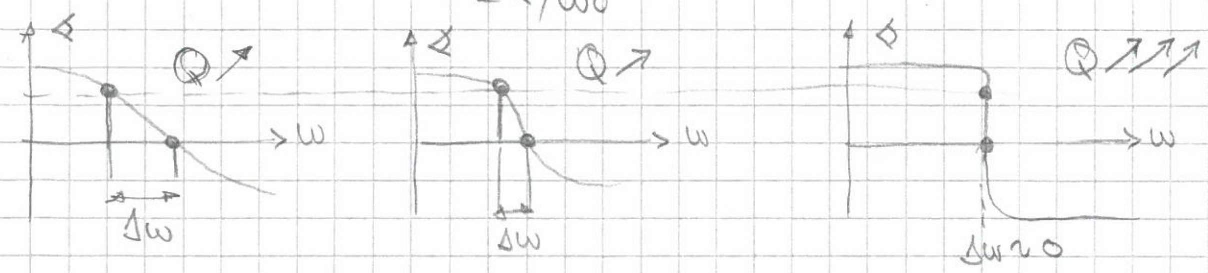
For ease of computing and understanding, we linear. approx. the zone near $\Delta\omega$ and $\Delta\varphi$:

$$\Delta\omega_0 \simeq \frac{\Delta\varphi}{\left.\frac{d\sphericalangle Z}{d\omega_0}\right|_{\omega_0=\omega_r}} \qquad \text{where } \sphericalangle Z = \operatorname{atan}\left\{\frac{\omega_r^2 - \omega_0^2}{\omega_0\omega_r/Q}\right\}$$



$$\Delta\omega_0 = \Delta\varphi \left.\frac{1}{\left[\frac{1}{1+\left(Q\cdot\frac{\omega_r^2-\omega_0^2}{\omega_0\omega_r}\right)^2}\right]\cdot\frac{Q}{\omega_r}\left[-\frac{2\omega_0^2+\omega_r^2-\omega_0^2}{\omega_0^2}\right]}\right|_{\omega_0=\omega_r}$$

We get $\Delta\omega_0 \simeq \frac{\Delta\varphi}{-2Q/\omega_0}$ → The larger Q, the lower $\Delta\omega$



For high Q, the rapid phase variation "desensitivizises" the oscillator to delays in the loop:

$$\Delta\omega_0 \simeq \frac{\Delta\varphi}{-\frac{2Q}{\omega_0}} \longrightarrow \frac{\Delta\omega_0}{\omega_0} = -\frac{\Delta\varphi}{2Q}$$

Higher Qs therefore stabilize frequency changes

Definition frequency stability $\triangleq \dfrac{\Delta\omega/\omega_0}{\Delta\varphi} = -\dfrac{1}{2Q}$

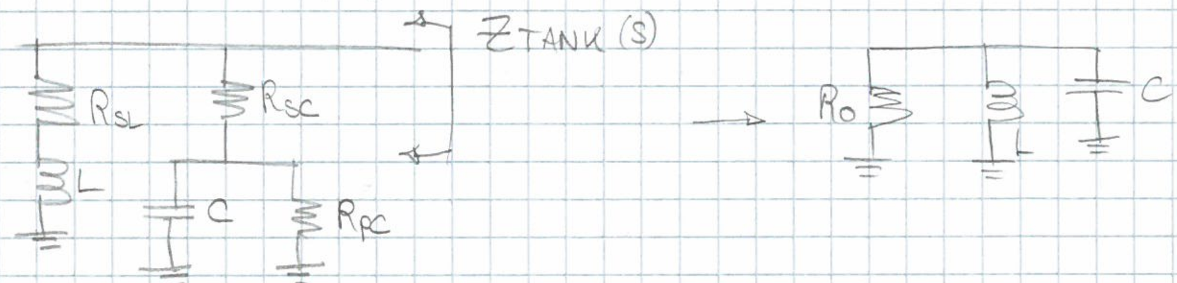
This result is valid for a basic LC oscillator, it changes with respect to topologies (e.g: ring oscillators don't have a Q factor).

Recall: Q represents the ratio between maximum energy stored in the tank and the energy dissipated per cycle (see previous oral answers to matched networks):

$Q = 2\pi \dfrac{E_r}{E_d} = \omega_0 RC = \dfrac{R}{\omega L}$ ⇝ for a // RLC reso

EX CURSUS

In a real tank, parasitic resistors are modeled like:



Assume small losses: $R_{SL} \ll \omega_0 L$ $\quad R_{SC} \ll \dfrac{1}{\omega_0 C}$ $\quad R_{PC} \gg \dfrac{1}{\omega_0 C}$

Then

$$Q_{SC} = \frac{1}{\omega_0 C R_{SC}} \qquad Q_{PC} = \omega_0 C R_{PC} \qquad Q_{SL} = \frac{\omega_0 L}{R_{SL}}$$

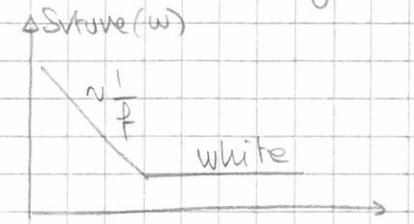
$$P_{SC} \simeq \frac{1}{2}(A_0 \omega_0 C)^2 R_{SC} \qquad P_{PC} \simeq \frac{1}{2}\frac{A_0^2}{R_{PC}} \qquad P_{SL} = \frac{1}{2}\left(\frac{A_0}{\omega_0 L}\right)^2 R_{SL}$$

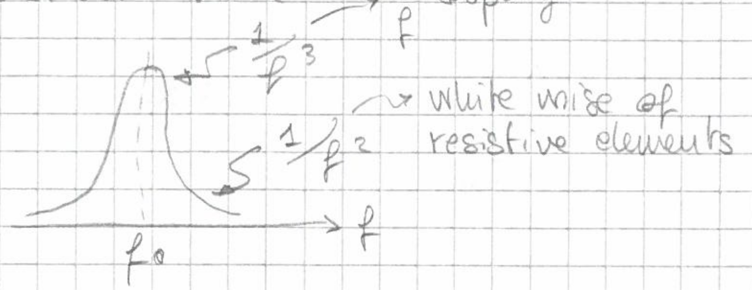
These three dissipated powers are equal to R_0 dissipated pwr:

$$P_{SL} + P_{SC} + P_{PC} = \frac{A_0^2}{2R_0} \longrightarrow \frac{1}{Q} = \frac{1}{Q_{SL}} + \frac{1}{Q_{SC}} + \frac{1}{Q_{PC}}$$

33) Phase noise in LC oscillators

Consider voltage noise on modulation Vtune $\rightarrow \frac{1}{f}$ coupling

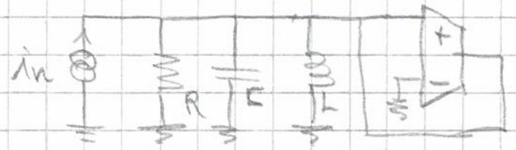




$$\mathcal{L}(\omega) \cong \frac{1}{2} S_\varphi(\omega) = \frac{1}{2} \frac{K_{VCO}^2}{\omega^2} \cdot S_{Vtune}(\omega)$$

We clearly see that: Voltage noise $\rightarrow$ FM noise $\rightarrow$ PM noise

Therefore output LO spectrum is anything but a δ



Tank will show a parasitic resistance modelled as $R \rightarrow S_{i\,noise}(f) = \frac{4KT}{R}$

We want to focus on a small range from osc frequency

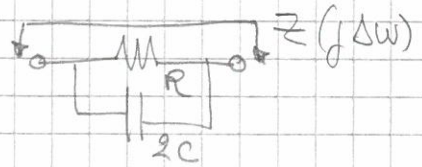
$\omega = \omega_r \pm \Delta\omega$, small offset from ω_r

$$Z(j\omega) = \frac{j\omega\omega_r/Q}{\omega_r^2 - \omega^2 + j\frac{\omega\omega_r}{Q}} \cdot R \underset{\omega = \omega_r \pm \Delta\omega}{=} \frac{R}{1 + j\frac{\Delta\omega}{\frac{\omega_r}{Q}} \cdot \frac{\Delta\omega \pm 2\omega_r}{\omega_r + \Delta\omega}}$$

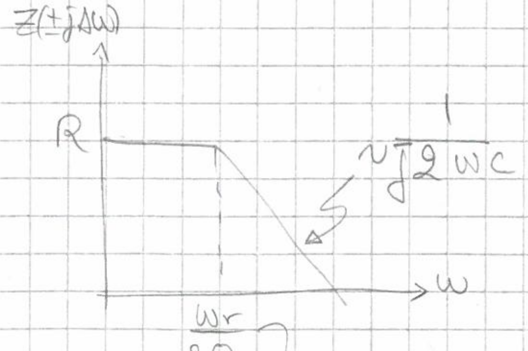
We want small $\Delta\omega$ analysis $\rightarrow \Delta\omega \ll \omega_r \rightarrow \frac{\Delta\omega \pm 2\omega_r}{\omega_r + \Delta\omega} \simeq 2$

$$Z(\pm j\Delta\omega) = \frac{R}{1 \pm j\frac{\Delta\omega}{\frac{\omega_r}{2Q}}} = \frac{R}{1 \pm j\,RC\,\Delta\omega} \rightarrow$$

$\hookrightarrow Q = \omega_r RC$



This is the equivalent impedance of the RLC tank when a small $\Delta\omega$ offset from ω_r is taken into account



$\rightarrow \frac{\omega_r}{2Q} = \frac{1}{2RC}$ based on $Q = \omega_r RC$ definition

We already know these results:

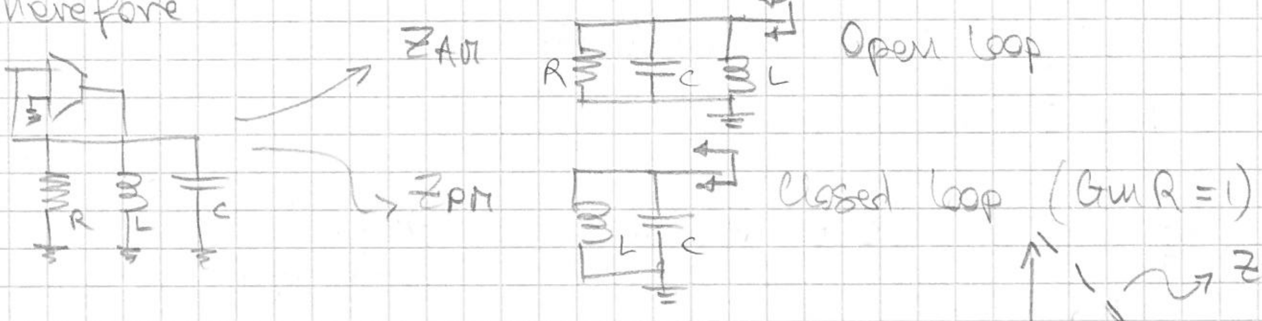
- Z_{TANK} @ resonance $= R$
- $-3dB$ BW is $\frac{\omega_r}{2Q} = \frac{1}{2RC}$ for a RLC TANK
- Slope is $-20dB/dec$

Now, consider $S_{in} = \frac{4KT}{R} \xrightarrow[\text{Theorem}]{\text{Rice's}}$ $\frac{2KT}{R}$ AM noise (in phase) / $\frac{2KT}{R}$ PM noise (in quadrature)

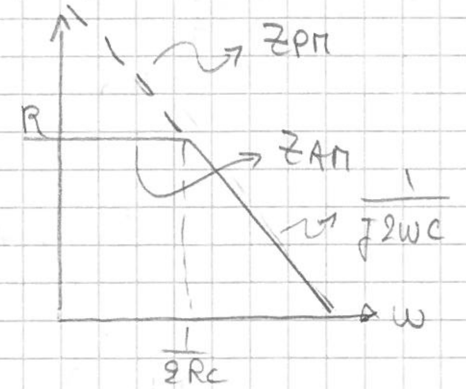
Oscillator system behaves differently:

- AM noise: transconductor hard limits its current, therefore there is no change in amplitude.
  Any amplitude modulation will therefore see an open loop of the osc $\rightarrow Z_{AM}(j\omega) = Z|_{OL}(j\omega) = Z_{RLC}(j\omega)$
- PM noise: transconductor can't distinguish between the osc phase and the noise induced on the phase.
  Loop is therefore closed and noise directly couples into it $\rightarrow$ R is compensated by loop Gm

Therefore



$$S_V(\omega) = \frac{2KT}{R}|Z_{AM}(\omega)|^2 + \frac{2KT}{R}|Z_{PM}(\omega)|^2$$



Where, for a small $\Delta\omega$ offset $\ll \frac{\omega_r}{2Q} = \frac{1}{2RC}$

$$|Z_{AM}(\omega)| = R^2 \quad |Z_{PM}(\omega)|^2 = \left|\frac{1}{j2\Delta\omega C}\right|^2 = \frac{1}{4\Delta\omega^2 C^2}$$

$$S_V(\omega) \simeq \frac{2KT}{R}\cdot R^2 + \frac{2KT}{R}\cdot\frac{1}{4\Delta\omega^2 C^2}\cdot\underbrace{\frac{R^2\omega_r^2}{R^2 C\omega_r^2}}_{Q^2} =$$

$$S_V(\omega) = 2KTR + \frac{1}{2}KTR\left(\frac{\omega_r}{Q}\right)^2\frac{1}{\Delta\omega^2} \underset{\uparrow}{\simeq} \frac{1}{2}KTR\left(\frac{\omega_r}{Q}\right)^2\frac{1}{\Delta\omega^2}$$

small Δω

34) Noise / Power Trade-off

Consider phase noise estimation:

small $\Delta\omega \rightarrow \frac{1}{\Delta\omega^2} \gg 1$

$$S_V(\Delta\omega) \simeq 2KTR + \frac{1}{2}KT \cdot \frac{1}{C^2} \cdot \frac{1}{R\Delta\omega^2} \simeq \frac{1}{2}KT\frac{1}{C^2}\frac{1}{R\Delta\omega^2}$$

To reduce S_V we could lower C value. This means that:

- $\omega_r = \frac{1}{\sqrt{LC}} \rightarrow \downarrow C \rightarrow \uparrow L \Rightarrow \uparrow R$ parasitic resistance of L
  $\rightarrow$ Q gets worse
- If $\uparrow R \rightarrow$ power dissipation increases $\rightarrow$ the small L values used in integrated RF circuits aren't that stable for higher temperatures
- higher temperatures increases thermal noise

Consider now the $\mathcal{L}(\Delta\omega)$:

$$\mathcal{L}(\Delta\omega) = \frac{S_V(\omega_r + \Delta\omega)}{P_{carrier}} = \frac{\frac{1}{2}KTR\left(\frac{\omega_r}{Q}\right)^2 \cdot \frac{F_a}{\Delta\omega^2}}{\frac{A_0^2}{2}} = \frac{\frac{1}{2}KT\left(\frac{\omega_r}{Q}\right)^2\frac{F_a}{\Delta\omega^2}}{\frac{A_0^2}{2R}}$$

Where $\frac{A_0^2}{2R} = P_{dissipated}$ and $P_{supply} = \eta P_{DISS} = P_{DC}$

Fa is an additional noise term factor that accounts for active elements noise

$$\mathcal{L}(\Delta\omega) = \frac{1}{2}\frac{KT}{\eta P_{DC}}\left(\frac{\omega_r}{Q}\right)^2\frac{F_a}{\Delta\omega^2}$$

Let us build a Figure of Merit that does not take into account osc frequency and dissipated power

Note $P_{DC,mW} = 10^3 P_{DC}[W]$

$$FoM|_{dB} = 10\log_{10}\left\{\frac{1}{\mathcal{L}(\Delta\omega)P_{DC,mW}} \cdot \left(\frac{\omega_{osc}}{\Delta\omega}\right)^2\right\}$$

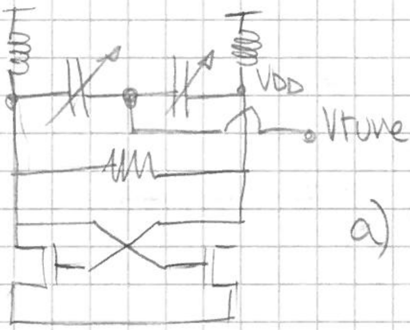
$$= 10\log_{10}\left\{10^{-3} \cdot \frac{2\eta}{KT}Q^2 \cdot \frac{1}{F_a}\right\}$$

IDEAL PSU $\quad$ noiseless active elements

Thermodynamic limit is for $\eta = 1 \quad F_a = 1$

$$FoM|_{dB} = 10\log_{10}\left\{\frac{2}{KT}Q^2\right\} - 30\,dB \longrightarrow \text{for } Q=10 \rightarrow FoM|_{min} = -197\,dB/Hz$$

35) VCO and FM noise on tuning voltage



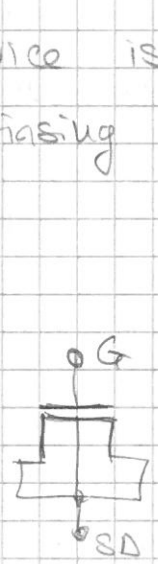
We need VCOs for channel selection.
Most common used device is varactors:

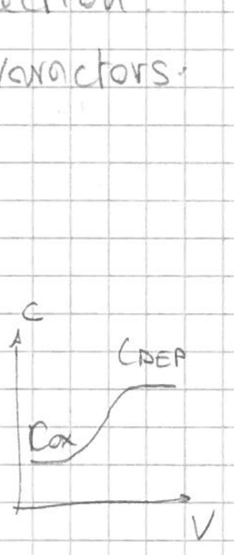
a) PN junction in reverse biasing

$$C = \frac{C_{d0}}{\left(1+\frac{V}{V_{j0}}\right)^m} \qquad m \sim \frac{1}{3} \div \frac{1}{2}$$

b) MOS junctions:

1) from inversion region to depletion
2) from accumulation " " depletion





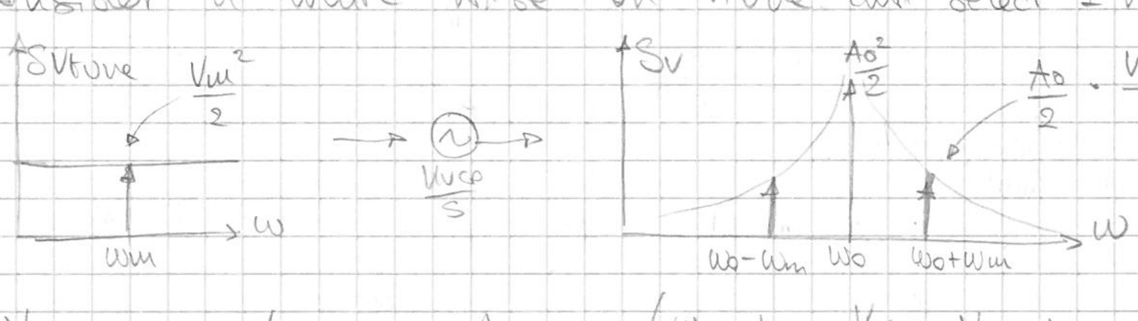
a) solution has DC current leakage but higher Q factors can be achieved

b) Very low DC leakage current

FM noise induced by V_{tune}

$$V_{tune} \rightarrow \boxed{\frac{K_{vco}}{s}} \rightarrow \sim \rightarrow \quad x_{out}(t) = A_0 \cos\left(\omega_{FR} t + \int_{-\infty}^{t} K_{vco} V_{tune}(t')\, dt'\right)$$

Integral translates to $\frac{1}{s}$ in Laplace domain.
Consider a white noise on V_{tune} and select 1 harmonic at ω_m:



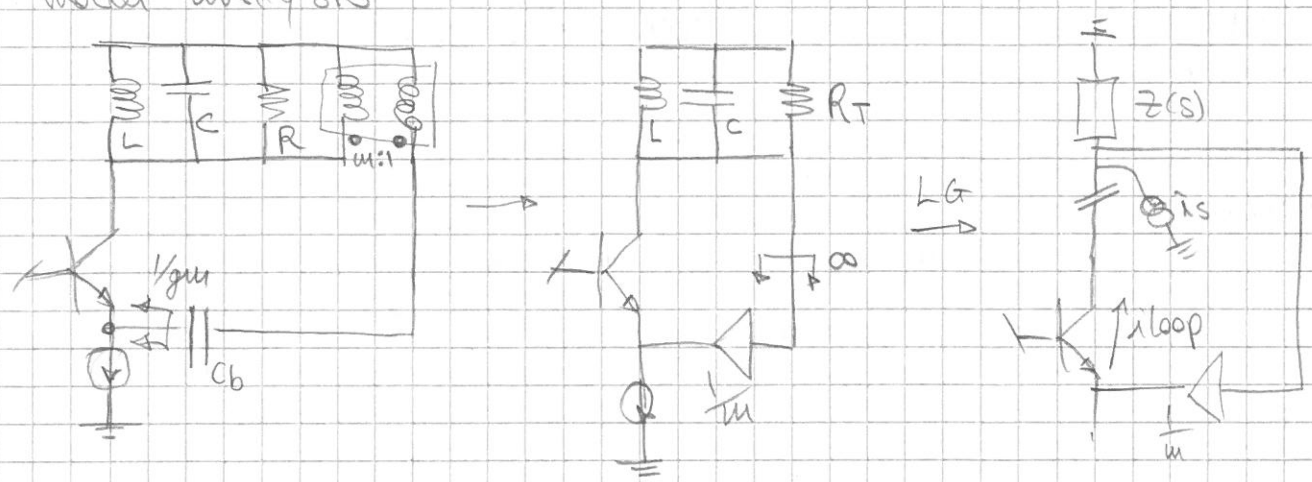
$$V_m \cos \omega_m t \longrightarrow A_0 \cos\left(\omega_{FR} t + \frac{K_{vco}}{\omega_m} V_m \sin \omega_m t\right)$$

$$\mathcal{L}(\omega_m) = \frac{S_\varphi}{2} = \frac{K_{vco}^2}{2\omega_m^2} \cdot \underbrace{S_{Vtune}(\omega_m)}_{\text{white noise spectrum}}$$

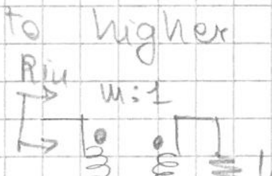
We see that integration of the white noise on V_{tune} generates a $\frac{1}{\omega^2}$ dependance on output phase spectrum

→ mod on V_{tune} generates FM mod on VCO

36) Single and differential LC osc topologies: LG and $R<0$ model analysis



We have a RLC tank + transconductor. Transformer is placed to higher the eq resistance ($\frac{1}{g_m}$ would lower Q factor)



$R_{in} = m^2 \cdot \frac{1}{g_m} \longrightarrow R_T = R // \frac{m^2}{g_m} = \frac{m^2 R}{m^2 + g_m R}$

We can easily derive $\underline{LG(s)}$

$H(s) = \dfrac{s\omega_r/Q}{s^2 + s\frac{\omega_r}{Q} + \omega_r^2}$ $\quad R_T = \dfrac{m^2 R}{m^2 + g_m R}$ $\quad Z(s) = H(s)\cdot R_T$

$$LG(s) = \frac{i_{loop}}{i_s} = Z(s) \cdot \frac{1}{m} \cdot \frac{1}{\frac{1}{g_m}} = \underline{\underline{Z(s)\frac{g_m}{m}}}$$

Same goes for negative resistance. Keep the same $Z(s)$

$i = -i_T = -g_m \frac{V}{m}$ $\quad \frac{V}{i} = -\frac{m}{g_m} = \underline{\underline{Z_a(s)}}$

- Barkhausen's condition

$LG(j\omega_0) = 1 \longrightarrow \angle = 0 \quad \omega_0 = \omega_r$

$\searrow |\cdot| = 1 \quad \frac{g_m}{m} R_T = 1$

- Negative resistance condition

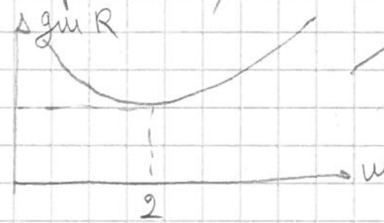
$Z(s) = -Z_a(s) \longrightarrow \Re e \Rightarrow R_T = \frac{m}{g_m} \rightarrow \frac{g_m}{m} R_T = 1$

$\searrow \Im m \Rightarrow L$ and C cancel out $\omega_0 = \omega_r = \frac{1}{\sqrt{LC}}$

Looking at $g_m \frac{R_T}{m} = 1$:

$$\frac{g_m}{m} \frac{m^2 R}{m^2 + g_m R} = 1 \;---\; \longrightarrow g_m R = \frac{m}{1 - 1/m}$$

We have an usual $g_m R$ (for LC osc) but we have a m dependancy

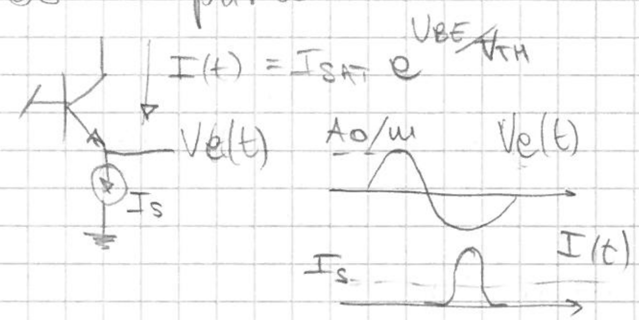


$\longrightarrow$ minimum is obtained for $m = 2$

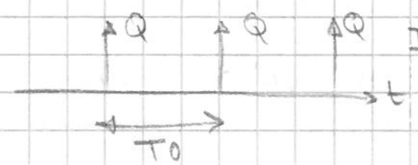
$$g_m R = \frac{2}{1 - \frac{1}{2}} = 4$$

- Small m $\rightarrow$ more voltage amplification but more losses
- Large m $\rightarrow$ large voltage attenuation but small losses

Osc amplitude :

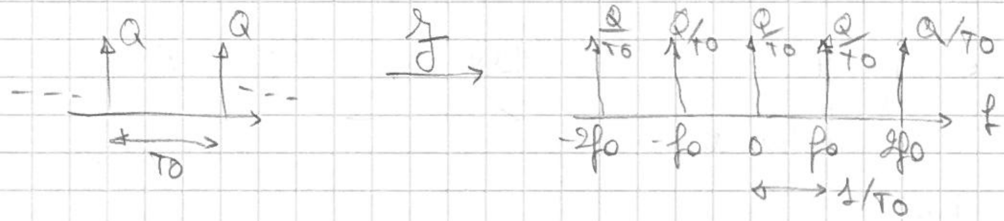


$I(t)$ increases exponentially only when $V_e(t)$ is at its most negative peak. We can approx to δ train



Consider :

- A_0 output peak voltage
- $\frac{A_0}{m}$ is the emitter voltage $\rightarrow$ $V_e(t)$ max voltage $\left/ \frac{A_0}{m} >> \frac{kT}{q} = V_{TH} \right)$



Remember that I_s is the bias current of the BJT

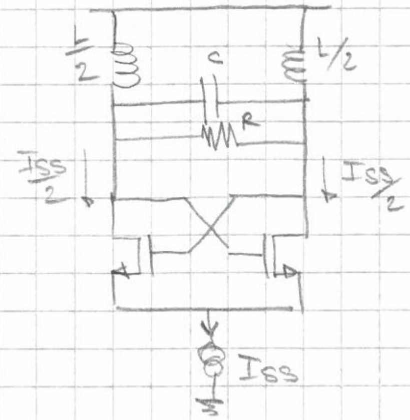
- We have two δ in $\pm f_0$ $\rightarrow$ $\overline{I_1} = 2Q/T$
- In 0 we have one δ $\rightarrow$ DC current $\rightarrow$ $I_s = Q/T_0$

So $g_{m_L} = \frac{\overline{I_1}}{\overline{V_1}} = \frac{2Q/T_0}{A_0/m} = \frac{2 I_s}{A_0/m}$

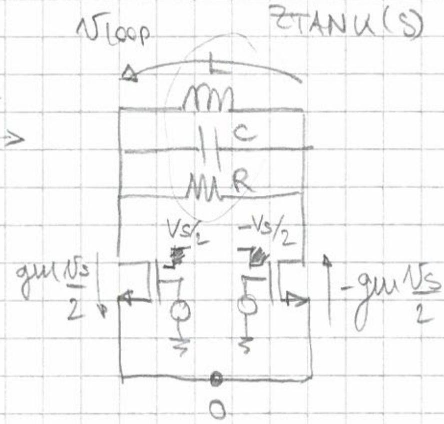
Oscillation condition for large signals:

$$g_{m_L} R = \frac{m}{1 - 1/m} \longrightarrow A_0 = 2 I_s R \left(1 - \frac{1}{m}\right) \overset{m=2}{=} I_s R$$

Differential oscillator



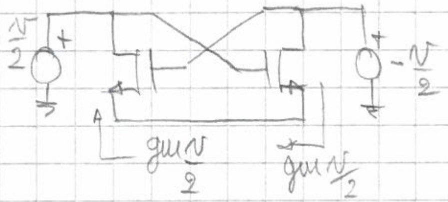
$\xrightarrow{LG}$



- LG(s) can easily be derived: $LG(s) = \frac{V_{loop}(s)}{V_s} = \frac{g_m}{2} Z_{TANK}(s)$

Oscillation condition $\begin{cases} \omega_0 = 1/\sqrt{LC} \\ \frac{g_m}{2} R = 1 \end{cases}$

- Negative resistance:



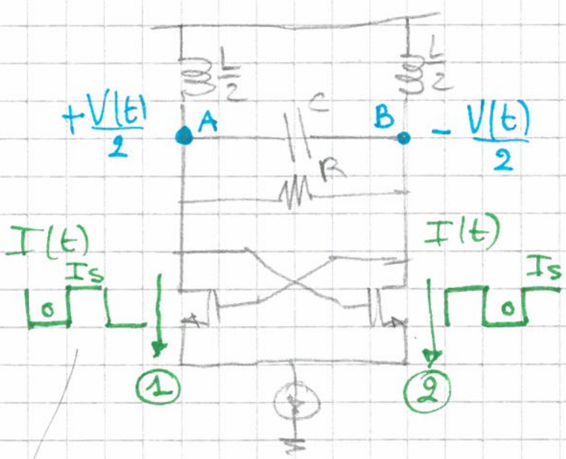
$\rightarrow Z_a(s) = \frac{\cancel{V}}{-g_m \frac{\cancel{V}}{2}} = -\frac{2}{g_m}$

$Z_a = -Z_{TANK} \rightarrow \frac{g_m}{2} R = 1$

$\omega_C = \frac{1}{\sqrt{LC}}$

Output amplitude:

Suppose abrupt switching of the FETs because of the large amplitude swing $(A_0 \gg \sqrt{2}\, V_{ov})$



① and ② are in anti-phase

$|G_{mh}| R = 1 \qquad G_{mh} = \frac{I_1}{V_1} = \frac{\frac{2}{\pi} I_{ss}}{A_0}$

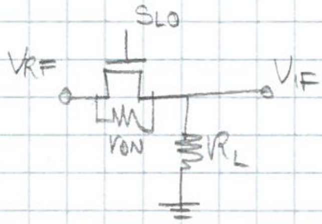
$A_0 = V_A - V_B =$ differential voltage amplitude

Therefore $A_0 = \frac{2}{\pi} I_{ss} \cdot R$

$\rightarrow \text{Harmonic}^{(1)} = \frac{2}{\pi} \Delta I_s = \frac{2}{\pi}(I_{ss} - 0) = \frac{2}{\pi} I_{ss}$

38) RTN passive mixers: conversion gain, noise

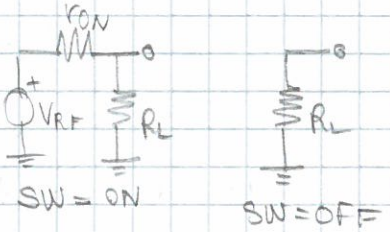
Consider



- $V_{RF}(t) = A\cos \omega_{RF} t$
- $S_{LO}(t) = \left[\frac{1}{2} + \frac{2}{\pi}\cos(\omega_{LO}t) + o.t\right]$

$$V_{IF} \overset{\sim}{=} \underbrace{\frac{R_L}{R_L + r_{ON}}}_{} S_{LO}(t)\cdot V_{RF}(t)$$

→ assume r_{ON} indipendent from biasing over time



$V_{IF}(t)$ = Linear, Time, Variant equation (if r_{ON} fixed):

$$V_{IF}(t) = \frac{R_L}{R_L + r_{ON}}\left[\frac{1}{2} + \frac{2}{\pi}\cos(\omega_{LO}t) + o.t\right]\cdot V_{RF}(t) =$$

→ Wanted freq. conversion

→ RF-to-IF feedthrough

$$V_{IF}(t) = \frac{R_L}{R_L + r_{ON}}\left[\underbrace{\frac{A}{2}\cos\omega_{RF}(t)}_{\text{RF-to-IF feedthrough}} + \underbrace{\frac{1}{2}A\cdot\frac{2}{\pi}\cos(\omega_{LO}-\omega_{RF})}_{\text{Wanted IF signal}} + \frac{1}{2}A\frac{2}{\pi}\cos(\omega_{LO}+\omega_{RF})t + o.t\right]$$

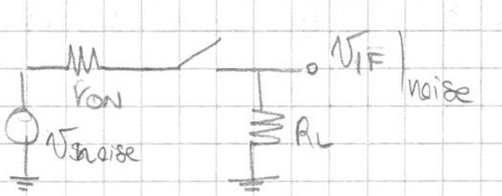
Conversion voltage gain $A_V \triangleq \frac{V_{IF}(\omega_{IF})}{V_{RF}(\omega_{RF})} = \left(\frac{R_L}{R_L + r_{ON}}\right)\frac{\frac{A}{\pi}}{A} = \frac{1}{\pi}\frac{R_L}{R_L + r_{ON}}$

$\rightarrow \omega_{LO} - \omega_{RF}$

$$\max A_V\Big|_{R_L \gg r_{ON}} = \frac{1}{\pi} \sim -10\,dB$$

Noise analysis for RTN passive mixer:

- MOSFET noise $\quad PSD\Big|_{r_{ON}} = 4KT\,r_{ON}$

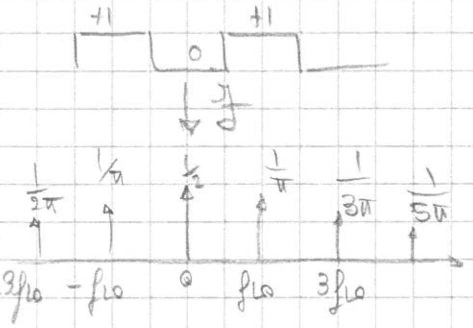


FET is in triode region:

$$v_{IF_n}(t) = v_n \cdot S_{LO}(t) \cdot \frac{R_L}{R_L + r_{ON}} \quad \rightsquigarrow \text{use PSDs + Fourier analysis}$$

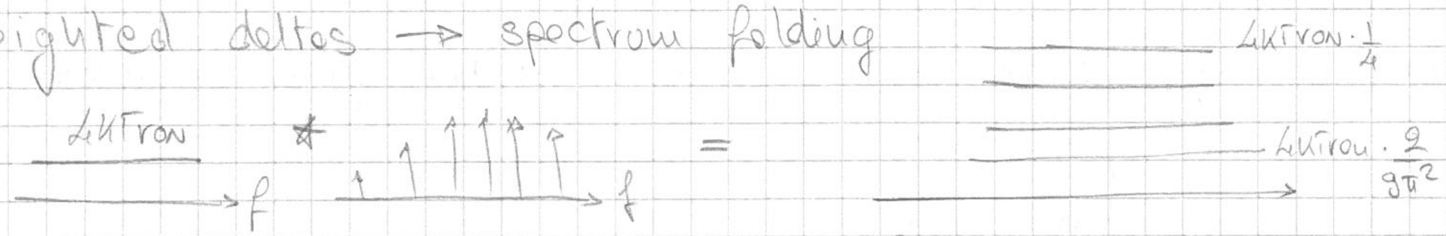
$$\mathcal{F}$$

$$PSD_{v_{IF}}(f) = PSD_{v_n}(f) * |S_{LO}(f)|^2 \left(\frac{R_L}{R_L + r_{ON}}\right)$$



$$PSD_{v_{IF}}(f) = 4KT r_{ON} * \sum_{K=-\infty}^{\infty} |c_K|^2 \delta(f - K f_{LO}) \left(\frac{R_L}{R_L + r_{ON}}\right)$$

In other words, convolution of white noise with infinite, weighted deltas → spectrum folding



But:

$$\sum_{K=-\infty}^{\infty} |c_K|^2 = \text{power of } S_{LO}(t) = \int_{-\infty}^{+\infty} |S_{LO}^2(f)|\,df \overset{\text{Parseval}}{=} \frac{1}{T_0}\int_0^{T_0} |S_{LO}^2(t)|\,dt$$

$$\frac{1}{T_0}\int_0^{T_0} |S_{LO}^2(t)|\,dt = \frac{1}{T_0}\int_0^{T_0/2} 1\,dt = \frac{1}{2}, \quad \text{so:}$$

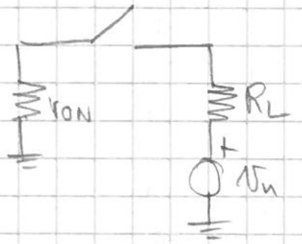
$$PSD_{v_{IF}}\Big|_{MOS} = 4KT r_{ON} \cdot \frac{1}{2} \left(\frac{R_L}{R_L + r_{ON}}\right)^2$$

Intuitive result: v_n is transferred to v_{IF} for only half of the period with $\left(\frac{R_L}{R_L + r_{ON}}\right)$ gain.

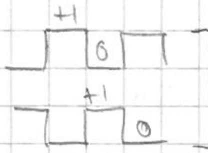
- R_L noise $\rightarrow$ $PSD = 4KTR_L$

Following the same procedure for MOSFET noise:

$$v_{IFn}(t) = v_n(t)\, S_{LO}(t)\, \frac{r_{ON}}{r_{ON}+R_L} + v_n(t)\, \overline{S_{LO}(t)}$$



Where $S_{LO}(t) =$ and $\overline{S_{LO}(t)} =$



$\rightarrow$ Necessary because we have two transfer functions for each half period

Same procedure + spectrum folding will lead to:

$$PSD_{v_{IF}}(f) = \underbrace{4KTR_L \cdot \frac{1}{2}\left(\frac{r_{ON}}{r_{ON}+R_L}\right)}_{\text{when SW = closed}} + \underbrace{4KTR_L \cdot \frac{1}{2}}_{\text{when SW = opened}}$$

Note: output noise varies over $\frac{1}{2}$ of the period $\rightarrow$ we say it is cyclo-stationary. Moreover, the narrowband filter at the output stage will average this noise over time

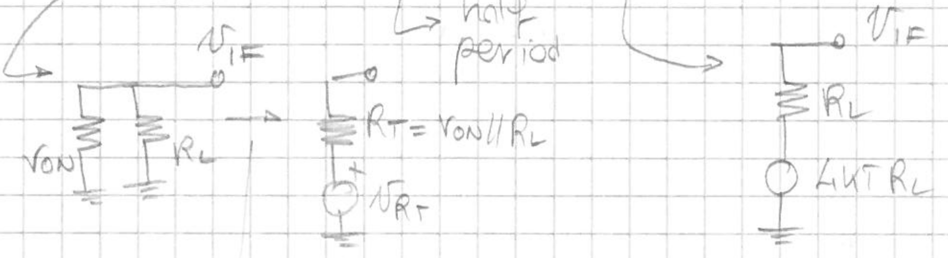
- FET r_{ON} + R_L noise:

$$PSD_{v_{IF}}\Big|_{TOT} = 2KTr_{on}\left(\frac{R_L}{R_L+r_{on}}\right)^2 + 2KTR_L\left(\frac{r_{ON}}{r_{ON}+R_L}\right)^2 + 2KTR_L$$

$$= 4KT(r_{ON} /\!/ R_L)\cdot\frac{1}{2} + 4KTR_L\cdot\frac{1}{2}$$

$\frac{1}{2}$ $\rightarrow$ half period

$\frac{1}{2}$ $\rightarrow$ half period



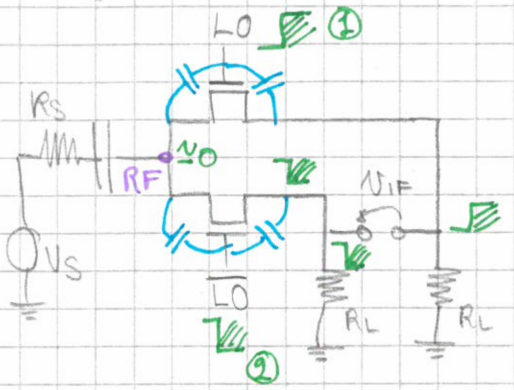
$\rightarrow$ Nyquist theorem

Note: see answer 40 for the justification of the use of a narrowband LPF to average $PSD|_{v_{IF}}$

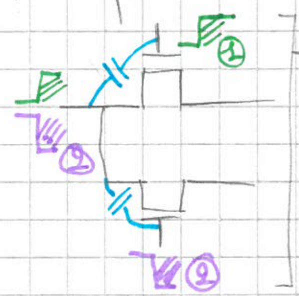
39) Single-balanced and double-balanced topologies, port-to-port isolation

- Single balanced

$$A_V = \frac{2}{\pi} \frac{R_L}{R_L + r_{on}}$$ double with respect to the RTN mixer



The doubled LO at RF port propagates its steps (①, ②) to the RF port through the parasitic capacitors:

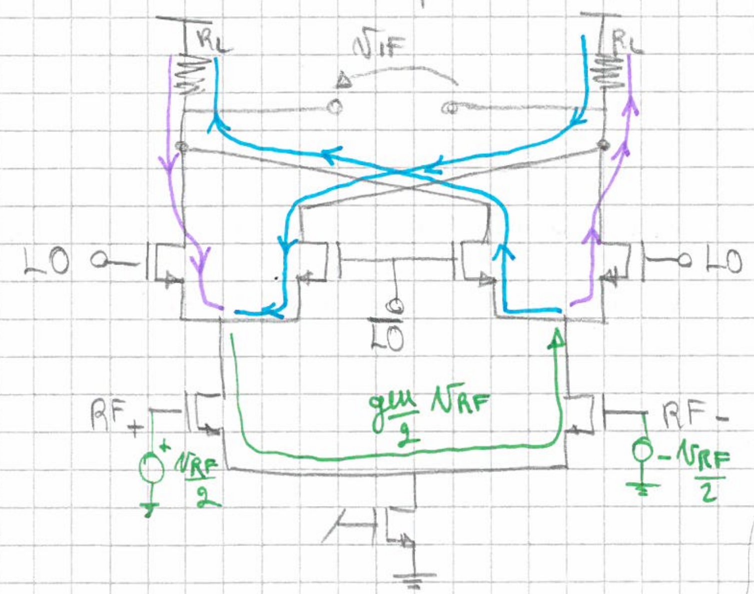


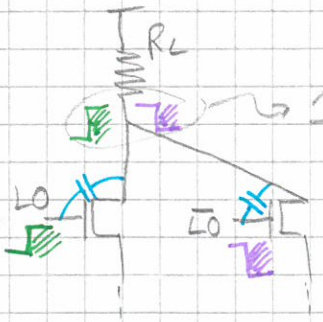
→ RF does not see any LO leakage

If LO duty cycle is 50%:

- zero RF-to-IF (double mixer has no DC current from RF to IF)
- zero LO-to-RF
- As we can see on v_{IF} node, LO will leak through: non-zero LO-to-IF

We can solve last point by using a double balanced mixer:



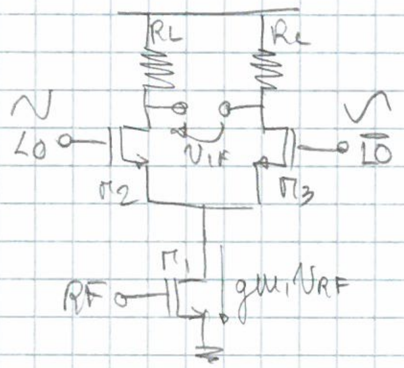


→ no LO-to-IF

Gain is basically unchanged but LO-to-IF is solved

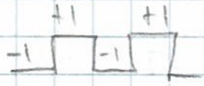
$$v_{IF} = \frac{g_{m_1}}{2} \cdot 2\, v_{RF} R_L\, X_{LO}(t) \longrightarrow A_V = \frac{2}{\pi} g_{m_1} R_L$$

40) Active CMOS mixers: conversion gain, noise, port-to-port isolation



Linear Time Variant model:

$$V_{IF}(t) = gm_1 \, V_{RF}(t) \, X_{LO}(t) \, R_L$$



Hypothesis:

- 50% duty cycle
- M_1 is in saturation
- Abrupt switching of M_2, M_3

$$A_V = \frac{V_{IF}(\omega_{IF})}{V_{RF}} \quad \text{where}$$

$$V_{RF}(t) = \underline{A} \cos \omega_{RF} t$$

$$V_{IF}(t) = gm_1 R_L A \cos \omega_{RF} t \left(\frac{4}{\pi} \cos \omega_{LO} t + \frac{4}{3\pi} \cos 3\omega_{LO} t + o.t \right)$$

$$= gm_1 R_L A \cdot \frac{2}{\pi} \cos(\omega_{RF} - \omega_{LO}) t + o.t \rightarrow \begin{cases} \omega_{RF} + \omega_{LO} \\ 3\omega_{LO} - \omega_{RF} \\ 3\omega_{LO} + \omega_{RF} \end{cases}$$

$$A_V = \frac{2}{\pi} gm_1 R_L \frac{\cancel{A}}{\cancel{A}} = \frac{2}{\pi} gm_1 R_L$$

Zero RF-to-IF if duty cycle is at 50%

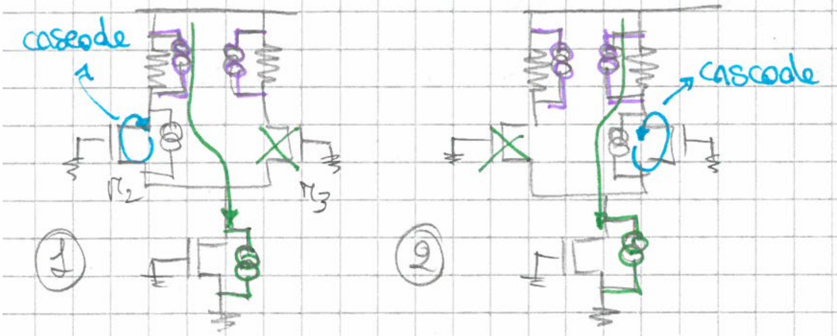
Zero LO-to-RF because of the $\overline{LO}$ and LO net balance on RF

non-zero LO-to-IF (we should use a double balanced mixer)

Note: the presence of o.t. (other terms) needs to be removed from the output by the use of a lowpass filter.
This output filter (narrowband) becomes especially important when PSD is computed and result will be expressed in an averaged noise of the mixer cyclo-stationary noise (averaged because of the LPF action

Noise in active single balanced mixer

- Unbalanced condition → abrupt switching of M_2, M_3



We can easily see that R_L noise is always there

$$PSD_{v_{IF}}\Big|_{R_L} = 2 \cdot 4KTR_L$$

↳ double load → double noise power

Scenario ①:

- M_3 is fully off
- M_2 is cascoded → noise current does not reach the output
- M_1 current noise is steered to M_2 and reaches the output

Scenario ②: same as ① but process is mirrored

Therefore: $PSD\Big|_{v_{IF}, M_2, M_3} = 0$ while for M_1

$$PSD\Big|_{v_{IF}, M_1} = R_L^2 \cdot 4KT\frac{\gamma}{\alpha} g_{m_1} \cdot \sum_{k=-\infty}^{+\infty} |c_k|^2$$ ↶ Same as previous answers

$$\sum_{k=-\infty}^{+\infty} |c_k|^2 = X_{LO} \text{ power} = \int_{-\infty}^{+\infty} |X_{LO}(f)|^2 df \underset{\uparrow}{=} \frac{1}{T_0}\int_0^{T_0} |x_{LO}(t)|^2 dt$$

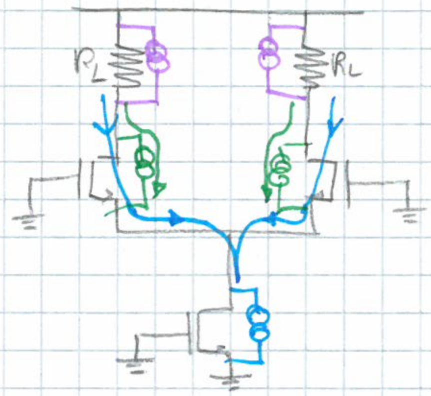
Parseval

$$\frac{1}{T_0}\int_0^{T_0} |x_{LO}(t)|^2 dt = \frac{1}{T_0}\left[\int_0^{T/2} (1)^2 dt + \int_0^{T/2} (-1)^2 dt\right] = 1$$

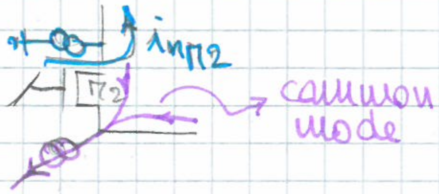
Intuitively: it does not matter whether M_1 noise current is steered left or right → M_1 noise current always reaches the output, therefore

$$PSD^{UNBAL} = 8KTR_L + 4KT\frac{\gamma}{\alpha} g_{m_1} R_L^2$$

- Balanced condition → M_2 and M_3 are both ON



- R_L noise reaches the output
- M_1 offers a common mode current → does not reach the output
- M_2, M_3 fully contribute to output current (can be easily derived by splitting noise in two contributions)

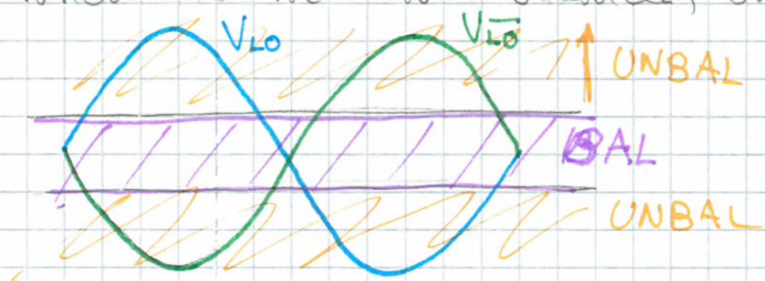


Therefore we can conclude that

$$PSD^{BAL} = \underbrace{8KTR_L}_{PSD|_{R_L}} + \underbrace{8KT\frac{\gamma}{\alpha} g_{m2} R_L^2}_{\text{contribution of } M_2 + M_3}$$

$\hookrightarrow g_{m2} = g_{m3}$

When do we have balanced, unbalanced scenarios?



Hypothesis:

- When M_2, M_3 are fully switching $PSD \simeq PSD|_{UNBAL}$
- Lowpass filtering on mixer's out will lead to an averaging of the cyclostationary noise

This way, we can define a duty cycle $D = t_w / T_0$

where t_w = time where circuit is balanced T_0 = wave period

Averaged out PSD will then be:

$$PSD|_{AVERAGE} = PSD|_{UNBAL}\left(1 - \frac{2t_w}{T_0}\right) + PSD|_{BAL} \cdot \frac{2t_w}{T_0} \; (*)$$

(*): in a period there are $2t_w$ in which PSD is balanced